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PRIMITIVITY IN MEREOLOGY. I

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Introduction This dissertation* deals with the Mereology, the formal system which Leśniewski constructed as a foundation for mathematics. The first chapter is introductory in nature; it sets forth some of the basic theorems of Mereology and it also offers a brief background of Ontology, Leśniewski's calculus of names. In this chapter the first of two important logical concepts, that of cardinality, is introduced.

The second important concept, that of primitivity, initially appears in the second chapter. The more familiar terms of Mereology are classified as primitive and non-primitive. The non-primitive terms are further classified under three headings. Co-primitive terms are those pairs of non-primitive terms which may be jointly used to define a primitive term. Independent terms are pairs of non-primitive terms; neither term of the pair is definable in terms of the other. Finally, dependent terms are those pairs of non-primitive terms which have one term definable by another, but not vice-versa. Much of the work with non-primitive terms is done with models, so many are displayed. Most of the terms in the second chapter are binary, but some ternary terms also appear. The third chapter deals with the possible elementary ternary relations on three individuals. After noting that a number of these relations are contradictory, we investigate the primitivity of the remainder. In Chapter IV we blend the notions of number and primitivity to yield a sequence of primitive terms, dependent by their definitions upon the cardinality of the name involved. These terms have the interesting property that the n-th term is primitive if and only if we assume the existence of at least $2^n - 1$ objects.

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CHAPTER I: THE PRELIMINARIES

1 *Ontological preliminaries* Leśniewski's mereology has as its logical basis two formal systems, ontology and protothetic. Ontology, which is discussed further in this chapter, is sometimes called the calculus of names. Protothetic is propositional calculus expanded to include quantification over propositions and proposition forming functors. We will begin this chapter with some of the basic definitions and theorems and then we will present, without proof, some of the results of Canty in [1]. These results will yield theorems interpretable as the Peano Postulates and hence will give us the natural numbers for later use.

We slightly expand ontology to include an axiom of infinity so that induction is possible. Using this axiom we can then formulate and prove analogs of the Peano Postulates, so important to the main body of work in the last chapter.

Ontology, without the axiom of infinity, has the following rather intuitive single axiom, due to Leśniewski

01 $[Aa] \therefore A \,\varepsilon\, a \;.\!\equiv\!: [\exists B] . B \,\varepsilon\, a : [CD] : C \,\varepsilon\, A . D \,\varepsilon\, A \;.\!\supset\!. C \,\varepsilon\, D : [C] :$
$C \,\varepsilon\, A \;.\!\supset\!. C \,\varepsilon\, a$

While there is no rule specifically stating how we should use capital or small letters, in general, we will use capital letters for individuals and small letters for general names, unless context dictates otherwise. We will write almost all functors in the informal way, e.g., $A \,\varepsilon\, a$ rather than $\varepsilon\{A, a\}$, both for reasons of economy and clarity.

We shall next introduce a large body of definitions and theorems in ontology. This will be done without proof since many of the properties exhibited are merely Boolean algebraic properties on general names and are either obvious or easily proved, *cf.* [1]. Very few of these properties are used in the mereological portions of the dissertation and, where they are so cited, we will merely write **ON** in the proof line without specifying which theorem or definition applies.

02 $[Aa] : A \,\varepsilon\, a \;.\!\supset\!. A \,\varepsilon\, A$
D01 $[a] : ! \{a\} \;.\!\equiv\!. [\exists A] . A \,\varepsilon\, a$
D02 $[a] \therefore \rightarrow \{a\} \;.\!\equiv\!: [AB] : A \,\varepsilon\, a . B \,\varepsilon\, a \;.\!\supset\!. A \,\varepsilon\, B$
03 $[A] : A \,\varepsilon\, A \;.\!\equiv\!. ! \{A\} . \rightarrow \{A\}$
D03 $[AB] : A = B \;.\!\equiv\!. A \,\varepsilon\, B . B \,\varepsilon\, A$
04 $[AB] \therefore A = B \;.\!\equiv\!: A \,\varepsilon\, A . B \,\varepsilon\, B : [\sigma] : \sigma\{A\} \;.\!\equiv\!. \sigma\{B\}$
D04 $[AB] : A \neq B \;.\!\equiv\!. A \,\varepsilon\, A . B \,\varepsilon\, B . \sim (A = B)$
D05 $[A] : A \,\varepsilon\, \vee \;.\!\equiv\!. A \,\varepsilon\, A$
D06 $[A] : A \,\varepsilon\, \wedge \;.\!\equiv\!. A \,\varepsilon\, A . \sim (A \,\varepsilon\, A)$
D07 $[ab] \therefore a \subset b \;.\!\equiv\!: [A] : A \,\varepsilon\, a \;.\!\supset\!. A \,\varepsilon\, b$
D08 $[ab] \therefore a \circ b \;.\!\equiv\!: [A] : A \,\varepsilon\, a \;.\!\equiv\!. A \,\varepsilon\, b$
05 $[ab] \therefore a \circ b \;.\!\equiv\!: [\sigma] : \sigma\{a\} \;.\!\equiv\!. \sigma\{b\}$
D09 $[ab] : a \Subset b \;.\!\equiv\!. a \subset b . \sim (b \subset a)$
D010 $[Aab] : A \,\varepsilon\, a \cap b \;.\!\equiv\!. A \,\varepsilon\, a . A \,\varepsilon\, b$
D011 $[Aab] \therefore A \,\varepsilon\, a \cup b \;.\!\equiv\!: A \,\varepsilon\, A : A \,\varepsilon\, a . \vee . A \,\varepsilon\, b$

D012 $[ab\sigma] \therefore a \overset{\sigma}{\infty} b \;.\!\equiv\!:\; [A] : A\,\varepsilon\, a \;.\!\supset\!.\; [\exists B] . B\,\varepsilon\, b . B\,\varepsilon\,\sigma(A) : [B] : B\,\varepsilon\, b \;.\!\supset\!.\; [\exists A] .$
$A\,\varepsilon\, a . B\,\varepsilon\,\sigma(A) : [ABC] : A\,\varepsilon\, a . B\,\varepsilon\, b . C\,\varepsilon\, a . B\,\varepsilon\,\sigma(A) . B\,\varepsilon\,\sigma(C) \;.\!\supset\!.\; A = C :$
$[ABC] : A\,\varepsilon\, a . B\,\varepsilon\, b . C\,\varepsilon\, b . B\,\varepsilon\,\sigma(A) . C\,\varepsilon\,\sigma(A) \;.\!\supset\!.\; B = C$

D013 $[ab] : a \infty b \;.\!\equiv\!.\; [\exists\sigma] . a \overset{\sigma}{\infty} b$

D014 $[Aab] : A\,\varepsilon\, a - b \;.\!\equiv\!.\; A\,\varepsilon\, a . \sim(A\,\varepsilon\, b)$

D015 $[\varphi] : !\,(\varphi) \;.\!\equiv\!.\; [\exists a] . \varphi\{a\}$

Note that φ is a proposition forming functor of one name argument and ∞ is inferentially equivalent, in the field of ontology, to ordinary equinumerosity.

D016 $[\varphi\psi] \therefore \varphi \subset \psi \;.\!\equiv\!:\; [a] : \varphi\{a\} \;.\!\supset\!.\; \psi\{a\}$

D017 $[\varphi\psi] \therefore \varphi \circ \psi \;.\!\equiv\!:\; [a] : \varphi\{a\} \;.\!\equiv\!.\; \psi\{a\}$

06 $[\varphi\psi] \therefore \varphi \circ \psi \;.\!\equiv\!:\; [\theta] : \theta(\varphi) \;.\!\equiv\!.\; \theta(\psi)$

07 $[a] . a \infty a$

08 $[ab] : a \infty b \;.\!\equiv\!.\; b \infty a$

09 $[abc] : a \infty b . b \infty c \;.\!\supset\!.\; a \infty c$

010 $[ab] \therefore a \infty b \;.\!\supset\!:\; !\,\{a\} \;.\!\equiv\!.\; !\,\{b\}$

011 $[AB] : A\,\varepsilon\, A . B\,\varepsilon\, B \;.\!\supset\!.\; A \infty B$

012 $[a] : a \infty \wedge \;.\!\equiv\!.\; a \circ \wedge$

013 $[ABab] : A\,\varepsilon\, A . B\,\varepsilon\, B . a \cup A \infty b \cup B . A \cap a \circ \wedge . B \cap b \circ \wedge \;.\!\supset\!.\; a \infty b$

D018 $[ab] : \infty \langle a \rangle \{b\} \;.\!\equiv\!.\; a \infty b$

014 $[a] . \infty \langle a \rangle \{a\}$

015 $[AB] : A\,\varepsilon\, A . B\,\varepsilon\, B \;.\!\supset\!.\; \infty \langle A \rangle \{B\}$

016 $[ab] : a \infty b \;.\!\equiv\!.\; \infty \langle a \rangle \circ \infty \langle b \rangle$

The following definitions, *D019, D020,* and *D021* are due to Canty and may be found in [1]. N says a predicate is numerical, i.e., it is true on equinumerous names. Q, which says a predicate is quantitative, if it is true on two names, says the names are equinumerous. Finally, CN, which reads "cardinal number" says of a predicate that it actually applies to some name and that it is both numerical and quantitative. Note that $\mathrm{Fin}\,\{a\}$ says a is an inductively finite set.

D019 $[\varphi] \therefore \mathrm{N}(\varphi) \;.\!\equiv\!:\; [ab] : \varphi(a) . a \infty b \;.\!\supset\!.\; \varphi(b)$

017 $[a] . \mathrm{N}(\infty \langle a \rangle)$

D020 $[\varphi] \therefore \mathrm{Q}(\varphi) \;.\!\equiv\!:\; [ab] : \varphi(a) . \varphi(b) \;.\!\supset\!.\; a \infty b$

018 $[a] . \mathrm{Q}(\infty \langle a \rangle)$

D021 $[\varphi] : \mathrm{CN}(\varphi) \;.\!\equiv\!.\; !\,(\varphi) . \mathrm{N}(\varphi) . \mathrm{Q}(\varphi)$

019 $[a] : \mathrm{CN}(\infty \langle a \rangle)$

020 $[\varphi] : \mathrm{CN}(\varphi) \;.\!\equiv\!.\; [\exists a] . \varphi \circ \infty \langle a \rangle$

021 $[\varphi a] : \varphi\{a\} . \mathrm{N}(\varphi) . \mathrm{Q}(\varphi) \;.\!\equiv\!.\; \infty \langle a \rangle \circ \varphi$

D022 $[a] :: \mathrm{Fin}\,\{a\} \;.\!\equiv\!\therefore\; [\varphi] \therefore \varphi\{\wedge\} : [Ab] : A\,\varepsilon\, a . \varphi\{b\} \;.\!\supset\!.\; \varphi\{b \cup A\} :\!\supset\!.\; \varphi\{a\}$

022 $\mathrm{Fin}\,\{\wedge\}$

023 $[A] : A\,\varepsilon\, A \;.\!\supset\!.\; \mathrm{Fin}\,\{A\}$

024 $[ab] : \mathrm{Fin}\,\{a\} . \mathrm{Fin}\,\{b\} \;.\!\supset\!.\; \mathrm{Fin}\,\{a \cup b\}$

025 $[ab] : \mathrm{Fin}\,\{a\} . b \subset a \;.\!\supset\!.\; \mathrm{Fin}\,\{b\}$

026 $[ab] : \mathrm{Fin}\,\{a\} . a \infty b \;.\!\supset\!.\; \mathrm{Fin}\,\{b\}$

027 $[a] : \mathrm{Fin}\,\{a\} \;.\!\supset\!.\; \infty \langle a \rangle \subset \mathrm{Fin}$

028 $[ab]:\mathrm{Fin}\{a\}.b\subset a.a\infty b.\supset.a\circ b$
$D023$ $[a]:0\{a\}.\equiv.a\circ\Lambda$
029 $\mathrm{CN}(0)$
030 $0\subset\mathrm{Fin}$
$D024$ $[a\varphi]:\mathrm{S}\langle\varphi\rangle\{a\}.\equiv.[\exists A].A\,\varepsilon\,a.\varphi\{a-A\}$
031 $[\varphi].\sim(0\circ\mathrm{S}\langle\varphi\rangle)$
032 $[\varphi\psi]:\varphi\circ\psi.\supset.\mathrm{S}\langle\varphi\rangle\circ\mathrm{S}\langle\psi\rangle$
033 $[a]:\mathrm{Fin}\{a\}.\supset.[\exists A].A\,\varepsilon\,A.\sim(A\,\varepsilon\,a)$ [Axiom]

D023 is Canty's definition of zero and *D024* that of successor. *033* is his axiom of infinity. With this we can establish the rest of the Peano Postulates. The reason we add this axiom is to obtain the principle of induction, which is a major tool in the proofs of Chapter IV.

034 $[\varphi]:\varphi\subset\mathrm{Fin}.\supset.\mathrm{S}\langle\varphi\rangle\subset\mathrm{Fin}$
035 $[\varphi]:\varphi\subset\mathrm{Fin}.\mathrm{CN}(\varphi).\supset.\mathrm{CN}(\mathrm{S}\langle\varphi\rangle)$
$D025$ $[a]:\mathrm{Inf}\{a\}.\equiv.\sim\mathrm{Fin}\{a\}$
036 $\mathrm{Inf}\{\mathrm{V}\}$
$D026$ $[\varphi]::\mathrm{Nn}(\varphi).\equiv\therefore[\theta]\therefore\theta(0):[\psi]:\theta(\psi).\supset.\theta(\mathrm{S}\langle\psi\rangle):\supset.\theta(\varphi)$
037 $\mathrm{Nn}(0)$
038 $[\varphi]:\mathrm{Nn}(\varphi).\supset.\sim(0\circ\mathrm{S}\langle\varphi\rangle)$
039 $[\varphi]:\mathrm{Nn}(\varphi).\supset.\mathrm{Nn}(\mathrm{S}\langle\varphi\rangle)$
040 $[\varphi\theta]\therefore\theta(0):[\psi]:\mathrm{Nn}(\psi).\theta(\psi).\supset.\theta(\mathrm{S}\langle\psi\rangle):\mathrm{Nn}(\varphi):\supset.\theta(\varphi)$

This is the classical form of mathematical induction.

041 $[\varphi]:\mathrm{Nn}(\varphi).\supset.\mathrm{CN}(\varphi)$
042 $[\varphi\psi]:\mathrm{Nn}(\varphi).\mathrm{Nn}(\psi).\mathrm{S}\langle\varphi\rangle\circ\mathrm{S}\langle\psi\rangle.\supset.\varphi\circ\psi$

Now we have the system of Peano. Consider *037-040* and *042*; these are precisely the Peano Postulates. In the following theorems, we will merely indicate the definition of addition.

$D027$ $[\theta]\therefore\mathrm{sm}\nparallel\theta\nparallel.\equiv:[\varphi]:\mathrm{Nn}(\varphi).\supset.\theta(\varphi\varphi0)$
$D028$ $[\theta]\therefore\mathrm{sm}_1\nparallel\theta\nparallel.\equiv:[\varphi\psi\chi]:\mathrm{Nn}(\varphi).\mathrm{Nn}(\psi).\mathrm{Nn}(\chi).\theta(\varphi\psi\chi).\supset.\theta(\mathrm{S}\langle\varphi\rangle\psi\,\mathrm{S}\langle\chi\rangle)$
$D029$ $[\varphi\psi\chi]\therefore\mathrm{Sm}(\varphi\psi\chi).\equiv:\mathrm{Nn}(\varphi).\mathrm{Nn}(\psi).\mathrm{Nn}(\chi):[\theta]:\mathrm{sm}\nparallel\theta\nparallel.\mathrm{sm}_1\nparallel\theta\nparallel.\supset.$
$\theta(\varphi\psi\chi)$
043 $[\varphi]:\mathrm{Nn}(\varphi).\supset.\mathrm{Sm}(\varphi\varphi0)$
044 $[\varphi]:\mathrm{Sm}(\varphi\psi\chi).\supset.\mathrm{Sm}(\mathrm{S}\langle\varphi\rangle\psi\,\mathrm{S}\langle\chi\rangle)$
045 $[\varphi\psi]:\mathrm{Nn}(\varphi).\mathrm{Nn}(\psi).\supset.[\exists\chi].\mathrm{Nn}(\chi).\mathrm{Sm}(\chi\varphi\psi)$
046 $[\varphi\psi\chi\eta]:\mathrm{Sm}(\psi\chi\varphi).\mathrm{Sm}(\eta\chi\varphi).\supset.\psi\circ\eta$
$D030$ $[\varphi\psi a]:+\langle\varphi\psi\rangle\{a\}.\equiv.\mathrm{Sm}(\infty\nparallel a\nparallel\varphi\psi)$

$+\langle\varphi\psi\rangle$ will be informally written $\varphi+\psi$.

047 $[\eta\varphi\psi]\therefore\mathrm{Nn}(\eta).\supset:\mathrm{Sm}(\eta\varphi\psi).\equiv.\eta\circ\varphi+\psi$
048 $[\varphi\psi]:\mathrm{Nn}(\varphi).\mathrm{Nn}(\psi).\supset.\mathrm{Nn}(\varphi+\psi)$
049 $[\varphi\psi\chi]:\varphi\circ\psi.\supset.\varphi+\chi\circ\psi+\chi$

Thus we have the basic properties of addition in the natural numbers. Similar properties for multiplication and exponentiation can also be established (see [1]). We will henceforth assume we have all the properties of the natural numbers and will proceed as if we were working with them. For example, we will use the notion of subtraction of natural number without specifically introducing a definition. Such things merely add bulk and not understanding since they have been done often before. In the following definitions we are informal in the sense that we use = not in its proper ontological setting but in the conventional mathematical mode.

D031 $[a] . \mathsf{Cd}\{a\} = 0 .\equiv. a \circ \Lambda$
D032 $[a] : \mathsf{Cd}\{a\} = 1 .\equiv. a \varepsilon a$
D033 $[a] \therefore \mathsf{Cd}\{a\} = n .\equiv: [A] : A \varepsilon a .\supset. \mathsf{Cd}\{a - A\} = n - 1$

Here we use a definition by induction, which is justified by our *040*. We will for the most part use the following alternate form of induction, which is inferentially equivalent.

050 $[\varphi\theta] \therefore \theta(0) : [\psi] : \mathsf{Nn}(\psi) . \psi < \varphi .\supset. \theta(\psi) : \mathsf{Nn}(\psi) .\supset. \theta(\varphi) :\supset. \theta(\varphi)$

This form is much more useful than *040*. Finally, we state the following elementary theorem of number theory merely because it is used so often in our proofs.

051 $[ab] : \mathsf{Nn}(a) . \mathsf{Nn}(b) . a > 1 . b > 1 .\supset. ab \geqslant a + b$

With this we conclude the section concerning ontology.

2 *Mereological preliminaries* Now we begin our formal discussion of mereology. As before we will begin by listing a large number of theorems and definitions without proving most of the theorems. We will use the axiom system for part, since much of our work is done using the term '**pr**'.

A1 $[ABC] : A \varepsilon \mathbf{pr}(B) . B \varepsilon \mathbf{pr}(C) .\supset. A \varepsilon \mathbf{pr}(C)$
A2 $[AB] : A \varepsilon \mathbf{pr}(B) .\supset. \sim(B \varepsilon \mathbf{pr}(A))$
A3 $[AB] : A \varepsilon \mathbf{pr}(B) .\supset. B \varepsilon B$

D1 $[AB] \therefore A \varepsilon \mathbf{el}(B) .\equiv. A \varepsilon A : A \varepsilon \mathbf{pr}(B) .\mathsf{v}. A = B$
D2 $[Aa] \therefore A \varepsilon \mathbf{Kl}(a) .\equiv: A \varepsilon A : [D] : D \varepsilon a .\supset. D \varepsilon \mathbf{el}(A) : [D] : D \varepsilon \mathbf{el}(A) .\supset.$
$[\exists EF] . E \varepsilon a . F \varepsilon \mathbf{el}(D) . F \varepsilon \mathbf{el}(E)$
A4 $[ABa] : A \varepsilon \mathbf{Kl}(a) . B \varepsilon \mathbf{Kl}(a) .\supset. A = B$
A5 $[Aa] : A \varepsilon a .\supset. [\exists B] . B \varepsilon \mathbf{Kl}(a)$

Note that *A4* and *A5* can be restated as follows:

A4' $[a] . \rightarrow \{\mathbf{Kl}(a)\}$
A5' $[a] : !\{a\} .\supset. !\{\mathbf{Kl}(a)\}$

T1 $[AB] \therefore A \varepsilon \mathbf{el}(B) .\equiv: A \varepsilon \mathbf{pr}(B) .\mathsf{v}. A = B$
T2 $[A] . \sim(A \varepsilon \mathbf{pr}(A))$
T3 $[A] : A \varepsilon A .\supset. A \varepsilon \mathbf{el}(A)$
T3.1 $[AB] : A \varepsilon \mathbf{el}(B) .\supset. B \varepsilon B$
T4 $[ABC] : A \varepsilon \mathbf{el}(B) . B \varepsilon \mathbf{el}(C) .\supset. A \varepsilon \mathbf{el}(C)$
T5 $[A] : A \varepsilon A .\supset. A \varepsilon \mathbf{Kl}(A)$
T6 $[A] : A \varepsilon \mathbf{el}(B) . B \varepsilon \mathbf{el}(A) .\supset. A = B$

T6.1 $[AB] \therefore A \,\varepsilon\, \mathbf{el}(B) \;.\!\equiv\!: A \,\varepsilon\, A : [D] : D \,\varepsilon\, \mathbf{el}(A) \;.\!\supset\!.\; [\exists F] . F \,\varepsilon\, \mathbf{el}(D) . F \,\varepsilon\, \mathbf{el}(B)$
T7 $[a] : ! \{a\} \;.\!\equiv\!.\; ! \{\mathbf{Kl}(a)\}$
T8 $[a] : a \circ \Lambda \;.\!\equiv\!.\; \mathbf{Kl}(a) \circ \Lambda$
T9 $[a] : a \circ \Lambda \;.\!\supset\!.\; \mathbf{el}(A) \circ \Lambda$
D3 $[A] : A \,\varepsilon\, \mathbf{Un} \;.\!\equiv\!.\; A \,\varepsilon\, \mathbf{Kl}(\mathrm{V})$
T10 $[A] : A \,\varepsilon\, A \;.\!\supset\!.\; A \,\varepsilon\, \mathbf{el}(\mathbf{Un})$
T10.1 $[A] : A \,\varepsilon\, \mathbf{Un} \;.\!\equiv\!.\; A = \mathbf{Un}$

Un is the collective class of all individuals. What follows next is a set of definitions and theorems proved by Clay in [2], which exhibit the Boolean algebraic properties of mereology. Some of these properties were known before Clay. Those concerning **+** and were known to Leśniewski and some of the properties of **×** were proved by Sobociński.

D4 $[ABC] : A \,\varepsilon\, B \mathbf{\vee} C \;.\!\equiv\!.\; A \,\varepsilon\, \mathbf{Kl}(B \cup C) . \rightarrow \{B\} . \rightarrow \{C\}$
D5 $[ABC] : A \,\varepsilon\, B \mathbf{\wedge} C \;.\!\equiv\!.\; B \,\varepsilon\, B . C \,\varepsilon\, C . A \,\varepsilon\, \mathbf{Kl}(\mathbf{el}(B) \cap \mathbf{el}(C))$
T11 $[ab] : a \subset b \;.\!\supset\!.\; \mathbf{Kl}(a) \subset \mathbf{el}(\mathbf{Kl}(b))$
T12 $[A] : \rightarrow \{A\} \;.\!\supset\!.\; A \circ \mathbf{Kl}(A)$
T13 $[ab] . \mathbf{Kl}(a \cup b) \circ \mathbf{Kl}(\mathbf{Kl}(a) \cup b)$
T14 $[BC] : \rightarrow \{B\} . \rightarrow \{C\} \;.\!\supset\!.\; B \mathbf{\vee} C \circ \mathbf{Kl}(\mathbf{el}(B) \cup \mathbf{el}(C))$
T15 $[AB] . A \mathbf{\wedge} B \circ B \mathbf{\wedge} A$
T16 $[AB] . A \mathbf{\vee} B \circ B \mathbf{\vee} A$
T17 $[AB] . A \mathbf{\wedge} B \subset \mathbf{el}(A)$
T18 $[AB] . A \mathbf{\wedge} B \subset \mathbf{el}(B)$
T19 $[AB] : \rightarrow \{A\} . \rightarrow \{B\} \;.\!\supset\!.\; A \subset \mathbf{el}(A \mathbf{\vee} B)$
T20 $[AB] : \rightarrow \{A\} . \rightarrow \{B\} \;.\!\supset\!.\; B \subset \mathbf{el}(A \mathbf{\vee} B)$
T21 $[AB] : \rightarrow \{A\} . \rightarrow \{B\} . \rightarrow \{C\} \;.\!\supset\!.\; A \mathbf{\wedge} (B \mathbf{\vee} C) \circ (A \mathbf{\wedge} B) \mathbf{\vee} (A \mathbf{\wedge} C)$
T22 $[ABC] : \rightarrow \{A\} . \rightarrow \{B\} . \rightarrow \{C\} \;.\!\supset\!.\; A \mathbf{\vee} (B \mathbf{\wedge} C) \circ (A \mathbf{\vee} B) \mathbf{\wedge} (A \mathbf{\vee} C)$

D6 $[AB] \therefore A \,\varepsilon\, \mathbf{Cm}(B) \;.\!\equiv\!: A \,\varepsilon\, A . B \,\varepsilon\, B . A \mathbf{\wedge} B \circ \Lambda . A \mathbf{\vee} B \,\varepsilon\, \mathbf{Un}$
D7 $[AB] \therefore A \,\varepsilon\, \mathbf{ex}(B) \;.\!\equiv\!: A \,\varepsilon\, A . B \,\varepsilon\, B : [D] : D \,\varepsilon\, \mathbf{el}(A) \;.\!\supset\!.\; \sim (D \,\varepsilon\, \mathbf{el}(B))$
T23 $[AB] : A \,\varepsilon\, \mathbf{Cm}(B) \;.\!\equiv\!.\; B \,\varepsilon\, \mathbf{Cm}(A)$
T23.1 $[AB] : A \,\varepsilon\, \mathbf{Cm}(B) \;.\!\supset\!.\; A \,\varepsilon\, \mathbf{ex}(B)$
T23.2 $[ABC] : A \,\varepsilon\, \mathbf{Cm}(B) . C \,\varepsilon\, \mathbf{pr}(A) \;.\!\supset\!.\; C \,\varepsilon\, \mathbf{ex}(B)$
T24 $[AB] : A \,\varepsilon\, \mathbf{ex}(B) \;.\!\equiv\!.\; A \,\varepsilon\, A . B \,\varepsilon\, B . A \mathbf{\wedge} B \circ \Lambda$
T25 $[AB] : A \,\varepsilon\, \mathbf{ex}(B) \;.\!\equiv\!.\; B \,\varepsilon\, \mathbf{ex}(A)$
T26 $[A] . \mathbf{Cm}(A) \circ \mathbf{Kl}(\mathbf{ex}(A))$
T27 $[A] . \mathbf{ex}(A) \circ \mathbf{el}(\mathbf{Cm}(A))$
T28 $[ABC] : A \,\varepsilon\, \mathbf{ex}(C) . B \,\varepsilon\, \mathbf{ex}(C) \;.\!\supset\!.\; A \mathbf{\vee} B \,\varepsilon\, \mathbf{ex}(C)$
PR $[ABC] : \mathrm{Hp}(2) \;.\!\supset\!.$
3. $A \mathbf{\wedge} C \circ \Lambda$. [1; *T24*]
4. $B \mathbf{\wedge} C \circ \Lambda$. [2; *T24*]
5. $(A \mathbf{\wedge} C) \mathbf{\vee} (B \mathbf{\wedge} C) \circ \Lambda$. [3; 4; *D4*; *T8*]
6. $(A \mathbf{\vee} B) \mathbf{\wedge} C \circ \Lambda$. [1; 2; *D7*; *T21*]
$A \mathbf{\vee} B \,\varepsilon\, \mathbf{ex}(C)$ [1; 2; *D4*; 6; *T24*]
T29 $[ABC] : A \mathbf{\vee} B \,\varepsilon\, \mathbf{ex}(C) . A \,\varepsilon\, A . B \,\varepsilon\, B \;.\!\supset\!.\; A \,\varepsilon\, \mathbf{ex}(C) . B \,\varepsilon\, \mathbf{ex}(C)$
PR $[ABC] : \mathrm{Hp}(3) \;.\!\supset\!.$
4. $C \,\varepsilon\, C$. [1; *D7*]
5. $(A \mathbf{\vee} B) \mathbf{\wedge} C \circ \Lambda$. [1; 2; 3; *D4*; *T24*]

6. $(A \wedge C) \mathbf{v} (B \wedge C) \circ \Lambda$. [5; 2; 3; *T21*]
7. $(A \wedge C) \circ \Lambda$. [6; *D4*; *T8*; 2; 4]
8. $(B \wedge C) \circ \Lambda$. [6; *D4*; *T8*; 2; 4]
$A \varepsilon \mathbf{ex}(C) . B \varepsilon \mathbf{ex}(C)$ [7; 8; 2; 3; 4; *T24*]

T30 $[ABC] : A \varepsilon A . B \varepsilon B .\supset. A \varepsilon \mathbf{ex}(C) . B \varepsilon \mathbf{ex}(C) .\equiv. A \mathbf{v} B \varepsilon \mathbf{ex}(C)$ [*T28*; *T29*]

D8 $[AB] : A \varepsilon \mathbf{lnk}(B) .\equiv. A \varepsilon A . [\exists CDE] . C \varepsilon \mathbf{el}(A) . C \varepsilon \mathbf{ex}(B) . D \varepsilon \mathbf{el}(B) . D \varepsilon \mathbf{ex}(A) . E \varepsilon \mathbf{el}(A) . E \varepsilon \mathbf{el}(B)$

T31 $[AB] : A \varepsilon \mathbf{lnk}(B) .\equiv. [\exists CDE] . C \varepsilon \mathbf{el}(A) . C \varepsilon \mathbf{ex}(B) . D \varepsilon \mathbf{el}(B) . D \varepsilon \mathbf{ex}(A) . E \varepsilon \mathbf{el}(A) . E \varepsilon \mathbf{el}(B)$ [*D8*]

T32 $[AB] : A \varepsilon \mathbf{lnk}(B) .\equiv. B \varepsilon \mathbf{lnk}(A)$ [*T31*]

Note, in the following theorem, the symbol **v** has the meaning of exclusive or, i.e., $p \mathbf{v} q .\equiv. p \vee q . \sim (p \cdot q)$. It is this characterization of unequal names, developed by Clay in [2], that enables us to do so much in later chapters. After *T33* we define **atm** and $A \varepsilon \mathbf{at}(B)$. This notion of atom naturally arises from the Boolean algebraic character of mereology. In [2], Clay has given a wealth of examples of atomic, as well as atomless, mereological models. For further results about atomic mereology see Sobociński [15] and Rickey [9]. The intuitive and in fact defined notion of atom is that of a name that has no part. $A \varepsilon \mathbf{at}(B)$ simply denotes that A is a minimal part of B.

T33 $[AB] \therefore A \neq B .\equiv: A \varepsilon \mathbf{pr}(B) .\vee. B \varepsilon \mathbf{pr}(A) .\vee. A \varepsilon \mathbf{lnk}(B) .\vee. A \varepsilon \mathbf{ex}(B)$

D9 $[AB] : A \varepsilon \mathbf{atm} .\equiv. A \varepsilon A . [B] . \sim (B \varepsilon \mathbf{pr}(A))$

T34 $[AB] : A \varepsilon \mathbf{atm} . B \varepsilon \mathbf{el}(A) .\supset. A = B$ [*D9*; *D1*]

D10 $[AB] : A \varepsilon \mathbf{at}(B) .\equiv. A \varepsilon A . A \varepsilon \mathbf{atm} . A \varepsilon \mathbf{el}(B)$

T35 $[AB] : A \varepsilon \mathbf{at}(B) .\equiv. A \varepsilon \mathbf{atm} . A \varepsilon \mathbf{el}(B)$ [*D10*]
T36 $[A] : A \varepsilon \mathbf{at}(A) .\equiv. A \varepsilon \mathbf{atm}$ [*T35*; *T3*]
T37 $[AB] : A \varepsilon \mathbf{atm} . B \varepsilon \mathbf{atm} . \sim (A = B) .\supset. A \varepsilon \mathbf{ex}(B)$
PR $[AB] \therefore \mathrm{Hp}(3) .\supset:$
4. $[D] : D \varepsilon \mathbf{el}(A) . D \varepsilon \mathbf{el}(B) .\supset. A = B :$ [1; 2; *T34*]
5. $[D] : D \varepsilon \mathbf{el}(A) .\supset. \sim (D \varepsilon \mathbf{el}(B)) :$ [3; 4]
$A \varepsilon \mathbf{ex}(B)$ [1; 2; 5; *D7*]
T38 $[AB] : A \varepsilon \mathbf{atm} . B \varepsilon B . \sim (A \varepsilon \mathbf{ex}(B)) .\supset. A \varepsilon \mathbf{el}(B)$
PR $[AB] : \mathrm{Hp}(3) .\supset.$
$[\exists C]$.
4. $C \varepsilon \mathbf{el}(A)$. [3; 1; 2; *D7*]
5. $C \varepsilon \mathbf{el}(B)$. [3; 1; 2; *D7*]
6. $C = A$. [1; 4; *T34*]
$A \varepsilon \mathbf{el}(B)$ [5; 6]
T39 $[AB] \therefore A \varepsilon \mathbf{atm} . B \varepsilon B .\supset: A \varepsilon \mathbf{ex}(B) .\vee. A \varepsilon \mathbf{el}(B)$ [*T33*; *T38*]
T40 $[ABC] : C \varepsilon (\mathbf{at}(A) \cup \mathbf{at}(B)) . A \varepsilon A . B \varepsilon B .\supset. C \varepsilon \mathbf{at}(A \mathbf{v} B)$
PR $[ABC] \therefore \mathrm{Hp}(3) .\supset:$
4. $C \varepsilon \mathbf{atm} :$ [1; *D10*]
5. $C \varepsilon \mathbf{el}(A) .\vee. C \varepsilon \mathbf{el}(B) :$ [1; *D10*]

6. $C \varepsilon (\mathbf{el}(A) \cup \mathbf{el}(B))$. [5; **ON**]
7. $C \varepsilon \mathbf{el}(\mathbf{Kl}(\mathbf{el}(A) \cup \mathbf{el}(B)))$. [6; *T5*; *T8*]
8. $C \varepsilon \mathbf{el}(A \mathbf{v} B)$. [7; 2; 3; *T14*]
$C \varepsilon \mathbf{at}(A \mathbf{v} B)$ [4; 8; *T35*]

T41 $[ABC] : C \varepsilon \mathbf{at}(A \mathbf{v} B) . A \varepsilon A . B \varepsilon B .\supset. C \varepsilon (\mathbf{at}(A) \cup \mathbf{at}(B))$

PR $[ABC] :: \mathrm{Hp}(3) .\supset\therefore$
4. $C \varepsilon \mathbf{atm}$: [1; *D10*]
5. $C \varepsilon \mathbf{el}(A) .\mathrm{v}. C \varepsilon \mathbf{ex}(A)$: [2; 4; *T39*]
6. $C \varepsilon \mathbf{el}(B) .\mathrm{v}. C \varepsilon \mathbf{ex}(B) \therefore$ [3; 4; *T39*]
7. $C \varepsilon \mathbf{el}(A) . C \varepsilon \mathbf{el}(B) :\mathrm{v}: C \varepsilon \mathbf{el}(A) . C \varepsilon \mathbf{ex}(B) :\mathrm{v}:$
$C \varepsilon \mathbf{ex}(A) . C \varepsilon \mathbf{el}(B) :\mathrm{v}: C \varepsilon \mathbf{ex}(A) . C \varepsilon \mathbf{ex}(B) \therefore$ [6; 7; **ON**]
8. $C \varepsilon \mathbf{el}(A) .\mathrm{v}. C \varepsilon \mathbf{el}(B) .\mathrm{v}. C \varepsilon \mathbf{ex}(A \mathbf{v} B)$: [7; 2; 3; *T30*]
9. $C \varepsilon \mathbf{el}(A) .\mathrm{v}. C \varepsilon \mathbf{el}(B)$: [1; *D10*; 8; *T39*]
$C \varepsilon (\mathbf{at}(A) \cup \mathbf{at}(B))$ [4; 9; *T35*]

T42 $[AB] : A \wedge B \circ \Lambda . A \varepsilon A . B \varepsilon B .\supset. \mathbf{at}(A) \cap \mathbf{at}(B) \circ \Lambda$

PR $[AB] : \mathrm{Hp}(3) .\supset.$
4. $\mathbf{Kl}(\mathbf{el}(A) \cap \mathbf{el}(B)) \circ \Lambda$. [1; 2; 3; *D5*]
5. $\mathbf{el}(A) \cap \mathbf{el}(B) \circ \Lambda$. [4; *T8*]
6. $\mathbf{at}(A) \subset \mathbf{el}(A)$.
7. $\mathbf{at}(B) \subset \mathbf{el}(B)$. [*D10*]
$\mathbf{at}(A) \cap \mathbf{at}(B) \circ \Lambda$. [5; 6; 7; **ON**]

T43 $[AB] : A \varepsilon A . B \varepsilon B .\supset. \mathbf{at}(A \mathbf{v} B) \circ \mathbf{at}(A) \cup \mathbf{at}(B)$ [*T40*; *T41*]

D11 $[ABC] : C \varepsilon A \backslash B .\equiv. C \varepsilon \mathbf{ex}(B) . A \varepsilon \mathbf{Kl}(B \cup C)$

T44 $[ABC] : C \varepsilon A \backslash B .\equiv. C \varepsilon C . B \varepsilon B . C \wedge B \circ \Lambda . A \varepsilon B \mathbf{v} C$ [*D11*; *D4*; *T19*]

T45 $[AB] : A \varepsilon \mathbf{pr}(B) .\equiv. B \backslash A \varepsilon B \backslash A$

T46 $[ABC] : A \varepsilon B \backslash C .\equiv. A \varepsilon \mathbf{Kl}(\mathbf{el}(B) \cap \mathbf{ex}(C)) . C \varepsilon \mathbf{el}(B)$

T47 $[AB] : A \varepsilon \mathbf{el}(B) .\supset. B \backslash A \circ B \wedge \mathbf{Cm}(A)$

PR $[AB] : \mathrm{Hp}(1) .\supset.$
2. $B \backslash A \circ \mathbf{Kl}(\mathbf{el}(B) \cap \mathbf{ex}(A))$. [1; *T46*]
3. $B \backslash A \circ \mathbf{Kl}(\mathbf{el}(B) \cap \mathbf{el}(\mathbf{Cm}(A)))$. [2; *T27*]
$B \backslash A \circ B \wedge \mathbf{Cm}(A)$ [1; *D1*; *A3*; *D5*; *D6*]

T48 $[AB] : A \varepsilon A . B \varepsilon B .\supset. A \mathbf{v} B \circ (A \wedge B) \mathbf{v} (A \backslash (A \wedge B)) \mathbf{v} (B \backslash (A \wedge B))$

PR $[AB] : \mathrm{Hp}(2) .\supset.$
3. $(A \wedge B) \mathbf{v} (A \backslash (A \wedge B)) \mathbf{v} (B \backslash (A \wedge B)) \circ (A \wedge B) \mathbf{v} (A \wedge \mathbf{Cm}(A \wedge B)) \mathbf{v}$
$(B \wedge \mathbf{Cm}(A \wedge B))$. [1; 2; *T47*; *T13*; *T14*]
4. $(A \wedge B) \mathbf{v} (A \backslash (A \wedge B)) \mathbf{v} (B \backslash (A \wedge B)) \circ (A \wedge B) \mathbf{v} ((A \mathbf{v} B) \wedge \mathbf{Cm}(A \wedge B))$.
[3; 1; 2; *T21*]
5. $(A \wedge B) \mathbf{v} (A \backslash (A \wedge B)) \mathbf{v} (B \backslash (A \wedge B)) \circ ((A \wedge B) \mathbf{v} (A \mathbf{v} B)) \wedge ((A \wedge B) \mathbf{v}$
$(\mathbf{Cm}(A \wedge B)))$. [4; *T22*]
6. $(A \wedge B) \mathbf{v} (A \backslash (A \wedge B)) \mathbf{v} (B \backslash (A \wedge B)) \circ ((A \wedge B) \mathbf{v} (A \mathbf{v} B)) \wedge \mathbf{Un}$. [5; *T4*; *D6*]
7. $(A \wedge B) \mathbf{v} (A \backslash (A \wedge B)) \mathbf{v} (B \backslash (A \wedge B)) \circ (A \wedge B) \mathbf{v} (A \mathbf{v} B)$. [6; *D5*]
$(A \wedge B) \mathbf{v} (A \backslash (A \wedge B)) \mathbf{v} (B \backslash (A \wedge B)) \circ A \mathbf{v} B$ [*T13*; *T14*; *T16*; 7]

T49 $[A] : A \varepsilon A . \sim (A \varepsilon \mathbf{atm}) .\supset. [\exists B] . B \varepsilon \mathbf{el}(A) . B \neq A$

PR $[A] : \mathrm{Hp}(2) .\supset.$
$[\exists B]$.

3. $B \varepsilon \mathbf{pr}(A)$. [1; 2; *D9*]

$[\exists B] . B \varepsilon \mathbf{el}(A) . B \neq A$ [3; *D1*]

T50 $[A] \therefore A \varepsilon \mathbf{atm} . \equiv : A \varepsilon A : [B] : B \varepsilon \mathbf{el}(A) . \supset . A = B$ [*T34*; *T49*]

These theorems form the basis for the rest of this dissertation. From time to time, new definitions and theorems will be made, but only where they are needed and appropriate. Throughout the dissertation, we make frequent use of the Boolean algebraic character of mereology. For this reason we shall adopt the convention of writing **BA** in a proof line, rather than specifying a perhaps large number of specific theorems. With this we conclude the introductory portion of mereology.

CHAPTER II: PRIMITIVE AND NON-PRIMITIVE TERMS

1 *The primitive terms* In this first section we indicate which of the better known terms of mereology are primitive. When we axiomatized our system, we assumed **pr** as primitive and we will continue to do so. It should be noted that Lejewski axiomatized mereology without the benefit of the definitions of **el** and **Kl** in [7]. In fact, he had a single axiom for **pr**. Grzegorczyk first formulated a single axiom for mereology in 1946. A simplification of his results appears in [12].

To simplify matters, once we have proved a term primitive, we may use that term to show that others are primitive. For example, suppose we show that **pr** may be defined in terms of **el**, then if **el** can be defined in terms of **Kl**, then **Kl** is also primitive. We will merely state the theorem which proves a term primitive in most cases. The terms up through **1.7** were all shown to be primitive by Leśniewski. We will, however, prove that **st** is primitive since the definition, due to Clay, is different from, but inferentially equivalent to, Leśniewski's. The proof is a simplification of Clay's proof.

1.1 The Term pr

This is our basic primitive term.

1.2 The Term el

T51 $[AB] : A \varepsilon \mathbf{pr}(B) . \equiv . A \varepsilon \mathbf{el}(B) . \sim (A = B)$

1.3 The Term Kl

T52 $[AB] : A \varepsilon \mathbf{el}(B) . \equiv . A \varepsilon A . B \varepsilon \mathbf{Kl}(A \cup B)$

1.4 The Term ex

T53 $[AB] \therefore A \varepsilon \mathbf{el}(B) . \equiv : A \varepsilon A . B \varepsilon B : [D] : D \varepsilon \mathbf{ex}(B) . \supset . D \varepsilon \mathbf{ex}(A)$

1.5 The Term +

D12 $[ABC] : A \varepsilon B + C . \equiv . A \varepsilon \mathbf{Kl}(B \cup C) . B \varepsilon \mathbf{ex}(C)$

T54 $[AB] : A \varepsilon \mathbf{ex}(B) . \equiv . A \varepsilon A . A + B \varepsilon A + B$

1.6 The Term \

T45 $[AB]: A \varepsilon \mathbf{pr}(B) .\equiv. A \varepsilon A . B\backslash A \varepsilon B\backslash A$

1.7 The Term **st**

D13 $[Aa]: A \varepsilon \mathbf{st}(a) .\equiv. A \varepsilon A . [\exists b] . b \subset a . A \varepsilon \mathbf{Kl}(b)$

T55 $[Aa]: A \varepsilon \mathbf{Kl}(a) .\supset. A \varepsilon \mathbf{st}(a)$ [*D13*]

T56 $[AB]: A \varepsilon \mathbf{pr}(B) .\supset. [\exists C] . C \varepsilon C . B \varepsilon \mathbf{st}(A \cup C) - (A \cup C)$

PR $[AB]: \mathrm{Hp}(1) .\supset.$

2. $B\backslash A \varepsilon B\backslash A .$ [1; *T45*]
3. $\sim (B \varepsilon A) .$ [1; *T33*]
4. $\sim (B \varepsilon B\backslash A) .$ [1; *T33*; *D11*]
5. $\sim (B \varepsilon A \cup (B\backslash A)) .$ [3; 4; **ON**]
6. $B \varepsilon \mathbf{Kl}(A \cup (B\backslash A)) .$ [2; *D11*]
7. $B \varepsilon \mathbf{st}(A \cup (B\backslash A)) .$ [6; *T55*]
8. $B \varepsilon \mathbf{st}(A \cup (B\backslash A)) - (A \cup (B\backslash A)) .$ [5; 7; **ON**]

$[\exists C] . C \varepsilon C . B \varepsilon \mathbf{st}(A \cup C) - (A \cup C)$ [2; 8]

T57 $[ABC]: A \varepsilon A . C \varepsilon C . B \varepsilon \mathbf{st}(A \cup C) - (A \cup C) .\supset. A \varepsilon \mathbf{pr}(B)$

PR $[ABC]: \mathrm{Hp}(3) .\supset.$

4. $\sim (B \varepsilon A) .$ [3; **ON**]
5. $\sim (B \varepsilon \mathbf{Kl}(A)) .$ [1; 4; *T5*]
6. $\sim (B \varepsilon C) .$ [3; **ON**]
7. $\sim (B \varepsilon \mathbf{Kl}(C)) .$ [2; 6; *T5*]
8. $\sim (B \circ \wedge) .$ [3; **ON**]
9. $B \varepsilon \mathbf{st}(A \cup C) .$ [3; **ON**]
10. $B \varepsilon \mathbf{Kl}(A \cup C) .$ [5; 7; 8; *D13*]
11. $\sim (A = B) .$ [4; 10]

$A \varepsilon \mathbf{pr}(B)$ [10; 1; *D2*; 11; *D1*]

T58 $[ABC]: A \varepsilon \mathbf{pr}(B) .\equiv. A \varepsilon A . [\exists C] . C \varepsilon C . B \varepsilon \mathbf{st}(A \cup C) - (A \cup C)$ [*T56*; *T57*]

Leśniewski defined a generalization of **+** by $[Aa]: A \varepsilon \mathbf{Sm}(a) .\equiv. A \varepsilon \mathbf{Kl}(a) .$ **dscr** $\{a\}$. The term **dscr** appears below. The idea of the **Sm** term is simply that of a disjoint union for more than two objects.

The next two terms were proved primitive by Sobociński. The first appeared in [13]; the second was unpublished.

1.8 The Term **dscr**

D14 $[a] :: \mathbf{dscr}\ \{a\} .\equiv\therefore [AB] \therefore A \varepsilon a . B \varepsilon a .\supset: A = B .\mathrm{v}. A \varepsilon \mathbf{ex}(B)$

T59 $[AB]: A \varepsilon \mathbf{ex}(B) . \equiv A \mp B . \mathbf{dscr}\ \{A \cup B\}$

1.9 The Term **∧** or **x**.

T60 $[AB]: A \varepsilon \mathbf{el}(B) .\equiv. A \varepsilon A . A \varepsilon A \wedge B$

The next five terms are due to Clay. A large portion of his [2] is devoted to showing that **w-dscr** is primitive. Weakly discrete, as he calls it, is just that property of a name which guarantees that distinct ontological names give rise to distinct mereological classes.

The next three terms, **lb**, **v**, and **glb** are merely analogs of the Boolean algebraic terms lower bound, join, and greatest lower bound. They were defined by Clay in [3] to further show the Boolean algebraic character of mereology. The next term, **ex-pr** will be explained in the discussion following the proof that it is primitive. That the last term, **cntr**, is primitive is immediate from the definition and the fact that **pr** is assumed primitive.

1.10 The Term **w-dscr**

D15 $[a] \therefore \textbf{w-dscr}\{a\} \mathrel{.{\equiv}:} [Ab] : A \mathbin{\varepsilon} a \,.\, b \subset a \,.\, A \mathbin{\varepsilon} \textbf{el}(\textbf{Kl}(b)) \mathrel{.{\supset}.} A \mathbin{\varepsilon} b$

T61 $[AB] :: A \mathbin{\varepsilon} \textbf{pr}(B) \mathrel{.{\equiv}\therefore} \rightarrow \{A\} . \rightarrow \{B\} . \sim(\textbf{w-dscr}\{A \cup B\}) \therefore [\exists ab] \therefore$ $\textbf{w-dscr}\{a \cup B\} : [C] : a \cup B \Subset C \mathrel{.{\supset}.} \sim(\textbf{w-dscr}\{C\}) : a \cup A \Subset b \,.$ $\textbf{w-dscr}\{b\}$

1.11 The Term **lb**

D16 $[Aa] \therefore A \mathbin{\varepsilon} \textbf{lb}(a) \mathrel{.{\equiv}:} A \mathbin{\varepsilon} A \,.\, !\{a\} : [D] : D \mathbin{\varepsilon} a \mathrel{.{\supset}.} A \mathbin{\varepsilon} \textbf{el}(D)$

T62 $[AB] : A \mathbin{\varepsilon} \textbf{el}(B) \mathrel{.{\equiv}.} B \mathbin{\varepsilon} B \,.\, A \mathbin{\varepsilon} \textbf{lb}(B)$

1.12 The Term **v**

T63 $[AB] : A \mathbin{\varepsilon} \textbf{el}(B) \mathrel{.{\equiv}.} A \mathbin{\varepsilon} A \,.\, B \mathbin{\varepsilon} A \,\textbf{v}\, B$

1.13 The Term **glb**

D17 $[Aa] : A \mathbin{\varepsilon} \textbf{glb}(a) \mathrel{.{\equiv}.} A \mathbin{\varepsilon} \textbf{lb}(a) \,.\, \textbf{lb}(a) \subset \textbf{el}(A)$

T64 $[AB] : A \mathbin{\varepsilon} \textbf{el}(B) \mathrel{.{\equiv}.} B \mathbin{\varepsilon} B \,.\, A \mathbin{\varepsilon} \textbf{glb}(A \cup B)$

1.14 The Term **ex-pr**

D18 $[AB] \therefore A \mathbin{\varepsilon} \textbf{ex-pr}(B) \mathrel{.{\equiv}:} A \mathbin{\varepsilon} A : A \mathbin{\varepsilon} \textbf{ex}(B) \mathrel{.{\vee}.} A \mathbin{\varepsilon} \textbf{pr}(B)$

T65 $[AB] : A \mathbin{\varepsilon} \textbf{pr}(B) \mathrel{.{\supset}.} A \mathbin{\varepsilon} \textbf{ex-pr}(B) \,.\, \sim(B \mathbin{\varepsilon} \textbf{ex-pr}(A))$
PR $[AB] : \mathrm{Hp}(1) \mathrel{.{\supset}.}$
2. $A \mathbin{\varepsilon} \textbf{ex-pr}(B) \,.$ [1; *D18*]
3. $\sim(B \mathbin{\varepsilon} \textbf{pr}(A)) \,.$ [1; *A2*]
4. $\sim(A \mathbin{\varepsilon} \textbf{ex}(B)) \,.$ [1; *T33*]
5. $\sim(B \mathbin{\varepsilon} \textbf{ex}(A)) \,.$ [4; *T25*]
6. $\sim(B \mathbin{\varepsilon} \textbf{ex-pr}(A)) \,.$ [3; 5; *D18*]
$A \mathbin{\varepsilon} \textbf{ex-pr}(B) \,.\, \sim(B \mathbin{\varepsilon} \textbf{ex-pr}(A))$ [2; 6]

T66 $[AB] : A \mathbin{\varepsilon} \textbf{ex-pr}(B) \,.\, \sim(B \mathbin{\varepsilon} \textbf{ex-pr}(A)) \mathrel{.{\supset}.} A \mathbin{\varepsilon} \textbf{pr}(B)$
PR $[AB] \therefore \mathrm{Hp}(2) \mathrel{.{\supset}:}$
3. $A \mathbin{\varepsilon} \textbf{ex}(B) \mathrel{.{\vee}.} A \mathbin{\varepsilon} \textbf{pr}(B) :$ [1; *D18*]
4. $B \mathbin{\varepsilon} B \,.$ [3; *D6*; *A3*]
5. $\sim(B \mathbin{\varepsilon} \textbf{ex}(A)) \,.$ [2; 4; *D18*]
6. $\sim(A \mathbin{\varepsilon} \textbf{ex}(B)) \,.$ [5; *T25*]
$A \mathbin{\varepsilon} \textbf{pr}(B)$ [3; 6]

T67 $[AB] : A \mathbin{\varepsilon} \textbf{pr}(B) \mathrel{.{\equiv}.} A \mathbin{\varepsilon} \textbf{ex-pr}(B) \,.\, \sim(B \mathbin{\varepsilon} \textbf{ex-pr}(A))$ [*T65*; *T66*]

Clay introduced this term in [2] and incorrectly stated it was not primitive. The introduction of this term follows some facts Clay mentioned about primitive terms. Chief among this is his demonstration that if λ is a primitive term such that $[A]: A \varepsilon A \,.\!\supset.\, \sim (A \varepsilon \lambda(A))$ then the negation of λ is also primitive. So now let us consider anew our *T33* and make the following definition

1.15.1 The Term **cntr**

D19 $\quad [AB]: A \varepsilon \mathbf{cntr}(B) \,.\!\equiv.\, B \varepsilon \mathbf{pr}(A)$

From *T33* we see that with **cntr** in the second disjunct, three of the four disjunctive terms are primitive. (The fourth, **lnk**, is shown not to be primitive in the next section.) What happens to their pairwise alternation, or even triple alternations? The triples are easily dealt with since any triple alternation is merely the negation of the fourth disjunct and hence, by the above discussion, it is primitive. We have shown that **ex-pr** is primitive and, similarly, if we defined **ex-cntr** it would also be primitive by *D19*. The third term **ex-lnk** is not primitive as is shown in the next section. The other three pairs are primitive since they are merely the negation of one of the cited pairs. The alternation of all four terms is a purely ontological term by *T33* and hence not primitive to mereology. This same type of investigation could be done with other pairs, triples, etc., of terms, and will be done, in part, in what follows. These four terms were singled out because of their special character, namely *T33*.

The last two primitive terms dealt with in this section are due to Lejewski and may be found in his [5] and [6]. He has also constructed single axioms for these two terms.

1.15 The Term **ov**

D20 $\quad [AB] \therefore A \varepsilon \mathbf{ov}(B) \,.\!\equiv:\, A \varepsilon A : [\exists C] . C \varepsilon \mathbf{el}(A) . C \varepsilon \mathbf{el}(B)$

T68 $\quad [AB]: A \varepsilon \mathbf{ex}(B) \,.\!\equiv.\, A \varepsilon A . B \varepsilon B . \sim (A \varepsilon \mathbf{ov}(B))$ $\quad$ [*D7*; *D20*]

1.16 The Term **elKl**

D21 $\quad [ABa]: A \varepsilon \mathbf{elKl}(B, a) \,.\!\equiv.\, A \varepsilon \mathbf{el}(B) . B \varepsilon \mathbf{Kl}(a)$

T69 $\quad [AB]: A \varepsilon \mathbf{el}(B) \,.\!\equiv.\, A \varepsilon \mathbf{elKl}(B, B)$ $\quad$ [*D21*; *T3*]

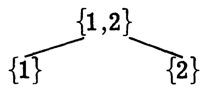
The term **ov**, standing for overlap, means that two individuals have at least some individual in common. The term **elKl** is a somewhat artificial construction used by Lejewski because it is relatively easy to construct a single axiom for it.

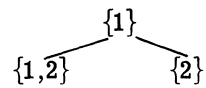
With this we conclude the section on the basic primitive terms. It should be noted that the majority of these terms are binary and some of the rest ternary. We will investigate more primitive ternary terms in the next chapter. The primitive terms introduced here are merely the best known and most used terms in current mereological research.

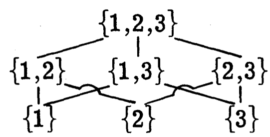
2 *The non-primitive terms*

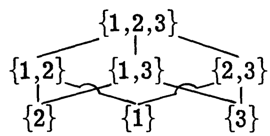
2.1 *The models* A theory is consistent if and only if it has a non-empty model. Furthermore, every model verifies every theorem of the theory.

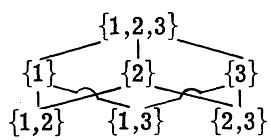
Clay has shown that Boolean algebras with zero deleted in which the members are individuals are models for mereology. The following six models are precisely this. The lines connecting two entries indicate the **pr** relation. Of the models the first five are Clay's, the last is new. The isomorphisms of the models are obvious.

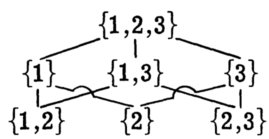
$\mathfrak{A}$


$\mathfrak{B}$


$\mathfrak{C}$


$\mathfrak{D}$


$\mathfrak{E}$


$\mathfrak{F}$


The five following terms were shown to be non-primitive by Clay in [2]. We shall merely indicate the proof of non-primitivity of a term by citing the two models where the term is unchanged but **pr** is different.

2.2 **Un**-$\mathfrak{C}$ and $\mathfrak{E}$

2.3 **Cm**-$\mathfrak{C}$ and $\mathfrak{E}$

2.4 **Ink**-$\mathfrak{C}$ and $\mathfrak{D}$

2.5 The Term Δ

D22 $[ABC]: A\,\varepsilon\, B \Delta C \;.\!\equiv.\; A\,\varepsilon\, \mathbf{Kl}((\mathbf{el}(B) \cap \mathbf{ex}(C)) \cup (\mathbf{ex}(B) \cap \mathbf{el}(C)))$

This is the mereological counterpart of symmetric difference introduced by Sobociński and investigated by Clay. It is not primitive by models $\mathfrak{A}$ and $\mathfrak{B}$.

2.6 The Term **wex**

D23 $[AB]: A\,\varepsilon\, \mathbf{wex}(B) \;.\!\equiv.\; A\,\varepsilon\, A \,.\, B\,\varepsilon\, B \,.\, \sim(A\,\varepsilon\,\mathbf{el}(B)) \,.\, \sim(B\,\varepsilon\,\mathbf{el}(A))$

This is the term promised in **1.14**. That it is equivalent to **ex-Ink** is shown by the following theorem of Clay:

T70 $[AB] \therefore A\,\varepsilon\,\mathbf{wex}(B) \;.\!\equiv:\; A\,\varepsilon\, A \,.\, B\,\varepsilon\, B : A\,\varepsilon\,\mathbf{ex}(B) \,.\!\vee\!.\, A\,\varepsilon\,\mathbf{Ink}(B)$

Models $\mathfrak{C}$ and $\mathfrak{E}$ show that **wex** is not primitive. These five preceding terms are the best known non-primitive terms.

3 *The independent terms* The proof of primitivity is essentially based on the notion that a particular term is definable in terms of another. In this section we call two terms independent if neither is definable in terms of the other. Two prove this, we find two pairs of models. In the first pair one term changes meaning while the other retains its meaning, in the second pair vice-versa. This being the case, we conclude that the terms are not definable in terms of each other.

3.1 *The Terms* Δ *and* **Un** In models $\mathfrak{A}$ and $\mathfrak{B}$ $\{1\} \Delta \{2\} = \{1,2\}$, $\{1\} \Delta \{1,2\} = \{2\}$ and $\{2\} \Delta \{1,2\} = \{1\}$, but in $\mathfrak{A}$, $\{1,2\} \varepsilon$ **Un** while in $\mathfrak{B}$, $\{1\} \varepsilon$ **Un**. In both $\mathfrak{C}$ and $\mathfrak{D}$ we have $\{1,2,3\} \varepsilon$ **Un** but in $\mathfrak{C}$ $\{3\} \Delta \{1,3\} = \{1\}$ while in $\mathfrak{D}$ $\{3\} \Delta \{1,3\} = \{2\}$.

3.2 *The Terms* Δ *and* **Cm** In models $\mathfrak{A}$ and $\mathfrak{B}$, Δ has the same meaning by **3.1**, but in $\mathfrak{A}$ **Cm**$(\{2\}) = \{1\}$ while in $\mathfrak{B}$ **Cm**$(\{2\}) = \{1,2\}$. In models $\mathfrak{C}$ and $\mathfrak{E}$ we have **Cm**$(\{1\}) = \{2,3\}$, **Cm**$(\{2\}) = \{1,3\}$ and **Cm**$(\{3\}) = \{1,2\}$ and by *T23*, this is enough, to show **Cm** has the same meaning. However, in $\mathfrak{C}$, $\{1\} \Delta \{1,2\} = \{2\}$ while in $\mathfrak{E}$ $\{1\} \Delta \{1,2\} = \{1,3\}$.

3.3 *The Terms* Δ *and* **Ink** In models $\mathfrak{C}$ and $\mathfrak{D}$, $\{1,2\} \varepsilon$ **Ink**$\{1,3\}$, $\{1,2\} \varepsilon$ **Ink**$\{2,3\}$, and $\{1,3\} \varepsilon$ **Ink**$\{2,3\}$ and these are the only such. In $\mathfrak{C}$, $\{3\} \Delta \{1,3\} = \{1\}$ while in $\mathfrak{D}$ $\{3\} \Delta \{1,3\} = \{2\}$. In $\mathfrak{C}$ we have $\{1,2\} \varepsilon$ **Ink**$\{1,3\}$ while in $\mathfrak{F}$ we have $\sim(\{1,2\} \varepsilon$ **Ink**$\{1,3\})$ but the following table shows that Δ has identical meaning in $\mathfrak{C}$ and $\mathfrak{F}$.

$\{1\} \Delta \{1,2\} = \{2\}$	$\{1\} \Delta \{2\} = \{1,2\}$
$\{1\} \Delta \{1,3\} = \{3\}$	$\{1\} \Delta \{3\} = \{1,3\}$
$\{1\} \Delta \{2,3\} = \{1,2,3\}$	$\{1\} \Delta \{1,2,3\} = \{2,3\}$
$\{2\} \Delta \{1,2\} = \{1\}$	$\{2\} \Delta \{3\} = \{2,3\}$
$\{2\} \Delta \{1,3\} = \{1,2,3\}$	$\{2\} \Delta \{1,2,3\} = \{1,3\}$
$\{2\} \Delta \{2,3\} = \{3\}$	$\{3\} \Delta \{1,2,3\} = \{1,2\}$
$\{3\} \Delta \{1,2\} = \{1,2,3\}$	$\{1,2\} \Delta \{1,3\} = \{2,3\}$
$\{3\} \Delta \{1,3\} = \{1\}$	$\{1,2\} \Delta \{2,3\} = \{1,3\}$
$\{3\} \Delta \{2,3\} = \{2\}$	$\{1,3\} \Delta \{2,3\} = \{1,2\}$
$\{1,2\} \Delta \{1,2,3\} = \{3\}$	$\{1,3\} \Delta \{1,2,3\} = \{2\}$

$\{2,3\} \Delta \{1,2,3\} = \{1\}$

3.4 *The Terms* **Ink** *and* **Un** In models $\mathfrak{A}$ and $\mathfrak{B}$ **Ink** is the same (vacuously) but in $\mathfrak{A}$ $\{1,2\} \varepsilon$ **Un** while in $\mathfrak{B}$, $\{1\} \varepsilon$ **Un**. On the other hand, $\{1,2,3\} \varepsilon$ **Un** in both $\mathfrak{D}$ and $\mathfrak{E}$ while $\{1,2\} \varepsilon$ **Ink**$\{2,3\}$ in $\mathfrak{D}$ but $\sim(\{1,2\} \varepsilon$ **Ink**$\{2,3\})$ in $\mathfrak{E}$.

Thus, we have shown that the above four pairs of terms are independent. In a later section we shall consider terms which, when used together, are primitive though neither, by itself, is primitive.

4 *The dependent terms* We shall next consider what we call dependent terms. A term is dependent upon another term if and only if the former is definable in terms of the latter.

4.1 Un depends on **Cm**

T71	$[AB]: A \varepsilon \mathbf{Un} . B \varepsilon B . \supset . \sim(B \varepsilon \mathbf{Cm}(A))$	
PR	$[AB]: \mathrm{Hp} . (2) . \supset .$	
3.	$B \varepsilon \mathbf{el}(A) .$	[1; 2; *T10*]
4.	$\sim(B \varepsilon \mathbf{ex}(A))$	[3; *T33*]
	$\sim(B \varepsilon \mathbf{Cm}(A))$	[1; 2; 4; *D6*; *D7*]
T72	$[AB]: A \varepsilon A . \sim(A \varepsilon \mathbf{Un}) . \supset . [\exists B] . B \varepsilon \mathbf{Cm}(A)$	
PR	$[AB]: \mathrm{Hp}(2) . \supset .$	
3.	$A \varepsilon \mathbf{el}(\mathbf{Un}) .$	[1; *T10*]

4.	$A \varepsilon \mathbf{pr}(\mathbf{Un})$.	[2; 3; *D1*]
	$[\exists C]$.	
5.	$\quad C \varepsilon \mathbf{Un}\backslash A$.	[4; *T45*]
6.	$\quad C \varepsilon \mathbf{ex}(A)$.	[5; *D11*]
7.	$!\{\mathbf{ex}(A)\}$.	[6; **ON**]
8.	$!\{\mathbf{Kl}(\mathbf{ex}(A))\}$.	[7; *T7*]
	$[\exists B] . B \varepsilon \mathbf{Cm}(A)$	[8; *T26*]
T73	$[A] \therefore A \varepsilon \mathbf{Un} .\equiv: A \varepsilon A : [B] : B \varepsilon B .\supset. \sim(B \varepsilon \mathbf{Cm}(A))$	[*T71*; *T72*]

In models $\mathfrak{C}$ and $\mathfrak{D}$, **Un** is the same but **Cm** differs, e.g., in $\mathfrak{C}$ $\{1\} \varepsilon$ $\mathbf{Cm}(\{2,3\})$ but in $\mathfrak{D}$ $\sim(\{1\} \varepsilon \mathbf{Cm}(\{2,3\}))$. Hence, **Cm** does not depend upon **Un**.

4.2 Un depends on wex

T74	$[AB] : A \varepsilon \mathbf{Un} . B \varepsilon B .\supset. \sim(B \varepsilon \mathbf{wex}(A))$	
PR	$[AB] : \mathrm{Hp}(2) .\supset.$	
3.	$B \varepsilon \mathbf{el}(A)$.	[1; 2; *T10*]
	$\sim(B \varepsilon \mathbf{wex}(A))$	[3; *D23*]
T75	$[A] : A \varepsilon A . \sim(A \varepsilon \mathbf{Un}) .\supset. [\exists B] . B \varepsilon \mathbf{wex}(A)$	
PR	$[A] : \mathrm{Hp}(2) .\supset.$	
3.	$A \varepsilon \mathbf{el}(\mathbf{Un})$.	[1; *T10*]
4.	$A \varepsilon \mathbf{pr}(\mathbf{Un})$.	[2; 3; *D1*]
	$[\exists C]$.	
5.	$\quad C \varepsilon \mathbf{Un}\backslash A$.	[4; *T45*]
6.	$\quad C \varepsilon \mathbf{ex}(A)$.	[5; *D11*]
7.	$\quad C \varepsilon \mathbf{wex}(A)$.	[6; *T70*]
	$[\exists B] . B \varepsilon \mathbf{wex}(A)$	[7]
T76	$[A] : A \varepsilon \mathbf{Un} .\equiv. A \varepsilon A . [B] . \sim(B \varepsilon \mathbf{wex}(A))$	[*T74*; *T75*]

In models $\mathfrak{C}$ and $\mathfrak{D}$ again $\{1\} \varepsilon \mathbf{wex}(\{2,3\})$ in $\mathfrak{C}$ but $\sim(\{1\}) \varepsilon \mathbf{wex}(\{2,3\})$ in $\mathfrak{D}$ but **Un** is the same in both models. Therefore, **wex** does not depend on **Un**.

4.3 Cm depends on wex

T77	$[ABC] : A \varepsilon \mathbf{Cm}(B) .\supset. A \varepsilon \mathbf{wex}(B)$.	
PR	$[AB] : \mathrm{Hp}(1) .\supset.$	
2.	$A \varepsilon \mathbf{ex}(B)$.	[*D6*; *T24*; 1]
	$A \varepsilon \mathbf{wex}(B)$.	[1; *D6*; 2; *T70*]
T78	$[ABC] : A \varepsilon \mathbf{Cm}(B) . C \varepsilon \mathbf{pr}(A) .\supset. C \varepsilon \mathbf{wex}(A)$	
PR	$[ABC] : \mathrm{Hp}(2) .\supset.$	
3.	$C \varepsilon \mathbf{el}(A)$.	[2; *D1*]
4.	$A \varepsilon \mathbf{el}(\mathbf{Cm}(B))$.	[1; *T26*; *A4*; *T3*]
5.	$C \varepsilon \mathbf{el}(\mathbf{Cm}(B))$.	[3; 4; *T4*]
6.	$C \varepsilon \mathbf{ex}(B)$.	[5; *T27*]
	$C \varepsilon \mathbf{wex}(B)$.	[6; *T70*]
T79	$[ABC] : A \varepsilon \mathbf{Cm}(B) . A \varepsilon \mathbf{pr}(C) . \sim(C \varepsilon \mathbf{Un}) .\supset. C \varepsilon \mathbf{wex}(B)$	
PR	$[ABC] : \mathrm{Hp}(3) .\supset.$	
4.	$A \varepsilon \mathbf{ex}(B)$.	[1; *D6*; *D7*]
5.	$\sim(A \varepsilon \mathbf{el}(B))$.	[4; *T33*; *D1*]

6. $A \varepsilon \mathbf{el}(C)$. [2; *D1*]
7. $\sim(C \varepsilon \mathbf{el}(B))$. [*T4*; 6; 5]
8. $C \varepsilon C$. [2; *A3*]
9. $C \varepsilon \mathbf{pr}(\mathbf{Un})$. [8; *T10*; 3]
10. $\mathbf{Un}\backslash C \varepsilon \mathbf{Un}\backslash C$. [9; *T45*]
11. $\mathbf{Un}\backslash C \varepsilon \mathbf{ex}(C)$. [10; *D11*]
12. $\sim(\mathbf{Un}\backslash C \varepsilon \mathbf{el}(C))$. [11; 3; *T33*; *D1*]
13. $\mathbf{Un}\backslash C \varepsilon \mathbf{ex}(A)$. [6; 11; 12; **BA**]
14. $\mathbf{Un}\backslash C \varepsilon \mathbf{el}(B)$. [13; 1; *T27*]
15. $\sim(B \varepsilon \mathbf{el}(C))$. [14; 13; *T4*]

$C \varepsilon \mathbf{wex}(B)$. [4; *D6*; 8; 7; 15; *D23*]

T80 $[ABC] \therefore A \varepsilon \mathbf{Cm}(B) : C \varepsilon\ \mathbf{ex}(A) .\mathrm{v}. C \varepsilon\ \mathbf{Ink}(A) :\supset. C \varepsilon \mathbf{wex}(A)$ [*T70*]

T81 $[ABC] \therefore A \varepsilon \mathbf{Cm}(B) . C \varepsilon C . \sim(C \varepsilon \mathbf{Un}) .\supset: C \varepsilon\ \mathbf{wex}(A) .\mathrm{v}. C \varepsilon \mathbf{wex}(B)$
[*T33*; *T77*-*T80*]

T82 $[ABC] :: A \varepsilon \mathbf{Cm}(B) .\supset\therefore A \neq B \therefore [C] \therefore C \varepsilon C . \sim(C \varepsilon \mathbf{Un}) .\supset:$
$C \varepsilon \mathbf{wex}(A) .\mathrm{v}. C \varepsilon \mathbf{wex}(B)$ [*D6*; *D5*, **ON**; *T81*]

T83 $[AB] :: \sim(A \varepsilon \mathbf{Cm}(B)) . A \neq B .\supset\therefore [\exists D] \therefore D \varepsilon \mathbf{el}(A) . D \varepsilon \mathbf{el}(B) :\mathrm{v}:$
$D \varepsilon \mathbf{ex}(A) . D \varepsilon \mathbf{ex}(B)$

PR $[AB] :: \mathrm{Hp}(2) .\supset\therefore$
3. $A \varepsilon A$. [2; **ON**]
4. $B \varepsilon B$: [2; **ON**]
5. $((A \wedge B) \circ \wedge) :\mathrm{v}. \sim((A \vee B) \varepsilon \mathbf{Un}) . A \vee B \varepsilon A \vee B$: [1; 3; 4; *D4*; *D6*]

$[\exists D]$:
6. $D \varepsilon \mathbf{el}(A \wedge B) .\mathrm{v}. D \varepsilon \mathbf{Un}\backslash(A \vee B)$: [5; *D5*; *T10*; *D1*; *T45*]
7. $D \varepsilon \mathbf{el}(A \wedge B) .\mathrm{v}. D \varepsilon \mathbf{ex}(A \vee B)$: [6; *D11*]

$[\exists D] \therefore D \varepsilon \mathbf{el}(A) . D \varepsilon \mathbf{el}(B) :\mathrm{v}: D \varepsilon \mathbf{ex}(A) . D \varepsilon \mathbf{ex}(B)$
[2; 7; *D5*; *T17*; *T18*; *T4*; *T30*]

T84 $[ABD] : D \varepsilon \mathbf{el}(A) . D \varepsilon \mathbf{el}(B) . A \neq B .\supset. D \varepsilon D . \sim(D \varepsilon \mathbf{Un})$.
$\sim(D \varepsilon \mathbf{wex}(A)) . \sim(D \varepsilon \mathbf{wex}(B))$

PR $[ABD] : \mathrm{Hp}(3) .\supset.$
4. $D \varepsilon D$. [1; **ON**]
5. $\sim(D \varepsilon \mathbf{Un})$. [1; 2; 3; *D3*]
6. $\sim(D \varepsilon \mathbf{wex}(A))$. [1; *D23*]
7. $\sim(D \varepsilon \mathbf{wex}(B))$. [2; *D23*]

$D \varepsilon D . \sim(D \varepsilon \mathbf{Un}) . \sim(D \varepsilon \mathbf{wex}(A)) . \sim(D \varepsilon \mathbf{wex}(B))$ [4-7]

T85 $[ABD] : D \varepsilon \mathbf{ex}(A) . D \varepsilon \mathbf{ex}(B) . A \neq B .\supset. A \vee B \varepsilon A \vee B . \sim((A \vee B) \varepsilon \mathbf{Un})$.
$\sim((A \vee B) \varepsilon \mathbf{wex}(A)) . \sim((A \vee B) \varepsilon \mathbf{wex}(B))$

PR $[ABD] : \mathrm{Hp}(3) .\supset.$
4. $A \vee B \varepsilon A \vee B$. [3; **ON**; *D4*]
5. $A \varepsilon \mathbf{el}(A \vee B)$. [3; 4; *T19*]
6. $\sim((A \vee B) \varepsilon \mathbf{wex}(A))$. [5; *D23*]
7. $B \varepsilon \mathbf{el}(A \vee B)$. [3; 4; *T20*]
8. $\sim((A \vee B) \varepsilon \mathbf{wex}(B))$. [7; *D23*]
9. $D \varepsilon \mathbf{ex}(A \vee B)$. [1; 2; 3; *T30*]
10. $\sim((A \vee B) \varepsilon \mathbf{Un})$. [9; *T10*; *T33*]

$A \vee B \varepsilon A \vee B . \sim((A \vee B) \varepsilon \mathbf{Un}) . \sim((A \vee B) \varepsilon \mathbf{wex}(A)) . \sim((A \vee B) \varepsilon \mathbf{wex}(B))$.
[4; 6; 8; 10]

T86	$[AB] \therefore \sim(A \,\varepsilon\, \mathbf{Cm}(B)) \,.\, A \neq B \,.\!\supset\!: [\exists C] : \sim(C \,\varepsilon\, \mathbf{Un}) \,.\, C \,\varepsilon\, C \,.$ $\sim(C \,\varepsilon\, \mathbf{wex}(A) \,.\mathrm{v}.\, C \,\varepsilon\, \mathbf{wex}(B))$	[*T83*-*T85*]
T87	$[AB] :: A \,\varepsilon\, \mathbf{Cm}(B) \,.\!\equiv\!\therefore A \neq B \therefore [C] \therefore C \,\varepsilon\, C \,.\sim(C \,\varepsilon\, \mathbf{Un}) \supset: C \,\varepsilon\, \mathbf{wex}(A) \,.\mathrm{v}.$ $C \,\varepsilon\, \mathbf{wex}(B)$	[*T82*; *T86*]
T88	$[AB] :: A \,\varepsilon\, \mathbf{Cm}(B) \,.\!\equiv\!\therefore A \neq B \therefore [C] \therefore C \,\varepsilon\, C \,.\, [\exists D] \,.\, D \,\varepsilon\, \mathbf{wex}(C) \,.\!\supset\!:$ $C \,\varepsilon\, \mathbf{wex}(A) \,.\mathrm{v}.\, C \,\varepsilon\, \mathbf{wex}(B)$	[*T87*; *T76*]

Models $\mathfrak{C}$ and $\mathfrak{F}$ show that **wex** is not dependent on **Cm**. $\mathbf{Cm}(\{1\}) = \{2,3\}$, $\mathbf{Cm}(\{2\}) = \{1,3\}$, and $\mathbf{Cm}(\{3\}) = \{1,2\}$ and by *T23* this is enough to show that **Cm** is the same in $\mathfrak{C}$ and $\mathfrak{F}$. In $\mathfrak{C}$ we have $\{1\} \,\varepsilon\, \mathbf{wex}\{2\}$ but in $\mathfrak{F}$ we have $\sim(\{1\} \,\varepsilon\, \mathbf{wex}\{2\})$ hence, **wex** differs in $\mathfrak{C}$ and $\mathfrak{F}$.

5 *The co-primitive terms* We use the word co-primitive to express the relationship of two terms neither of which is primitive but when they are used jointly can define a primitive term. The method of proof is as before, but instead of using just a single term to define the previous term, we use two terms. For example, in **5.1** below we define **ex** in terms of **lnk** and **wex**. It is possible to use just a single term, defined by the two terms, and show that this is primitive. We will do this in **5.1** but after that one example we will not. The important concept is that two non-primitive terms together can be primitive, not that new primitive terms are definable.

5.1 The Terms **lnk** and **wex**

T89	$[AB] : A \,\varepsilon\, \mathbf{ex}(B) \,.\!\supset\!.\, A \,\varepsilon\, \mathbf{wex}(B) \,.\sim(A \,\varepsilon\, \mathbf{lnk}(B))$	
PR	$[AB] : \mathrm{Hp}(1) \,.\!\supset\!.$	
2.	$A \,\varepsilon\, \mathbf{wex}(B) \,.$	[1; *T70*]
3.	$\sim(A \,\varepsilon\, \mathbf{lnk}(B)) \,.$	[1; *T33*]
	$A \,\varepsilon\, \mathbf{wex}(B) \,.\sim(A \,\varepsilon\, \mathbf{lnk}(B)) \,.$	[2; 3]
T90	$[AB] : A \,\varepsilon\, \mathbf{wex}(B) \,.\sim(A \,\varepsilon\, \mathbf{lnk}(B)) \,.\!\supset\!.\, A \,\varepsilon\, \mathbf{ex}(B)$	
PR	$[AB] \therefore \mathrm{Hp}(2) \,.\!\supset\!:$	
3.	$A \,\varepsilon\, \mathbf{lnk}(B) \,.\mathrm{v}.\, A \,\varepsilon\, \mathbf{ex}(B) :$	[1; *T70*]
	$A \,\varepsilon\, \mathbf{ex}(B) \,.$	[2; 3; **ON**]
T91	$[AB] : A \,\varepsilon\, \mathbf{ex}(B) \,.\!\equiv\!.\, A \,\varepsilon\, \mathbf{wex}(B) \,.\sim(A \,\varepsilon\, \mathbf{lnk}(B))$	[*T89*; *T90*]
D24	$[ABCD] \therefore \mathbf{lw}(ABCD) \,.\!\equiv\!: A \,\varepsilon\, A \,.\, B \,\varepsilon\, B \,.\, C \,\varepsilon\, C \,.\, D \,\varepsilon\, D : A \,\varepsilon\, \mathbf{lnk}(B) \,.\mathrm{v}.$ $C \,\varepsilon\, \mathbf{wex}(D) \,.$	
T92	$[C] \,.\sim(C \,\varepsilon\, \mathbf{lnk}(C))$	[*D8*]
T93	$[C] \,.\sim(C \,\varepsilon\, \mathbf{wex}(C))$	[*D23*]
T94	$[AB] : A \,\varepsilon\, \mathbf{lnk}(B) \,.\!\equiv\!.\, [C] \,.\, \mathbf{lw}(ABCC)$	[*T93*; *D24*]
T95	$[AB] : A \,\varepsilon\, \mathbf{wex}(B) \,.\!\equiv\!.\, [C] \,.\, \mathbf{lw}(CCAB)$	[*T92*; *D24*]
T96	$[AB] : A \,\varepsilon\, \mathbf{ex}(B) \,.\!\equiv\!.\, [C] \,.\, \mathbf{lw}(CCAB) \,.\sim(\mathbf{lw}(ABCC))$	[*T91*; *T94*; *T95*]

Thus, *T96* shows we could define a primitive term by another (quaternary) term which is therefore primitive. This process is applicable throughout the rest of the section but will not be done for the reason cited above.

5.2 The Terms **lnk** and **Cm**

T97 $[AB] : B \,\varepsilon\, \mathbf{lnk}(\mathbf{Cm}(A)) \,.\!\supset\!.\, \sim(B \,\varepsilon\, \mathbf{ex}(A)) \,.\sim(A \,\varepsilon\, \mathbf{cntr}(B))$

PR $[AB]:\ \mathrm{Hp}(1)\ .\supset.$
$[\exists C]$.
3. $C\,\varepsilon\,\mathbf{el}(B)$.
4. $C\,\varepsilon\,\mathbf{ex}(\mathbf{Cm}(A))$. [1; *T31*]
5. $C\,\varepsilon\,\mathbf{el}(A)$. [4; *D6*; *D7*]
6. $\sim(B\,\varepsilon\,\mathbf{ex}(A))$. [2; 5; *D7*]
$[\exists D]$.
7. $D\,\varepsilon\,\mathbf{el}(B)$.
8. $D\,\varepsilon\,\mathbf{el}(\mathbf{Cm}(A))$. [1; *T31*]
9. $D\,\varepsilon\,\mathbf{ex}(A)$. [8; *T27*]
10. $\sim(D\,\varepsilon\,\mathbf{el}(A))$. [9; *T33*]
11. $\sim(A\,\varepsilon\,\mathbf{cntr}(B))$. [7; 10; *T4*; *D19*; *D1*]
$\sim(B\,\varepsilon\,\mathbf{ex}(A))\,.\sim(A\,\varepsilon\,\mathbf{cntr}(B))$. [6; 11]

T98 $[AB]:A\,\varepsilon\,\mathbf{el}(B)\ .\supset.\ \mathbf{Cm}(B)\subset\mathbf{el}(\mathbf{Cm}(A))$ [*D6*; *T63*; **BA**]

T99 $[AB]:A\,\varepsilon\,\mathbf{pr}(B)\,.\,\mathbf{Cm}(B)\,\varepsilon\,\mathbf{Cm}(B)\ .\supset.\ \sim(A\,\varepsilon\,\mathbf{lnk}(B))\,.\,B\,\varepsilon\,\mathbf{lnk}(\mathbf{Cm}(A))$

PR $[AB]:\mathrm{Hp}(2)\ .\supset.$
3. $\sim(A\,\varepsilon\,\mathbf{lnk}(B))$. [1; *T33*]
4. $A\,\varepsilon\,\mathbf{el}(B)$. [1; *D1*]
5. $A\,\varepsilon\,\mathbf{ex}(\mathbf{Cm}(A))$. [*D6*; *T24*]
6. $B\backslash A\,\varepsilon\,B\backslash A$. [1; *T45*]
7. $B\backslash A\,\varepsilon\,\mathbf{el}(B)$.
8. $B\backslash A\,\varepsilon\,\mathbf{ex}(A)$. [6; *D11*]
9. $B\backslash A\,\varepsilon\,\mathbf{el}(\mathbf{Cm}(A))$. [8; *T27*]
10. $\mathbf{Cm}(B)\,\varepsilon\,\mathbf{ex}(B)$. [2; *D6*; *T24*]
11. $\mathbf{Cm}(B)\,\varepsilon\,\mathbf{el}(\mathbf{Cm}(A))$. [4; 2; *T98*]
12. $B\,\varepsilon\,\mathbf{lnk}(\mathbf{Cm}(A))$. [4; 5; 7; 9; 10; 11; *T31*]
$\sim(A\,\varepsilon\,\mathbf{lnk}(B))\,.\,B\,\varepsilon\,\mathbf{lnk}(\mathbf{Cm}(A))$. [3; 12]

T100 $[AB]::A\,\varepsilon\,\mathbf{pr}(B)\ .\supset\therefore\ A\neq B\,.\,\mathbf{Cm}(B)\circ\wedge\ :\mathrm{v}:\ \mathbf{Cm}(B)\,\varepsilon\,\mathbf{Cm}(B)\,.$
$\sim(A\,\varepsilon\,\mathbf{lnk}(B))\,.\,B\,\varepsilon\,\mathbf{lnk}(\mathbf{Cm}(A))$ [*T87*; *T33*; *T10*]

T101 $[AB]:A\neq B\,.\,\mathbf{Cm}(B)\circ\wedge\ .\supset.\ A\,\varepsilon\,\mathbf{pr}(B)$

PR $[AB]:\mathrm{Hp}(2)\ .\supset.$
3. $B\,\varepsilon\,\mathbf{Un}$. [2; *D3*; *D6*]
4. $A\,\varepsilon\,\mathbf{el}(B)$. [1; 3; *T10*]
$A\,\varepsilon\,\mathbf{pr}(B)$. [1; 4; *D1*]

T102 $[AB]:\mathbf{Cm}(B)\,\varepsilon\,\mathbf{Cm}(B)\,.\sim(A\,\varepsilon\,\mathbf{lnk}(B))\,.\,B\,\varepsilon\,\mathbf{lnk}(\mathbf{Cm}(A))\ .\supset.\ A\,\varepsilon\,\mathbf{pr}(B)$

PR $[AB]:\mathrm{Hp}(3)\ .\supset.$
4. $A\neq B$. [3; *D8*; *D6*; **BA**]
5. $\sim(B\,\varepsilon\,\mathbf{ex}(A))$. [3; *T97*]
6. $\sim(A\,\varepsilon\,\mathbf{ex}(B))$. [5; *T25*]
7. $\sim(A\,\varepsilon\,\mathbf{cntr}(B))$. [3; *T97*]
$A\,\varepsilon\,\mathbf{pr}(B)$ [2; 4; 5; 6; 7; *T33*]

T103 $[AB]::A\,\varepsilon\,\mathbf{pr}(B)\ .\equiv\therefore\ A\neq B\,.\,\mathbf{Cm}(B)\circ\wedge\ :\mathrm{v}:\ \mathbf{Cm}(B)\,\varepsilon\,\mathbf{Cm}(B)\,.$
$\sim(A\,\varepsilon\,\mathbf{lnk}(B))\,.\,B\,\varepsilon\,\mathbf{lnk}(\mathbf{Cm}(A))$ [*T100*-*T102*]

5.3 The Terms Δ and wex

T104 $[AB]:A\,\varepsilon\,\mathbf{pr}(B)\ .\supset.\ A\neq B\,.\sim(A\,\varepsilon\,\mathbf{wex}(B))\,.\,A\,\Delta\,B\,\varepsilon\,\mathbf{wex}(B)$

PR $[AB]:\mathrm{Hp}(1)\ .\supset.$

2. $A \neq B$. [1; *T2*; *A3*]
3. $A\, \varepsilon\, \mathbf{el}(B)$. [1; *D1*]
4. $\sim(A\, \varepsilon\, \mathbf{wex}(B))$. [3; *D23*]
5. $B \backslash A\, \varepsilon\, B \backslash A$. [1; *T45*]
6. $B \backslash A = A \Delta B$. [5; *D22*; *T46*]
7. $B \backslash A\, \varepsilon\, \mathbf{ex}(A)$. [4; *D11*]
8. $B \backslash A\, \varepsilon\, \mathbf{wex}(A)$. [7; *T70*]
9. $A \Delta B\, \varepsilon\, \mathbf{wex}(A)$. [6; 8]

$A = B . \sim(A\, \varepsilon\, \mathbf{wex}(B)) . A \Delta B\, \varepsilon\, \mathbf{wex}(A)$

T105 $[AB] : A\, \varepsilon\, \mathbf{cntr}(B) .\supset. \sim((A \Delta B)\, \varepsilon\, \mathbf{wex}(A))$

PR $[AB] : \mathrm{Hp}(1) .\supset.$

2. $B\, \varepsilon\, \mathbf{pr}(A)$. [1; *D19*]
3. $A \backslash B\, \varepsilon\, A \backslash B$. [2; *T45*]
4. $A \Delta B = A \backslash B$. [3; *D11*; *D22*; *T46*]
5. $A \backslash B\, \varepsilon\, \mathbf{el}(A)$. [3; *D11*]
6. $A \Delta B\, \varepsilon\, \mathbf{el}(A)$. [4; 5]

$\sim((A \Delta B)\, \varepsilon\, \mathbf{wex}(A))$ [6; *D23*]

T106 $[AB] : A \neq B . \sim(A\, \varepsilon\, \mathbf{wex}(B)) . A \Delta B\, \varepsilon\, \mathbf{wex}(A) .\supset. A\, \varepsilon\, \mathbf{pr}(B)$

PR $[AB] : \mathrm{Hp}(3) .\supset.$

4. $\sim(A\, \varepsilon\, \mathbf{cntr}(B))$. [3; *T105*]

$$\left.\begin{array}{ll} 5. & \sim(A\, \varepsilon\, \mathbf{ex}(B)) . \\ 6. & \sim(A\, \varepsilon\, \mathbf{lnk}(B)) . \end{array}\right\} \quad [2; T70]$$

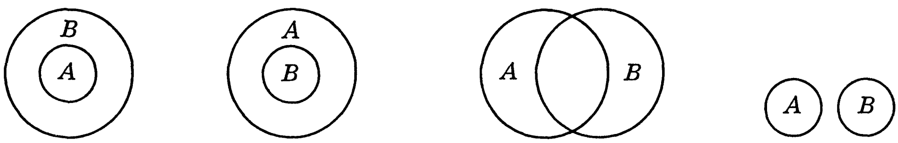
$A\, \varepsilon\, \mathbf{pr}(B)$. [1; 4; 5; 6; *T33*]

T107 $[AB] : A\, \varepsilon\, \mathbf{pr}(B) .\equiv. A \varepsilon A . \sim(A\, \varepsilon\, \mathbf{wex}(B)) . A \Delta B\, \varepsilon\, \mathbf{wex}(A) . A \neq B$. [*T104*; *T106*]

This concludes the chapter on primitive and non-primitive terms.

CHAPTER III: THE TERNARY TERMS

1 *Preliminary remarks* In Chapter II, we mentioned that some of the primitive terms defined there were actually ternary terms, i.e., they were relations on three individuals. From Theorem *T33* we concluded that the possible arrangements for two distinct objects A and B were as follows:



$A\, \varepsilon\, \mathbf{pr}(B)$ $A\, \varepsilon\, \mathbf{cntr}(B)$ $A\, \varepsilon\, \mathbf{lnk}(B)$ $A\, \varepsilon\, \mathbf{ex}(B)$

These arrangements, of course, are valid if there are three distinct individuals instead of two. We merely must state the relationship that holds between pairs. In what follows we will do just this; we will not however, express any stronger relation than that possible from *T33*, nor will we consider relations with arguments which are general names.

For example, the following arrangement is possible for three distinct individuals.

Here we have that $A\,\varepsilon\,\mathbf{pr}(B)$, $B\,\varepsilon\,\mathbf{ex}(C)$, and $A\,\varepsilon\,\mathbf{ex}(C)$. With this in mind we adopt the following notation. We will let the letters **c**, **e**, **l**, and **p** stand for **cntr**, **ex**, **lnk**, and **pr**, respectively. We will use the expression $\mathrm{xyz}(ABC)$ to mean that $A\,\varepsilon\,\mathrm{x}(B)$, $B\,\varepsilon\,\mathrm{y}(C)$, and $A\,\varepsilon\,\mathrm{z}(C)$ where x, y, and z vary over **c**, **e**, **l**, and **p**. For example, in the previous diagram we would write **pee**(ABC) to indicate that arrangement.

There are four relations possible between two distinct names and there are three distinct names being considered, so there will be sixty-four arrangements and hence, definitions. We will use the following scheme:

S1 $[ABC]:\mathrm{xyz}(ABC)\ .\!\equiv\!.\ A\,\varepsilon\,\mathrm{x}(B)\,.\,B\,\varepsilon\,\mathrm{y}(C)\,.\,A\,\varepsilon\,\mathrm{z}(C)$

Note that from this scheme, if the term is defined, A, B, and C must be distinct by *T33*. Some of these arrangements prove to be impossible and we will treat these first. For convenience we will list all sixty-four potential cases and refer to them by number. We will abbreviate the cases with just their initial letters.

C1.	**ccc**	*C17.*	**ecc**	*C33.*	**lcc**	*C49.*	**pcc**
C2.	**cce**	*C18.*	**ece**	*C34.*	**lce**	*C50.*	**pce**
C3.	**ccl**	*C19.*	**ecl**	*C35.*	**lcl**	*C51.*	**pcl**
C4.	**ccp**	*C20.*	**ecp**	*C36.*	**lcp**	*C52.*	**pcp**
C5.	**cec**	*C21.*	**eec**	*C37.*	**lec**	*C53.*	**pec**
C6.	**cee**	*C22.*	**eee**	*C38.*	**lee**	*C54.*	**pee**
C7.	**cel**	*C23.*	**eel**	*C39.*	**lel**	*C55.*	**pel**
C8.	**cep**	*C24.*	**eep**	*C40.*	**lep**	*C56.*	**pep**
C9.	**clc**	*C25.*	**elc**	*C41.*	**llc**	*C57.*	**plc**
C10.	**cle**	*C26.*	**ele**	*C42.*	**lle**	*C58.*	**ple**
C11.	**cll**	*C27.*	**ell**	*C43.*	**lll**	*C59.*	**pll**
C12.	**clp**	*C28.*	**elp**	*C44.*	**llp**	*C60.*	**plp**
C13.	**cpc**	*C29.*	**epc**	*C45.*	**lpc**	*C61.*	**ppc**
C14.	**cpe**	*C30.*	**epe**	*C46.*	**lpe**	*C62.*	**ppe**
C15.	**cpl**	*C31.*	**epl**	*C47.*	**lpl**	*C63.*	**ppl**
C16.	**cpp**	*C32.*	**epp**	*C48.*	**lpp**	*C64.*	**ppp**

2 *The impossible relations* The cases treated here are contradictory by nature of their definitions. We will give the case number, state the definition in full, and then show that such a relation is contradictory.

C2 $[ABC]:\mathbf{cce}(ABC)\ .\!\equiv\!.\ A\,\varepsilon\,\mathbf{cntr}(B)\,.\,B\,\varepsilon\,\mathbf{cntr}(C)\,.\,A\,\varepsilon\,\mathbf{ex}(C)$

T108 $[ABC]:\mathbf{cce}(ABC)\ .\!\supset\!.\ \wedge\,\varepsilon\,\wedge$

PR $[ABC]:\mathrm{Hp}(1)\ .\!\supset\!.$

$$\left.\begin{array}{ll}2. & A\,\varepsilon\,\mathbf{cntr}(B)\,.\\ 3. & B\,\varepsilon\,\mathbf{cntr}(C)\,.\\ 4. & A\,\varepsilon\,\mathbf{ex}(C)\,.\end{array}\right\}\qquad [1;\ C2]$$

5. $A \,\varepsilon\, \mathbf{cntr}(C)$. [2; 3; *D19*; *A1*]
$\wedge \,\varepsilon\, \wedge$ [4; 5; *T33*; **ON**]

C3 $[ABC]: \mathbf{ccl}(ABC) \;.\!\equiv.\; A \,\varepsilon\, \mathbf{cntr}(B) \,.\, B \,\varepsilon\, \mathbf{cntr}(C) \,.\, A \,\varepsilon\, \mathbf{lnk}(C)$

T109 $[ABC]: \mathbf{ccl}(ABC) \;.\!\supset.\; \wedge \,\varepsilon\, \wedge$
PR $[ABC]: \mathrm{Hp}(1) \;.\!\supset.$
2. $A \,\varepsilon\, \mathbf{cntr}(B)$.
3. $B \,\varepsilon\, \mathbf{cntr}(C)$. [1; *C3*]
4. $A \,\varepsilon\, \mathbf{lnk}(C)$.
5. $A \,\varepsilon\, \mathbf{cntr}(C)$. [2; 3; *D19*; *A1*]
$\wedge \,\varepsilon\, \wedge$ [4; 5; *T33*; **ON**]

C4 $[ABC]: \mathbf{ccp}(ABC) \;.\!\equiv.\; A \,\varepsilon\, \mathbf{cntr}(B) \,.\, B \,\varepsilon\, \mathbf{cntr}(C) \,.\, A \,\varepsilon\, \mathbf{pr}(C)$

T110 $[ABC]: \mathbf{ccp}(ABC) \;.\!\supset.\; \wedge \,\varepsilon\, \wedge$
PR $[ABC]: \mathrm{Hp}(1) \;.\!\supset.$
2. $A \,\varepsilon\, \mathbf{cntr}(B)$.
3. $B \,\varepsilon\, \mathbf{cntr}(C)$. [1; *C4*]
4. $A \,\varepsilon\, \mathbf{pr}(C)$.
5. $A \,\varepsilon\, \mathbf{cntr}(C)$. [2; 3; *D19*; *A1*]
$\wedge \,\varepsilon\, \wedge$ [4; 5; *T33*; **ON**]

C8 $[ABC]: \mathbf{cep}(ABC) \;.\!\equiv.\; A \,\varepsilon\, \mathbf{cntr}(B) \,.\, B \,\varepsilon\, \mathbf{ex}(C) \,.\, A \,\varepsilon\, \mathbf{pr}(C)$

T111 $[ABC]: \mathbf{cep}(ABC) \;.\!\supset.\; \wedge \,\varepsilon\, \wedge$
PR $[ABC]: \mathrm{Hp}(1) \;.\!\supset.$
2. $A \,\varepsilon\, \mathbf{cntr}(B)$.
3. $B \,\varepsilon\, \mathbf{ex}(C)$. [1; *C8*]
4. $A \,\varepsilon\, \mathbf{pr}(C)$.
5. $C \,\varepsilon\, \mathbf{cntr}(A)$. [4; *D19*]
6. $C \,\varepsilon\, \mathbf{cntr}(B)$. [2; 5; *D19*; *A1*]
7. $C \,\varepsilon\, \mathbf{ex}(B)$. [3; *T25*]
$\wedge \,\varepsilon\, \wedge$ [6; 7; *T33*; **ON**]

C10 $[ABC]: \mathbf{cle}(ABC) \;.\!\equiv.\; A \,\varepsilon\, \mathbf{cntr}(B) \,.\, B \,\varepsilon\, \mathbf{lnk}(C) \,.\, A \,\varepsilon\, \mathbf{ex}(C)$

T112 $[ABC]: \mathbf{cle}(ABC) \;.\!\supset.\; \wedge \,\varepsilon\, \wedge$
PR $[ABC]: \mathrm{Hp}(1) \;.\!\supset.$
2. $A \,\varepsilon\, \mathbf{cntr}(B)$.
3. $B \,\varepsilon\, \mathbf{lnk}(C)$. [1; *C10*]
4. $A \,\varepsilon\, \mathbf{ex}(C)$.
5. $B \,\varepsilon\, \mathbf{el}(A)$. [2; *D19*; *D1*]
$[\exists D]$.
6. $D \,\varepsilon\, \mathbf{el}(B)$.
7. $D \,\varepsilon\, \mathbf{el}(C)$. [3; *D8*]
8. $D \,\varepsilon\, \mathbf{el}(A)$. [5; 7; *T4*]
9. $\sim(A \,\varepsilon\, \mathbf{ex}(C))$. [7; 8; *D7*]
$\wedge \,\varepsilon\, \wedge$ [4; 9; **ON**]

C12 $[ABC]: \mathbf{clp}(ABC) \;.\!\equiv.\; A \,\varepsilon\, \mathbf{cntr}(B) \,.\, B \,\varepsilon\, \mathbf{lnk}(C) \,.\, A \,\varepsilon\, \mathbf{pr}(C)$.

T113 $[ABC]: \mathbf{clp}(ABC) \mathbin{.\supset.} \wedge \varepsilon \wedge$

PR $[ABC]: \mathrm{Hp}(1) \mathbin{.\supset.}$

$$\left.\begin{array}{ll} 2. & A \varepsilon \mathbf{cntr}(B) . \\ 3. & B \varepsilon \mathbf{lnk}(C) . \\ 4. & A \varepsilon \mathbf{pr}(C) . \end{array}\right\} \quad [1; C12]$$

5. $B \varepsilon \mathbf{pr}(A)$. [2; *D19*]
6. $B \varepsilon \mathbf{pr}(C)$. [4; 5; *A1*]

$\wedge \varepsilon \wedge$ [3; 6; *T33*; **ON**]

C14 $[ABC]: \mathbf{cpe}(ABC) \mathbin{.\equiv.} A \varepsilon \mathbf{cntr}(B) . B \varepsilon \mathbf{pr}(C) . A \varepsilon \mathbf{ex}(C)$

T114 $[ABC]: \mathbf{cpe}(ABC) \mathbin{.\supset.} \wedge \varepsilon \wedge$

PR $[ABC]: \mathrm{Hp}(1) \mathbin{.\supset.}$

$$\left.\begin{array}{ll} 2. & A \varepsilon \mathbf{cntr}(B) . \\ 3. & B \varepsilon \mathbf{pr}(C) . \\ 4. & A \varepsilon \mathbf{ex}(C) . \end{array}\right\} \quad [1; C14]$$

5. $B \varepsilon \mathbf{pr}(A)$. [2; *D19*]
6. $B \varepsilon \mathbf{el}(C)$. [3; *D1*]
7. $B \varepsilon \mathbf{el}(A)$. [5; *D1*]
8. $\sim (A \varepsilon \mathbf{ex}(C))$. [6; 7; *D7*]

$\wedge \varepsilon \wedge$ [4; 8; **ON**]

C17 $[ABC]: \mathbf{ecc}(ABC) \mathbin{.\equiv.} A \varepsilon \mathbf{ex}(B) . B \varepsilon \mathbf{cntr}(C) . A \varepsilon \mathbf{cntr}(C) .$

T115 $[ABC]: \mathbf{ecc}(ABC) \mathbin{.\supset.} \wedge \varepsilon \wedge$

PR $[ABC]: \mathrm{Hp}(1) \mathbin{.\supset.}$

$$\left.\begin{array}{ll} 2. & A \varepsilon \mathbf{ex}(B) . \\ 3. & B \varepsilon \mathbf{cntr}(C) . \\ 4. & A \varepsilon \mathbf{cntr}(C) . \end{array}\right\} \quad [1; C17]$$

5. $C \varepsilon \mathbf{el}(B)$. [3; *D19*; *D1*]
6. $C \varepsilon \mathbf{el}(A)$. [4; *D19*; *D1*]
7. $\sim (A \varepsilon \mathbf{ex}(B))$. [4; 5; *D7*]

$\wedge \varepsilon \wedge$ [2; 6; **ON**]

C19 $[ABC]: \mathbf{ecl}(ABC) \mathbin{.\equiv.} A \varepsilon \mathbf{ex}(B) . B \varepsilon \mathbf{cntr}(C) . A \varepsilon \mathbf{lnk}(C)$

T116 $[ABC]: \mathbf{ecl}(ABC) \mathbin{.\supset.} \wedge \varepsilon \wedge$

PR $[ABC]: \mathrm{Hp}(1) \mathbin{.\supset.}$

$$\left.\begin{array}{ll} 2. & A \varepsilon \mathbf{ex}(B) . \\ 3. & B \varepsilon \mathbf{cntr}(C) . \\ 4. & A \varepsilon \mathbf{lnk}(C) . \end{array}\right\} \quad [1; C19]$$

$[\exists D]$.

$$\left.\begin{array}{ll} 5. & D \varepsilon \mathbf{el}(A) . \\ 6. & D \varepsilon \mathbf{el}(C) . \end{array}\right\} \quad [4; D8]$$

7. $D \varepsilon \mathbf{el}(B)$. [3; 6; *D19*; *D1*; *T4*]
8. $\sim (A \varepsilon \mathbf{ex}(B))$. [5; 7; *D7*]

$\wedge \varepsilon \wedge$ [2; 8; **ON**]

C20 $[ABC]: \mathbf{ecp}(ABC) \mathbin{.\equiv.} A \varepsilon \mathbf{ex}(B) . B \varepsilon \mathbf{cntr}(C) . A \varepsilon \mathbf{pr}(C)$

T117 $[ABC]: \mathbf{ecp}(ABC) \mathbin{.\supset.} \wedge \varepsilon \wedge$

PR	$[ABC]:\mathrm{Hp}(1) .\supset.$	
2.	$A \varepsilon \mathbf{ex}(B) .$	
3.	$B \varepsilon \mathbf{cntr}(C) .$	[1; *C20*]
4.	$A \varepsilon \mathbf{pr}(C) .$	
5.	$C \varepsilon \mathbf{pr}(B) .$	[3; *D19*]
6.	$A \varepsilon \mathbf{pr}(B) .$	[4; 5; *A1*]
	$\wedge \varepsilon \wedge$	[2; 6; *T33*; **ON**]

C25 $[ABC]:\mathbf{elc}(ABC) .\equiv. A \varepsilon \mathbf{ex}(B) . B \varepsilon \mathbf{lnk}(C) . A \varepsilon \mathbf{cntr}(C)$

T118	$[ABC]:\mathbf{elc}(ABC) .\supset. \wedge \varepsilon \wedge$	
PR	$[ABC]:\mathrm{Hp}(1) .\supset.$	
2.	$A \varepsilon \mathbf{ex}(B) .$	
3.	$B \varepsilon \mathbf{lnk}(C) .$	[1; *C25*]
4.	$A \varepsilon \mathbf{cntr}(C) .$	
5.	$C \varepsilon \mathbf{el}(A) .$	[4; *D19*; *D1*]
	$[\exists D] .$	
6.	$D \varepsilon \mathbf{el}(B) .$	[3; *D8*]
7.	$D \varepsilon \mathbf{el}(C) .$	
8.	$D \varepsilon \mathbf{el}(A) .$	[5; 7; *T4*]
9.	$\sim (A \varepsilon \mathbf{ex}(B)) .$	[6; 8; *D7*]
	$\wedge \varepsilon \wedge$	[2; 9; **ON**]

C29 $[ABC]:\mathbf{epc}(ABC) .\equiv. A \varepsilon \mathbf{ex}(B) . B \varepsilon \mathbf{pr}(C) . A \varepsilon \mathbf{cntr}(C)$

T119	$[ABC]:\mathbf{epc}(ABC) .\supset. \wedge \varepsilon \wedge$	
PR	$[ABC]:\mathrm{Hp}(1) .\supset.$	
2.	$A \varepsilon \mathbf{ex}(B) .$	
3.	$B \varepsilon \mathbf{pr}(C) .$	[1; *C29*]
4.	$A \varepsilon \mathbf{cntr}(C) .$	
5.	$C \varepsilon \mathbf{pr}(A) .$	[4; *D19*]
6.	$B \varepsilon \mathbf{pr}(A) .$	[3; 5; *A1*]
7.	$A \varepsilon \mathbf{cntr}(B) .$	[6; *D19*]
	$\wedge \varepsilon \wedge$	[2; 7; *T33*; **ON**]

C36 $[ABC]:\mathbf{lcp}(ABC) .\equiv. A \varepsilon \mathbf{lnk}(B) . B \varepsilon \mathbf{cntr}(C) . A \varepsilon \mathbf{pr}(C)$

T120	$[ABC]:\mathbf{lcp}(ABC) .\supset. \wedge \varepsilon \wedge$	
PR	$[ABC]:\mathrm{Hp}(1) .\supset.$	
2.	$A \varepsilon \mathbf{lnk}(B) .$	
3.	$B \varepsilon \mathbf{cntr}(C) .$	[1; *C36*]
4.	$A \varepsilon \mathbf{pr}(C) .$	
5.	$C \varepsilon \mathbf{pr}(B) .$	[3; *D19*]
6.	$A \varepsilon \mathbf{pr}(B) .$	[4; 5; *A1*]
	$\wedge \varepsilon \wedge$	[2; 6; *T33*; **ON**]

C40 $[ABC]:\mathbf{lep}(ABC) .\equiv. A \varepsilon \mathbf{lnk}(B) . B \varepsilon \mathbf{ex}(C) . A \varepsilon \mathbf{pr}(C)$

T121	$[ABC]:\mathbf{lep}(ABC) .\supset. \wedge \varepsilon \wedge$	
PR	$[ABC]:\mathrm{Hp}(1) .\supset.$	

2.	$A \varepsilon \mathbf{lnk}(B)$.	[1; *C40*]
3.	$B \varepsilon \mathbf{lnk}(C)$.	
4.	$A \varepsilon \mathbf{pr}(C)$.	
	$[\exists D]$.	
5.	$D \varepsilon \mathbf{el}(A)$.	[2; *D8*]
6.	$D \varepsilon \mathbf{el}(B)$.	
7.	$D \varepsilon \mathbf{el}(C)$.	[4; 5; *D1*; *T4*]
8.	$\sim (B \varepsilon \mathbf{ex}(C))$.	[6; 7; *D7*]
	$\Lambda \varepsilon \Lambda$	[3; 8; **ON**]

C45 $[ABC] : \mathbf{lpc}(ABC) .\equiv. A \varepsilon \mathbf{lnk}(B) . B \varepsilon \mathbf{pr}(C) . A \varepsilon \mathbf{cntr}(C)$.

T122	$[ABC] : \mathbf{lpc}(ABC) .\supset. \Lambda \varepsilon \Lambda$	
PR	$[ABC] : \mathrm{Hp}(1) .\supset.$	
2.	$A \varepsilon \mathbf{lnk}(B)$.	[1; *C45*]
3.	$B \varepsilon \mathbf{pr}(C)$.	
4.	$A \varepsilon \mathbf{cntr}(C)$.	
5.	$C \varepsilon \mathbf{pr}(A)$.	[4; *D19*]
6.	$B \varepsilon \mathbf{pr}(A)$.	[3; 5; *A1*]
7.	$A \varepsilon \mathbf{cntr}(B)$.	[6; *D19*]
	$\Lambda \varepsilon \Lambda$	[2; 7; *T33*; **ON**]

C46 $[ABC] : \mathbf{lpe}(ABC) .\equiv. A \varepsilon \mathbf{lnk}(B) . B \varepsilon \mathbf{pr}(C) . A \varepsilon \mathbf{ex}(C)$

T123	$[ABC] : \mathbf{lpe}(ABC) .\supset. \Lambda \varepsilon \Lambda$	
PR	$[ABC] : \mathrm{Hp}(1) .\supset.$	
2.	$A \varepsilon \mathbf{lnk}(B)$.	[1; *C46*]
3.	$B \varepsilon \mathbf{pr}(C)$.	
4.	$A \varepsilon \mathbf{ex}(C)$.	
	$[\exists D]$.	
5.	$D \varepsilon \mathbf{el}(A)$.	[2; *D8*]
6.	$D \varepsilon \mathbf{el}(B)$.	
7.	$D \varepsilon \mathbf{el}(C)$.	[3; 6; *D1*; *T4*]
8.	$\sim (A \varepsilon \mathbf{ex}(C))$.	[5; 7; *D7*]
	$\Lambda \varepsilon \Lambda$	[4; 8; **ON**]

C53 $[ABC] : \mathbf{pec}(ABC) .\equiv. A \varepsilon \mathbf{pr}(B) . B \varepsilon \mathbf{ex}(C) . A \varepsilon \mathbf{cntr}(C)$

T124	$[ABC] : \mathbf{pec}(ABC) .\supset. \Lambda \varepsilon \Lambda$	
PR	$[ABC] : \mathrm{Hp}(1) .\supset.$	
2.	$A \varepsilon \mathbf{pr}(B)$.	[1; *C53*]
3.	$B \varepsilon \mathbf{ex}(C)$.	
4.	$A \varepsilon \mathbf{cntr}(C)$.	
5.	$C \varepsilon \mathbf{pr}(A)$.	[4; *D19*]
6.	$C \varepsilon \mathbf{pr}(B)$.	[2; 5; *A1*]
7.	$B \varepsilon \mathbf{cntr}(C)$.	[6; *D19*]
	$\Lambda \varepsilon \Lambda$	[3; 7; *T33*; **ON**]

C55 $[ABC] : \mathbf{pel}(ABC) .\equiv. A \varepsilon \mathbf{pr}(B) . B \varepsilon \mathbf{ex}(C) . A \varepsilon \mathbf{pr}(C)$

T125 $[ABC] : \mathbf{pel}(ABC) .\supset. \Lambda \varepsilon \Lambda$

PR $[ABC]:\mathrm{Hp}(1)\ .\supset.$

2. $A\ \varepsilon\ \mathbf{pr}(B)\ .$
3. $B\ \varepsilon\ \mathbf{ex}(C)\ .$ [1; *C55*]
4. $A\ \varepsilon\ \mathbf{lnk}(C)\ .$

$[\exists D]\ .$

5. $D\ \varepsilon\ \mathbf{el}(A)\ .$ [4; *D8*]
6. $D\ \varepsilon\ \mathbf{el}(C)\ .$
7. $D\ \varepsilon\ \mathbf{el}(B)\ .$ [2; 5; *D1*; *T4*]
8. $\sim(B\ \varepsilon\ \mathbf{ex}(C))\ .$ [6; 7; *D7*]

$\wedge\ \varepsilon\ \wedge$ [3; 8; **ON**]

C56 $[ABC]:\mathbf{pep}(ABC)\ .\equiv.\ A\ \varepsilon\ \mathbf{pr}(B)\ .\ B\ \varepsilon\ \mathbf{ex}(C)\ .\ A\ \varepsilon\ \mathbf{pr}(C)$

T126 $[ABC]:\mathbf{pep}(ABC)\ .\supset.\ \wedge\ \varepsilon\ \wedge$

PR $[ABC]:\mathrm{Hp}(1)\ .\supset.$

2. $A\ \varepsilon\ \mathbf{pr}(B)\ .$
3. $B\ \varepsilon\ \mathbf{ex}(C)\ .$ [1; *C56*]
4. $A\ \varepsilon\ \mathbf{pr}(C)\ .$
5. $A\ \varepsilon\ \mathbf{el}(B)\ .$ [2; *D1*]
6. $A\ \varepsilon\ \mathbf{el}(C)\ .$ [4; *D1*]
7. $\sim(B\ \varepsilon\ \mathbf{ex}(C))\ .$ [5; 6; *D7*]

$\wedge\ \varepsilon\ \wedge$ [3; 7; **ON**]

C57 $[ABC]:\mathbf{plc}(ABC)\ .\equiv.\ A\ \varepsilon\ \mathbf{pr}(B)\ .\ B\ \varepsilon\ \mathbf{lnk}(C)\ .\ A\ \varepsilon\ \mathbf{cntr}(C)$

T127 $[ABC]:\mathbf{plc}(ABC)\ .\supset.\ \wedge\ \varepsilon\ \wedge$

PR $[ABC]:\mathrm{Hp}(1)\ .\supset.$

2. $A\ \varepsilon\ \mathbf{pr}(B)\ .$
3. $B\ \varepsilon\ \mathbf{lnk}(C)\ .$ [1; *C57*]
4. $A\ \varepsilon\ \mathbf{cntr}(C)\ .$
5. $C\ \varepsilon\ \mathbf{pr}(A)\ .$ [4; *D19*]
6. $C\ \varepsilon\ \mathbf{pr}(B)\ .$ [2; 5; *A1*]
7. $B\ \varepsilon\ \mathbf{cntr}(C)\ .$ [6; *D19*]

$\wedge\ \varepsilon\ \wedge$ [3; 7; *T33*; **ON**]

C61 $[ABC]:\mathbf{ppc}(ABC)\ .\equiv.\ A\ \varepsilon\ \mathbf{pr}(B)\ .\ B\ \varepsilon\ \mathbf{pr}(C)\ .\ A\ \varepsilon\ \mathbf{cntr}(C)$

T128 $[ABC]:\mathbf{ppc}(ABC)\ .\supset.\ \wedge\ \varepsilon\ \wedge$

PR $[ABC]:\mathrm{Hp}(1)\ .\supset.$

2. $A\ \varepsilon\ \mathbf{pr}(B)\ .$
3. $B\ \varepsilon\ \mathbf{pr}(C)\ .$ [1; *C61*]
4. $A\ \varepsilon\ \mathbf{cntr}(C)\ .$
5. $A\ \varepsilon\ \mathbf{pr}(C)\ .$ [2; 3; *A1*]

$\wedge\ \varepsilon\ \wedge$ [4; 5; *T33*; **ON**]

C62 $[ABC]:\mathbf{ppe}(ABC)\ .\equiv.\ A\ \varepsilon\ \mathbf{pr}(B)\ .\ B\ \varepsilon\ \mathbf{pr}(C)\ .\ A\ \varepsilon\ \mathbf{ex}(C)$

T129 $[ABC]:\mathbf{ppe}(ABC)\ .\supset.\ \wedge\ \varepsilon\ \wedge$

PR $[ABC]:\mathrm{Hp}(1)\ .\supset.$

2. $A\ \varepsilon\ \mathbf{pr}(B)\ .$
3. $B\ \varepsilon\ \mathbf{pr}(C)\ .$ [1; *C62*]
4. $A\ \varepsilon\ \mathbf{ex}(C)\ .$

5.	$A\,\varepsilon\,\mathbf{pr}(C)$.	[2; 3; *A1*]
	$\wedge\,\varepsilon\,\wedge$	[4; 5; *T33*; **ON**]

C63 $[ABC]:\mathbf{ppl}(ABC) \mathrel{.{\equiv}.} A\,\varepsilon\,\mathbf{pr}(B)\,.\,B\,\varepsilon\,\mathbf{pr}(C)\,.\,A\,\varepsilon\,\mathbf{lnk}(C)$

T130	$[ABC]:\mathbf{ppl}(ABC) \mathrel{.{\supset}.} \wedge\,\varepsilon\,\wedge$	
PR	$[ABC]:\mathrm{Hp}(1) \mathrel{.{\supset}.}$	
2.	$A\,\varepsilon\,\mathbf{pr}(B)$.	
3.	$B\,\varepsilon\,\mathbf{pr}(C)$.	[1; *C63*]
4.	$A\,\varepsilon\,\mathbf{lnk}(C)$.	
5.	$A\,\varepsilon\,\mathbf{pr}(C)$.	[2; 3; *A1*]
	$\wedge\,\varepsilon\,\wedge$	[4; 5; *T33*; **ON**]

These theorems show that defining the above terms leads to a contradiction, therefore, the relations are impossible and cannot serve as primitive terms.

3 *The primitive ternary relations* In this section we will consider those cases in which the ternary relations are primitive. In some cases there will be certain restrictions on the number of names. We will begin each section with a list of terms which are equivalent and then prove that just one is primitive. From this we conclude that all are primitive.

3.1 The Term $\mathbf{ppp}(ABC)$ is primitive

C64 $[ABC]:\mathbf{ppp}(ABC) \mathrel{.{\equiv}.} A\,\varepsilon\,\mathbf{pr}(B)\,.\,B\,\varepsilon\,\mathbf{pr}(C)\,.\,A\,\varepsilon\,\mathbf{pr}(C)$
C1 $[ABC]:\mathbf{ccc}(ABC) \mathrel{.{\equiv}.} A\,\varepsilon\,\mathbf{cntr}(B)\,.\,B\,\varepsilon\,\mathbf{cntr}(C)\,.\,A\,\varepsilon\,\mathbf{cntr}(C)$
C13 $[ABC]:\mathbf{cpc}(ABC) \mathrel{.{\equiv}.} A\,\varepsilon\,\mathbf{cntr}(B)\,.\,B\,\varepsilon\,\mathbf{pr}(C)\,.\,A\,\varepsilon\,\mathbf{pr}(C)$
C16 $[ABC]:\mathbf{cpp}(ABC) \mathrel{.{\equiv}.} A\,\varepsilon\,\mathbf{cntr}(B)\,.\,B\,\varepsilon\,\mathbf{pr}(C)\,.\,A\,\varepsilon\,\mathbf{pr}(C)$
C49 $[ABC]:\mathbf{pcc}(ABC) \mathrel{.{\equiv}.} A\,\varepsilon\,\mathbf{pr}(B)\,.\,B\,\varepsilon\,\mathbf{cntr}(C)\,.\,A\,\varepsilon\,\mathbf{cntr}(C)$
C52 $[ABC]:\mathbf{pcp}(ABC) \mathrel{.{\equiv}.} A\,\varepsilon\,\mathbf{pr}(B)\,.\,B\,\varepsilon\,\mathbf{cntr}(C)\,.\,A\,\varepsilon\,\mathbf{pr}(C)$

T131	$[ABC]:\mathbf{ppp}(ABC) \mathrel{.{\equiv}.} \mathbf{ccc}(CBA)$	[*D19*; *C64*; *C1*]
T132	$[ABC]:\mathbf{ppp}(ABC) \mathrel{.{\equiv}.} \mathbf{cpc}(CAB)$	[*D19*; *C64*; *C13*]
T133	$[ABC]:\mathbf{ppp}(ABC) \mathrel{.{\equiv}.} \mathbf{cpp}(BAC)$	[*D19*; *C64*; *C16*]
T134	$[ABC]:\mathbf{ppp}(ABC) \mathrel{.{\equiv}.} \mathbf{pcc}(BCA)$	[*D19*; *C64*; *C49*]
T135	$[ABC]:\mathbf{ppp}(ABC) \mathrel{.{\equiv}.} \mathbf{pcp}(ACB)$	[*D19*; *C64*; *C52*]

Thus, we may consider the above cases as one.

T136	$[ABC]\therefore \mathbf{ppp}(ABC)\,.\mathrm{v}.\,\mathbf{ppp}(ACB)\,.\mathrm{v}.\,\mathbf{ppp}(CAB) \mathrel{:{\supset}.} A\,\varepsilon\,\mathbf{pr}(B)$	[*C64*]
T137	$[AB]:A\,\varepsilon\,\mathbf{pr}(B)\,.\sim(B\,\varepsilon\,\mathbf{Un}) \mathrel{.{\supset}.} [\exists C]\,.\,\mathbf{ppp}(ABC)$	
PR	$[AB]:\mathrm{Hp}(2) \mathrel{.{\supset}.}$	
3.	$B\,\varepsilon\,\mathbf{pr}(\mathbf{Un})$.	[*A3*; 1; *T10*; 2; *D1*]
4.	$A\,\varepsilon\,\mathbf{pr}(\mathbf{Un})$.	[*A1*; 1; 3]
	$[\exists C]\,.\,\mathbf{ppp}(ABC)$	[1; 2; 3; *C64*]
T138	$[AB]:A\,\varepsilon\,\mathbf{pr}(B)\,.\,B\,\varepsilon\,\mathbf{Un}\,.\sim(A\,\varepsilon\,\mathbf{atm}) \mathrel{.{\supset}.} [\exists C]\,.\,\mathbf{ppp}(CAB)$	
PR	$[AB]:\mathrm{Hp}(3) \mathrel{.{\supset}.}$	
	$[\exists C]$.	
4.	$C\,\varepsilon\,\mathbf{pr}(A)$.	[1; 3; *D9*]
5.	$C\,\varepsilon\,\mathbf{pr}(B)$.	[*A1*; 4; 1]
	$[\exists C]\,.\,\mathbf{ppp}(CAB)$	[1; 4; 5; *C64*]

T139 $[AB]: A \varepsilon \mathbf{pr}(B) . B \varepsilon \mathbf{Un} . A \varepsilon \mathbf{atm} . \mathrm{Cd}\{\vee\} > 3 .\supset. [\exists C]: \mathbf{ppp}(ACB)$
PR $[AB]: \mathrm{Hp}(4) .\supset.$
5. $B\backslash A \varepsilon B\backslash A .$ [1; *T45*]
6. $B\backslash A \varepsilon \mathbf{ex}(A) .$ [5; *D11*]
7. $(B\backslash A) \wedge A \circ \wedge .$ [6; *T24*]
8. $(B\backslash A) \vee A = B .$ [*D11*; *D4*]
9. $(B\backslash A) \vee A \varepsilon \mathbf{Un} .$ [8; 2]
10. $\sim (B\backslash A \varepsilon \mathbf{atm}) .$ [9; 3; 4]
$[\exists D] .$
11. $D \varepsilon \mathbf{pr}(B\backslash A) .$ [10; 5; *D9*]
12. $D \vee A \varepsilon \mathbf{pr}((B\backslash A) \vee A) .$ [11; **BA**]
13. $D \vee A \varepsilon \mathbf{pr}(\mathbf{Un}) .$ [12; 9]
14. $A \varepsilon \mathbf{pr}(D \vee A) .$ [3; *T19*; 1]
$[\exists C] . \mathbf{ppp}(ACB)$ [1; 2; 13; 14]
T140 $[AB] \therefore \mathrm{Cd}\{\vee\} > 3 . A \varepsilon \mathbf{pr}(B) .\supset: [\exists C]: \mathbf{ppp}(ABC) .\vee. \mathbf{ppp}(ACB) .\vee. \mathbf{ppp}(CAB) .$ [*T137*-*T139*]
T141 $[AB] \therefore \mathrm{Cd}\{\vee\} > 3 .\supset: A \varepsilon \mathbf{pr}(B) .\equiv: [\exists C]: \mathbf{ppp}(ABC) .\vee. \mathbf{ppp}(ACB) .\vee. \mathbf{ppp}(CAB)$ [*T136*; *T140*]

Therefore, under the slight restriction that there are more than three individuals we have that $\mathbf{ppp}(ABC)$ is primitive. Also, by *T131*-*T135* we have that these terms too, are primitive.

3.2 The Term pee(ABC) is primitive

C54 $[ABC]: \mathbf{pee}(ABC) .\equiv. A \varepsilon \mathbf{pr}(B) . B \varepsilon \mathbf{ex}(C) . A \varepsilon \mathbf{ex}(C)$
C6 $[ABC]: \mathbf{cee}(ABC) .\equiv. A \varepsilon \mathbf{cntr}(B) . B \varepsilon \mathbf{ex}(C) . A \varepsilon \mathbf{ex}(C)$
C18 $[ABC]: \mathbf{ece}(ABC) .\equiv. A \varepsilon \mathbf{ex}(B) . B \varepsilon \mathbf{cntr}(C) . A \varepsilon \mathbf{ex}(C)$
C21 $[ABC]: \mathbf{eec}(ABC) .\equiv. A \varepsilon \mathbf{ex}(B) . B \varepsilon \mathbf{ex}(C) . A \varepsilon \mathbf{cntr}(C)$
C24 $[ABC]: \mathbf{eep}(ABC) .\equiv. A \varepsilon \mathbf{ex}(B) . B \varepsilon \mathbf{ex}(C) . A \varepsilon \mathbf{pr}(C)$
C30 $[ABC]: \mathbf{epe}(ABC) .\equiv. A \varepsilon \mathbf{ex}(B) . B \varepsilon \mathbf{pr}(C) . A \varepsilon \mathbf{ex}(C)$

T142 $[ABC]: \mathbf{pee}(ABC) .\equiv. \mathbf{cee}(BAC)$ [*C6*; *C54*]
T143 $[ABC]: \mathbf{pee}(ABC) .\equiv. \mathbf{ece}(CBA)$ [*C18*; *C54*]
T144 $[ABC]: \mathbf{pee}(ABC) .\equiv. \mathbf{eec}(BCA)$ [*C21*; *C54*]
T145 $[ABC]: \mathbf{pee}(ABC) .\equiv. \mathbf{eep}(ACB)$ [*C24*; *C54*]
T146 $[ABC]: \mathbf{pee}(ABC) .\equiv. \mathbf{epe}(CAB)$ [*C30*; *C54*]
T147 $[ABC] \therefore \mathbf{pee}(ACB) .\vee. \mathbf{pee}(CAB) .\vee. \mathbf{pee}(CBA) :\supset. A \varepsilon \mathbf{ex}(B)$ [*C54*]
T148 $[AB]: A \varepsilon \mathbf{ex}(B) . \sim (A \varepsilon \mathbf{Cm}(B)) .\supset. [\exists C] . \mathbf{pee}(ACB)$
PR $[AB]: \mathrm{Hp}(2) .\supset.$
3. $A \varepsilon \mathbf{pr}(\mathbf{Cm}(B)) .$ [1; *T27*; 2; *D1*]
4. $\mathbf{Cm}(B) \varepsilon \mathbf{ex}(B) .$ [3; *D6*; *T24*]
$[\exists C] . \mathbf{pee}(ACB)$ [1; 3; 4; *C54*]
T148.1 $[ABCD]: A \varepsilon \mathbf{Cm}(B) . C \varepsilon \mathbf{ex}(A) . D \varepsilon \mathbf{el}(C) .\supset. [\exists F] . F \varepsilon \mathbf{el}(D) . F \varepsilon \mathbf{el}(B)$
PR $[ABCD] :: \mathrm{Hp}(3) .\supset \therefore$
4. $A \vee B = \mathbf{Un} .$ [1; *D6*; *T10.1*]
5. $C \varepsilon \mathbf{el}(A \vee B) .$ [2; 4; *T10*; **ON**]
6. $\rightarrow \{A\} .$ [1; **ON**]
7. $\rightarrow \{B\} .$ [1; *T23*; **ON**]

8. $A \mathbf{v} B \circ \mathbf{Kl}(\mathbf{el}(A) \cup \mathbf{el}(B))$. [6; 7; *T14*]
9. $C \varepsilon \mathbf{el}(\mathbf{Kl}(\mathbf{el}(A) \cup \mathbf{el}(B)))$. [5; 8; **ON**]
10. $D \varepsilon \mathbf{el}(\mathbf{Kl}(\mathbf{el}(A) \cup \mathbf{el}(B)))$. [3; 9; *T4*]
11. $\mathbf{Kl}(\mathbf{el}(A) \cup \mathbf{el}(B)) \varepsilon \mathbf{Kl}(\mathbf{el}(A) \cup \mathbf{el}(B))$ ∴ [10; *T3.1*]
$[\exists EF]$ ∴
12. $E \varepsilon \mathbf{el}(A) \cup \mathbf{el}(B)$.
13. $F \varepsilon \mathbf{el}(E)$. [11; 10; *D2*]
14. $F \varepsilon \mathbf{el}(D)$.
15. $F \varepsilon \mathbf{el}(C)$. [14; 3; *T4*]
16. $\sim(F \varepsilon \mathbf{el}(A))$: [2; 15; *D7*]
17. $F \varepsilon \mathbf{el}(A)$ .v. $F \varepsilon \mathbf{el}(B)$: [13; 12; *T4*; **ON**]
18. $F \varepsilon \mathbf{el}(B)$ ∴ [16; 17]
$[\exists F]$. $F \varepsilon \mathbf{el}(D)$. $F \varepsilon \mathbf{el}(B)$ [14; 18]

T148.2 $[ABC]$ ∴ $A \varepsilon \mathbf{Cm}(B)$: $[C]$: $C \varepsilon C$.⊃. $\sim(C \varepsilon \mathbf{pr}(A))$. $\sim(C \varepsilon \mathbf{pr}(B))$:
$C \varepsilon \mathbf{ex}(A)$.⊃. $C = B$
PR $[ABC]$ ∴ Hp(3) :⊃:
4. $C \varepsilon C$. [3; **ON**]
5. $\sim(C \varepsilon \mathbf{pr}(B))$: [2; 4]
6. $[D]$: $D \varepsilon \mathbf{el}(C)$.⊃. $[\exists F]$. $F \varepsilon \mathbf{el}(D)$. $D \varepsilon \mathbf{el}(B)$: [1; 2; *T148.1*]
7. $C \varepsilon \mathbf{el}(B)$. [4; 6; *T6.1*]
$C = B$ [7; 5; *D1*]

T148.3 $[ABC]$ ∴ $[C]$: $C \varepsilon C$.⊃. $\sim(C \varepsilon \mathbf{pr}(A))$. $\sim(C \varepsilon \mathbf{pr}(B))$: $A \varepsilon \mathbf{pr}(C)$.
$C \varepsilon \mathbf{el}(\mathbf{Kl}(\mathbf{el}(A) \cup \mathbf{el}(B)))$. $\sim(B \varepsilon \mathbf{el}(C))$. $D \varepsilon \mathbf{el}(C)$.⊃.
$[\exists F]$. $F \varepsilon \mathbf{el}(D)$. $F \varepsilon \mathbf{el}(A)$
PR $[ABC]$ :: Hp(5) :⊃∴
6. $D \varepsilon \mathbf{el}(\mathbf{Kl}(\mathbf{el}(A) \cup \mathbf{el}(B)))$. [5; 3; *T4*]
7. $\mathbf{Kl}(\mathbf{el}(A) \cup \mathbf{el}(B)) \varepsilon \mathbf{Kl}(\mathbf{el}(A) \cup \mathbf{el}(B))$. [6; *T3.1*]
$[\exists EF]$ ∴
8. $E \varepsilon \mathbf{el}(A) \cup \mathbf{el}(B)$.
9. $F \varepsilon \mathbf{el}(E)$. [7; 6; *D2*]
10. $F \varepsilon \mathbf{el}(D)$.
11. $\sim(F \varepsilon \mathbf{pr}(B))$: [1; 10; **ON**]
12. $F \varepsilon \mathbf{el}(A)$ .v. $F = B$: [9; 8; *T4*; 11; *D1*; **ON**]
13. $F \varepsilon \mathbf{el}(A)$ .v. $B \varepsilon \mathbf{el}(C)$: [12; 10; 5; *T4*]
14. $F \varepsilon \mathbf{el}(A)$ ∴ [13; 4]
$[\exists F]$. $F \varepsilon \mathbf{el}(D)$. $F \varepsilon \mathbf{el}(A)$ [10; 14]

T148.4 $[ABC]$ ∴ $[C]$: $C \varepsilon C$.⊃. $\sim(C \varepsilon \mathbf{pr}(A))$. $\sim(C \varepsilon \mathbf{pr}(B))$: $A \varepsilon \mathbf{pr}(C)$.
$C \varepsilon \mathbf{el}(\mathbf{Kl}(\mathbf{el}(A) \cup \mathbf{el}(B)))$. $\sim(B \varepsilon \mathbf{el}(C))$.⊃. $B \varepsilon \mathbf{el}(C)$
PR $[ABC]$ ∴ Hp(4) :⊃:
5. $C \varepsilon C$: [3; **ON**]
6. $[D]$: $D \varepsilon \mathbf{el}(C)$.⊃. $[\exists F]$. $F \varepsilon \mathbf{el}(D)$. $F \varepsilon \mathbf{el}(A)$: [1; 2; 3; 4; *T148.3*]
7. $C \varepsilon \mathbf{el}(A)$. [5; 6; *T6.1*]
8. $A = C$. [2; 7; *D1*; *T6*]
$B \varepsilon \mathbf{el}(C)$ [2; 8; *T2*]

T148.5 $[ABC]$ ∴ $A \varepsilon \mathbf{Cm}(B)$: $[C]$: $C \varepsilon C$.⊃. $\sim(C \varepsilon \mathbf{pr}(A))$. $\sim(C \varepsilon \mathbf{pr}(B))$:
$A \varepsilon \mathbf{pr}(C)$.⊃. $C = \mathbf{Un}$
PR $[ABC]$ ∴ Hp(3) :⊃:

4.	$C \varepsilon C$.	[3; *A3*]
5.	$\rightarrow \{A\}$.	[1; **ON**]
6.	$\rightarrow \{B\}$.	[1; *T23*; **ON**]
7.	$A \mathbf{v} B = \mathbf{Un}$.	[1; *D6*; *T10.1*]
8.	$A \mathbf{v} B \circ \mathbf{Kl}(\mathbf{el}(A) \cup \mathbf{el}(B))$.	[6; 7; *T14*]
9.	$C \varepsilon \mathbf{el}(A \mathbf{v} B)$.	[4; 7; *T10*; **ON**]
10.	$C \varepsilon \mathbf{el}(\mathbf{Kl}(\mathbf{el}(A) \cup \mathbf{el}(B)))$.	[8; 9; **ON**]
11.	$\mathbf{Kl}(\mathbf{el}(A) \cup \mathbf{el}(B)) \varepsilon \mathbf{Kl}(\mathbf{el}(A) \cup \mathbf{el}(B))$:	[10; *T3.1*]
12.	$[D] : D \varepsilon \mathbf{el}(A) \cup \mathbf{el}(B) \ .\supset. \ D \varepsilon \mathbf{el}(C)$:	[3; *D1*; *T148.4*; *T4*]
13.	$[D] : D \varepsilon \mathbf{el}(C) \ .\supset. \ [\exists EF] . E \varepsilon \mathbf{el}(A) \cup \mathbf{el}(B) . F \varepsilon \mathbf{el}(E) . F \varepsilon \mathbf{el}(D)$:	[10; 11; *D1*; *D2*]
14.	$C \varepsilon \mathbf{Kl}(\mathbf{el}(A) \cup \mathbf{el}(B))$.	[4; 12; 13; *D2*]
15.	$C \varepsilon A \mathbf{v} B$.	[8; 15; **ON**]
	$C = \mathbf{Un}$	[7; 15; *T10.1*]

T148.6 $[ABC] \therefore [C] : C \varepsilon C \ .\supset. \ \sim(C \varepsilon \mathbf{pr}(A)) . \sim(C \varepsilon \mathbf{pr}(B)) : C \varepsilon \mathbf{Ink}(A) \ .\supset.$ $\sim(C \varepsilon \mathbf{Ink}(A))$

PR	$[ABC] : \mathrm{Hp}(2) \ .\supset.$	
	$[\exists D]$.	
3.	$D \varepsilon \mathbf{el}(A)$.	[2; *D8*]
4.	$D \varepsilon \mathbf{ex}(C)$.	[2; *D8*]
5.	$\sim(D \varepsilon \mathbf{pr}(A))$.	[3; 1; **ON**]
6.	$D = A$.	[3; 5; *D1*]
7.	$A \varepsilon \mathbf{ex}(C)$.	[4; 6]
	$[\exists E]$.	
8.	$E \varepsilon \mathbf{el}(C)$.	[2; *D8*]
9.	$E \varepsilon \mathbf{el}(A)$.	[2; *D8*]
10.	$\sim(E \varepsilon \mathbf{el}(C)$.	[7; 9; *D7*]
	$\sim(C \varepsilon \mathbf{Ink}(A))$	[8; 10]

T148.7 $[ABC] \therefore A \varepsilon \mathbf{Cm}(B) : [C] : C \varepsilon C \ .\supset. \ \sim(C \varepsilon \mathbf{pr}(A)) . \sim(C \varepsilon \mathbf{pr}(B)) : C \varepsilon C \ .\supset:$ $C = A . \mathrm{v} . C = B . \mathrm{v} . C = \mathbf{Un}$

PR	$[ABC] \therefore \mathrm{Hp}(3) \ :\supset:$	
4.	$\sim(C \varepsilon \mathbf{pr}(A))$	[2; 3]
5.	$A \varepsilon A$:	[1; **ON**]
6.	$C = A . \mathrm{v} . C \varepsilon \mathbf{pr}(A) . \mathrm{v} . C \varepsilon \mathbf{ex}(A) . \mathrm{v} . A \varepsilon \mathbf{pr}(C) . \mathrm{v} . C \varepsilon \mathbf{Ink}(A)$:	[3; 5; *T33*]
	$C = A . \mathrm{v} . C = B . \mathrm{v} . C = \mathbf{Un}$	[6; 4; *T148.2*; *T148.5*; *T148.6*]

T149 $[AB] \therefore A \varepsilon \mathbf{Cm}(B) : [C] : C \varepsilon C \ .\supset. \ \sim(C \varepsilon \mathbf{pr}(A)) . \sim(C \varepsilon \mathbf{pr}(B)) \ :\supset.$ $\mathrm{Cd}\{\mathrm{V}\} \leqslant 3$

PR	$[AB] :: \mathrm{Hp}(2) \ :\supset\therefore$	
3.	$[C] \therefore C \varepsilon C \ .\supset: C = A . \mathrm{v} . C = B . \mathrm{v} . C = \mathbf{Un}$:	[1; 2; *T148.7*]
	$\mathrm{Cd}\{\mathrm{V}\} \leqslant 3$	[3; **ON**]

T150 $[ABC] : A \varepsilon \mathbf{Cm}(B) . C \varepsilon \mathbf{pr}(A) \ .\supset \mathbf{pee}(CAB)$

PR	$[ABC] : \mathrm{Hp}(2) \ .\supset.$	
3.	$A \varepsilon \mathbf{ex}(B)$.	[1; *T23.1*]
4.	$C \varepsilon \mathbf{ex}(B)$.	[1; 2; *T23.2*]
	$\mathbf{pee}(CAB)$	[2; 3; 4; *C54*]

T151 $[ABC] : A \varepsilon \mathbf{Cm}(B) . C \varepsilon \mathbf{pr}(B) \ .\supset \mathbf{pee}(CBA)$ [*T150*; *T23*]

T152 $[AB] \therefore A \varepsilon \mathbf{Cm}(B) . \mathrm{Cd}\{\mathrm{V}\} > 3 \ .\supset: [\exists C] : \mathbf{pee}(CAB) . \mathrm{v} . \mathbf{pee}(CBA)$

PR	$[AB] \therefore \mathrm{Hp}(2) \;.\supset:$	
3.	$\sim(\mathsf{Cd}\{\mathsf{V}\} \leqslant 3)\,.$	[2; **ON**]
	$[\exists C]:$	
4.	$\quad C\,\varepsilon\,\mathbf{pr}(A)\,.\mathrm{v}.\,C\,\varepsilon\,\mathbf{pr}(B):$	[3; *T149*]
	$[\exists C]:\mathbf{pee}(CAB)\,.\mathrm{v}.\,\mathbf{pee}(CBA)$	[4; 1; *T150*; *T151*]
T153	$[AB]::\mathsf{Cd}\{\mathsf{V}\} > 3 \;.\supset\therefore\; A\,\varepsilon\,\mathbf{ex}(B) \;.\equiv:\; [\exists C]:\mathbf{pee}(ACB)\,.\mathrm{v}.$ $\mathbf{pee}(CAB)\,.\mathrm{v}.\,\mathbf{pee}(CBA)$	[*T148*; *T152*; *T147*]

Once again under the assumption that there are more than three objects, we see that **pee**(ABC) is primitive. So too for those terms proved equivalent in *T142-T146*.

3.3 The Term **ple** is primitive

C58	$[ABC]:\mathbf{ple}(ABC) \;.\equiv.\; A\,\varepsilon\,\mathbf{pr}(B)\,.\,B\,\varepsilon\,\mathbf{lnk}(C)\,.\,A\,\varepsilon\,\mathbf{ex}(C)$	
C7	$[ABC]:\mathbf{cel}(ABC) \;.\equiv.\; A\,\varepsilon\,\mathbf{cntr}(B)\,.\,B\,\varepsilon\,\mathbf{ex}(C)\,.\,A\,\varepsilon\,\mathbf{lnk}(C)$	
C28	$[ABC]:\mathbf{elp}(ABC) \;.\equiv.\; A\,\varepsilon\,\mathbf{ex}(B)\,.\,B\,\varepsilon\,\mathbf{lnk}(C)\,.\,A\,\varepsilon\,\mathbf{pr}(C)$	
C31	$[ABC]:\mathbf{epl}(ABC) \;.\equiv.\; A\,\varepsilon\,\mathbf{ex}(B)\,.\,B\,\varepsilon\,\mathbf{pr}(C)\,.\,A\,\varepsilon\,\mathbf{lnk}(C)$	
C34	$[ABC]:\mathbf{lce}(ABC) \;.\equiv.\; A\,\varepsilon\,\mathbf{lnk}(B)\,.\,B\,\varepsilon\,\mathbf{cntr}(C)\,.\,A\,\varepsilon\,\mathbf{ex}(C)$	
C37	$[ABC]:\mathbf{lec}(ABC) \;.\equiv.\; A\,\varepsilon\,\mathbf{lnk}(B)\,.\,B\,\varepsilon\,\mathbf{ex}(C)\,.\,A\,\varepsilon\,\mathbf{cntr}(C)$	
T154	$[ABC]:\mathbf{ple}(ABC) \;.\equiv.\; \mathbf{cel}(BAC)$	[*C58*; *C7*]
T155	$[ABC]:\mathbf{ple}(ABC) \;.\equiv.\; \mathbf{elp}(ACB)$	[*C58*; *C28*]
T156	$[ABC]:\mathbf{ple}(ABC) \;.\equiv.\; \mathbf{epl}(CAB)$	[*C58*; *C31*]
T157	$[ABC]:\mathbf{ple}(ABC) \;.\equiv.\; \mathbf{lce}(CBA)$	[*C58*; *C34*]
T158	$[ABC]:\mathbf{ple}(ABC) \;.\equiv.\; \mathbf{lec}(BCA)$	[*C58*; *C37*]
T159	$[ABCD]:\mathbf{ple}(ABD)\,.\,\mathbf{ple}(CDB) \;.\supset.\; A\,\varepsilon\,\mathbf{ex}(C)$	
PR	$[ABCD]:\mathrm{Hp}(2) \;.\supset.$	
3.	$A\,\varepsilon\,\mathbf{pr}(B)\,.$	[1; *C58*]
4.	$C\,\varepsilon\,\mathbf{ex}(B)\,.$	[2; *C58*]
5.	$B\,\varepsilon\,\mathbf{ex}(C)\,.$	[4; *T25*]
	$A\,\varepsilon\,\mathbf{ex}(C)$	[3; 5; **BA**]
T160	$[ABCD]\therefore\mathbf{ple}(ABC)\,.\mathrm{v}.\,\mathbf{ple}(CBA)\,.\mathrm{v}:\mathbf{ple}(ABD)\,.\,\mathbf{ple}(CDB) \;:\supset.\; A\,\varepsilon\,\mathbf{ex}(C)$	[*C58*; *T25*; *T159*]
T161	$[AC]:A\,\varepsilon\,\mathbf{ex}(C)\,.\sim(A\,\varepsilon\,\mathbf{atm}) \;.\supset.\; [\exists B]\,.\,\mathbf{ple}(CBA)$	
PR	$[AC]:\mathrm{Hp}(2) \;.\supset.$	
3.	$C\,\varepsilon\,\mathbf{ex}(A)\,.$	[1; *T25*]
	$[\exists E]\,.$	
4.	$\quad E\,\varepsilon\,\mathbf{pr}(A)\,.$	[1; 2; *D9*]
5.	$\quad A\backslash E\,\varepsilon\,A\backslash E\,.$	[4; *T45*]
6.	$\quad A\backslash E\,\varepsilon\,\mathbf{el}(A)\,.$	[4; 5; *D11*]
7.	$\quad E\,\varepsilon\,\mathbf{el}(A)\,.$	[4; *D1*]
8.	$\quad A\backslash E\,\varepsilon\,\mathbf{ex}(C)\,.$	[1; 6; **BA**]
9.	$\quad E\,\mathbf{v}\,C\,\varepsilon\,E\,\mathbf{v}\,C\,.$	[3; 4; *D4*]
10.	$\quad E\,\varepsilon\,\mathbf{el}(E\,\mathbf{v}\,C)\,.$	[4; 9; *T20*]
11.	$\quad C\,\varepsilon\,\mathbf{el}(E\,\mathbf{v}\,C)\,.$	[1; 9; *T21*]
12.	$\quad C\,\varepsilon\,\mathbf{pr}(E\,\mathbf{v}\,C)\,.$	[11; 1; 4; *D1*]
13.	$\quad A\backslash E\,\varepsilon\,\mathbf{ex}(E)\,.$	[5; *D11*]
14.	$\quad A\backslash E\,\varepsilon\,\mathbf{ex}(E\,\mathbf{v}\,C)\,.$	[8; 13; *T30*]

15.	$E\,\mathbf{v}\,C\ \varepsilon\ \mathbf{lnk}(A)$.	[3; 11; 7; 10; 6; 14; *T31*]
	$[\exists B]\,.\,\mathbf{ple}(CBA)$	[12; 15; 3; *C58*]
T162	$[AC]:A\ \varepsilon\ \mathbf{ex}(C)\,.\sim(C\ \varepsilon\ \mathbf{atm})\ .\supset.\ [\exists B]\,.\,\mathbf{ple}(ABC)$	[*T161*; *T25*]
T163	$[AB]:A\ \varepsilon\ \mathbf{atm}\,.\,B\ \varepsilon\ \mathbf{atm}\,.\,A\ \varepsilon\ \mathbf{Cm}(B)\ .\supset.\ \mathrm{Cd}\,\{\mathsf{V}\}=3$	
PR	$[AB]::\mathrm{Hp}(3)\ .\supset\therefore$	
4.	$A\,\mathbf{v}\,B\ \varepsilon\ \mathbf{Un}$.	[3; *D6*]
5.	$A\,\mathbf{v}\,B\ \varepsilon\ \mathsf{V}$.	[4; **ON**]
6.	$A\ \varepsilon\ \mathsf{V}$.	[1; **ON**]
7.	$B\ \varepsilon\ \mathsf{V}$:	[2; **ON**]
8.	$[C]:C\ \varepsilon\ \mathbf{pr}(\mathbf{Un})\ .\supset.\ C\ \varepsilon\ \mathbf{pr}(A\,\mathbf{v}\,B)\therefore$	[4]
9.	$[C]\therefore C\ \varepsilon\ \mathbf{pr}(A\,\mathbf{v}\,B)\ .\supset:\ C=A\,.\mathrm{v}.\,C=B\therefore$	[1; 2; 3; *T37*; **BA**; *T50*]
10.	$[C]\therefore C\ \varepsilon\ \mathbf{el}(\mathbf{Un})\ .\supset:\ C\ \varepsilon\ \mathbf{Un}\,.\mathrm{v}.\,C\ \varepsilon\ A\,.\mathrm{v}.\,C\ \varepsilon\ B\therefore$	[*D1*; 9]
11.	$[C]\therefore C\ \varepsilon\ C\ .\supset:\ C\ \varepsilon\ \mathbf{Un}\,.\mathrm{v}.\,C\ \varepsilon\ A\,.\mathrm{v}.\,C\ \varepsilon\ B\therefore$	[*T10*; 10]
	$\mathrm{Cd}\,\{\mathsf{V}\}=3$	[11; *D033*]
T164	$[AC]:A\ \varepsilon\ \mathbf{ex}(C)\,.\sim(A\ \varepsilon\ \mathbf{Cm}(C))\ .\supset.\ [\exists DE]\,.\,\mathbf{ple}(ADE)\,.\,\mathbf{ple}(CED)$	
PR	$[AC]:\mathrm{Hp}(2)\ .\supset.$	
3.	$A\ \varepsilon\ \mathbf{el}(\mathbf{Cm}(C))$.	[1; *T27*]
4.	$A\ \varepsilon\ \mathbf{pr}(\mathbf{Cm}(C))$.	[2; 3; *D1*]
5.	$C\ \varepsilon\ \mathbf{el}(\mathbf{Cm}(A))$.	[1; *T25*; *T27*]
6.	$\sim(C\ \varepsilon\ \mathbf{Cm}(A))$.	[2; *T23*]
7.	$C\ \varepsilon\ \mathbf{pr}(\mathbf{Cm}(A))$.	[5; 6; *D1*]
8.	$A\ \varepsilon\ \mathbf{ex}(\mathbf{Cm}(A))$.	[1; *D6*; *T24*]
9.	$C\ \varepsilon\ \mathbf{ex}(\mathbf{Cm}(C))$.	[1; *D6*; *T24*]
10.	$\mathbf{Cm}(A)\backslash C\ \varepsilon\ \mathbf{Cm}(A)\backslash C$.	[7; *T45*]
11.	$\mathbf{Cm}(A)\backslash C\ \varepsilon\ \mathbf{el}(\mathbf{Cm}(A))$.	[10; *T46*]
12.	$\mathbf{Cm}(A)\backslash C\ \varepsilon\ \mathbf{ex}(C)$.	[10; *D11*]
13.	$\mathbf{Cm}(A)\backslash C\ \varepsilon\ \mathbf{el}(\mathbf{Cm}(C))$.	[12; *T27*]
14.	$\mathbf{Cm}(A)\ \varepsilon\ \mathbf{lnk}(\mathbf{Cm}(C))$.	[3; 8; 5; 9; 11; 13; *T31*]
15.	$\mathbf{Cm}(C)\ \varepsilon\ \mathbf{lnk}(\mathbf{Cm}(A))$.	[14; *T32*]
16.	$\mathbf{ple}(A\ \mathbf{Cm}(C)\ \mathbf{Cm}(A))$.	[4; 15; 8; *C58*]
17.	$\mathbf{ple}(C\ \mathbf{Cm}(A)\ \mathbf{Cm}(C))$.	[7; 14; 9; *C58*]
	$[\exists DE]\,.\,\mathbf{ple}(ADE)\,.\,\mathbf{ple}(CED)$	[16; 17]
T165	$[AC]:A\ \varepsilon\ \mathbf{ex}(C)\,.\,\mathrm{Cd}\,\{\mathsf{V}\}>3\ .\supset:\ [\exists B]:\mathbf{ple}(ABC)\,.\mathrm{v}.\,\mathbf{ple}(CBA):\mathrm{v}:$	
	$[\exists DE]\,.\,\mathbf{ple}(ADE)\,.\,\mathbf{ple}(CED)$	[*T161*-*T164*]
T166	$[AC]::\mathrm{Cd}\,\{\mathsf{V}\}>3\ .\supset\therefore A\ \varepsilon\ \mathbf{ex}(C)\ .\equiv:\ [\exists B]:\mathbf{ple}(ABC)\,.\mathrm{v}.\,\mathbf{ple}(CBA):\mathrm{v}:$	
	$[\exists DE]\,.\,\mathbf{ple}(ADE)\,.\,\mathbf{ple}(CED)$	[*T160*; *T165*]

Again, with a somewhat restricted assumption on the number of objects, we get that $\mathbf{ple}(ABC)$ and all those terms equivalent to it are primitive.

3.4 The Term **pce** is primitive

C50	$[ABC]:\mathbf{pce}(ABC)\ .\equiv.\ A\ \varepsilon\ \mathbf{pr}(B)\,.\,B\ \varepsilon\ \mathbf{cntr}(C)\,.\,A\ \varepsilon\ \mathbf{ex}(C)$	
C5	$[ABC]:\mathbf{cec}(ABC)\ .\equiv.\ A\ \varepsilon\ \mathbf{cntr}(B)\,.\,B\ \varepsilon\ \mathbf{ex}(C)\,.\,A\ \varepsilon\ \mathbf{cntr}(C)$	
C32	$[ABC]:\mathbf{epp}(ABC)\ .\equiv.\ A\ \varepsilon\ \mathbf{ex}(B)\,.\,B\ \varepsilon\ \mathbf{pr}(C)\,.\,A\ \varepsilon\ \mathbf{pr}(C)$	
T167	$[ABC]:\mathbf{pce}(ABC)\ .\equiv.\ \mathbf{cec}(BAC)$	[*C50*; *C5*]
T168	$[ABC]:\mathbf{pce}(ABC)\ .\equiv.\ \mathbf{epp}(ACB)$	[*C5*; *C32*]

T169	$[ABC]:\mathbf{pce}(ABC)\ .\supset.\ A\,\varepsilon\,\mathbf{pr}(B)$	[*C50*]
T170	$[AB]:\ A\,\varepsilon\,\mathbf{pr}(B)\ .\supset.\ [\exists C]\,.\,\mathbf{pce}(ABC)$	
PR	$[AB]:\mathrm{Hp}(1)\ .\supset.$	
2.	$B\backslash A\,\varepsilon\,B\backslash A\,.$	[1; *T45*]
3.	$B\backslash A\,\varepsilon\,\mathbf{ex}(A)\,.$	[2; *D11*]
4.	$A\,\varepsilon\,\mathbf{ex}(B\backslash A)\,.$	[3; *T25*]
5.	$B\backslash A\,\varepsilon\,\mathbf{el}(B)\,.$	[2; *D11*]
6.	$B\backslash A\,\varepsilon\,\mathbf{pr}(B)\,.$	[1; 5; *D11*; *T33*]
7.	$\mathbf{pce}(ABB\backslash A)\,.$	[1; 6; *D19*; 4; *C50*]
	$[\exists C]\,.\,\mathbf{pce}(ABC)$	[7]
T171	$[AB]:A\,\varepsilon\,\mathbf{pr}(B)\ .\equiv.\ [\exists C]\,.\,\mathbf{pce}(ABC]$	[*T170*; *C50*]

Notice that **pce** generalizes the terms $\backslash$ and $+$, that is, $\backslash$ and $+$ are just **pce** with more conditions added. For example, consider the following theorem:

T172 $[ABC]:A\,\varepsilon\,B\backslash C\ .\equiv.\ \mathbf{pce}(ABC)\,.\,A\,\mathbf{v}\,C = B$ [*D11*; *C50*; *D5*]

Thus, we have three more equivalent, ternary primitive terms.

3.5 The Term **plp** is primitive

C60	$[ABC]:\mathbf{plp}(ABC)\ .\equiv.\ A\,\varepsilon\,\mathbf{pr}(B)\,.\,B\,\varepsilon\,\mathbf{lnk}(C)\,.\,A\,\varepsilon\,\mathbf{pr}(C)$	
C15	$[ABC]:\mathbf{cpl}(ABC)\ .\equiv.\ A\,\varepsilon\,\mathbf{cntr}(B)\,.\,B\,\varepsilon\,\mathbf{pr}(C)\,.\,A\,\varepsilon\,\mathbf{lnk}(C)$	
C33	$[ABC]:\mathbf{lcc}(ABC)\ .\equiv.\ A\,\varepsilon\,\mathbf{lnk}(B)\,.\,B\,\varepsilon\,\mathbf{cntr}(C)\,.\,A\,\varepsilon\,\mathbf{cntr}(C)$	
T173	$[ABC]:\mathbf{plp}(ABC)\ .\equiv.\ \mathbf{cpl}(BAC)$	[*C60*; *C15*]
T174	$[ABC]:\mathbf{plp}(ABC)\ .\equiv.\ \mathbf{lcc}(BCA)$	[*C60*; *C33*]
T175	$[AB]:A\,\varepsilon\,\mathbf{pr}(B)\,.\sim(B\,\varepsilon\,\mathbf{Un})\ .\supset.\ [\exists C]\,.\,\mathbf{plp}(ABC)$	
PR	$[AB]:\mathrm{Hp}(2)\ .\supset.$	
3.	$B\backslash A\,\varepsilon\,B\backslash A\,.$	[1; *T45*]
4.	$\mathbf{Cm}(B)\,\varepsilon\,\mathbf{Cm}(B)\,.$	[*A3*; 1; 2; *D6*]
5.	$A\,\mathbf{v}\,\mathbf{Cm}(B)\,\varepsilon\,A\,\mathbf{v}\,\mathbf{Cm}(B)\,.$	[1; 4; *D4*]
6.	$A\,\varepsilon\,\mathbf{el}(B)\,.$	[1; *D1*]
7.	$A\,\varepsilon\,\mathbf{el}(A\,\mathbf{v}\,\mathbf{Cm}(B))\,.$	[1; 5; *T19*]
8.	$B\backslash A\,\varepsilon\,\mathbf{el}(B)\,.$	[3; *D11*]
9.	$B\backslash A\,\varepsilon\,\mathbf{ex}(A)\,.$	[3; *D11*]
10.	$B\backslash A\,\varepsilon\,\mathbf{ex}(\mathbf{Cm}(B))\,.$	[8; **BA**; *D7*; *D6*]
11.	$B\backslash A\,\varepsilon\,\mathbf{ex}(A\,\mathbf{v}\,\mathbf{Cm}(B))\,.$	[*D7*; 9; 10; *T30*]
12.	$\mathbf{Cm}(B)\,\varepsilon\,\mathbf{el}(A\,\mathbf{v}\,\mathbf{Cm}(B))\,.$	[4; 5; *T20*]
13.	$\mathbf{Cm}(B)\,\varepsilon\,\mathbf{ex}(B)\,.$	[3; 5]
14.	$B\,\varepsilon\,\mathbf{lnk}(A\,\mathbf{v}\,\mathbf{Cm}(B))\,.$	[6; 7; 8; 11; 12; 13; *T31*]
15.	$A\,\varepsilon\,\mathbf{pr}(A\,\mathbf{v}\,\mathbf{Cm}(B))$	[4; 7; *D1*]
	$[\exists C]\,.\,\mathbf{plp}(ABC)$	[1; 14; 15; *C60*]
T176	$[ABC]:B\,\varepsilon\,\mathbf{Un}\ .\supset.\ \sim\mathbf{plp}(BAC)\,.\sim\mathbf{plp}(ABC)$	
PR	$[ABC]:\mathrm{Hp}(1)\ .\supset.$	
2.	$\sim(B\,\varepsilon\,\mathbf{pr}(C))\,.$	[1; *D3*; *T10*]
3.	$\sim(\mathbf{plp}(BAC))\,.$	[2; *C60*]
4.	$\sim(A\,\varepsilon\,\mathbf{lnk}(B))\,.$	[1; *T10*; *T33*]
5.	$\sim\mathbf{plp}(ABC)\,.$	[4; *C60*]
	$\sim\mathbf{plp}(BAC)\,.\sim\mathbf{plp}(ABC)$	[3; 5]

T177 $[ABC]::A\,\varepsilon\,\mathsf{pr}(B)\ .\supset\therefore\ [\exists C].\,\mathsf{plp}(ABC):\mathrm{v}:A \neq B\,.\,[DE].\sim\mathsf{plp}(BDE)\,.$
$\sim\mathsf{plp}(DBE)$ [*T33*; *T175*; *T176*]

T178 $[ABC]:\mathsf{plp}(ABC)\ .\supset.\ A\,\varepsilon\,\mathsf{pr}(B)$ [*C60*]

T179 $[AB]:\mathsf{ppp}(AB\mathsf{Un})\ .\supset.\ [\exists C].\,\mathsf{plp}(ABC)$

PR $[AB]:\mathrm{Hp}(1)\ .\supset.$

2. $A\,\varepsilon\,\mathsf{pr}(B)\,.$ [1; *C64*]

3. $B\,\varepsilon\,\mathsf{pr}(\mathsf{Un})\,.$ [1; *C64*]

4. $\sim(B\,\varepsilon\,\mathsf{Un})\,.$ [3; *T33*]

$[\exists C].\,\mathsf{plp}(ABC)$ [2; 4; *T175*]

T180 $[B]\therefore B\,\varepsilon\,B\,.\sim(B\,\varepsilon\,\mathsf{Un})\,.\,\mathrm{Cd}\{\mathsf{V}\}>3\ .\supset:\ [\exists C]:\mathsf{ppp}(CB\mathsf{Un})\,.\mathrm{v}.\,\mathsf{ppp}(BC\mathsf{Un})$

PR $[B]\therefore \mathrm{Hp}(3)\ .\supset:$

4. $B\,\varepsilon\,\mathsf{pr}(\mathsf{Un}):$ [1; 2; *D1*; *T10*]

$[\exists C]:\mathsf{ppp}(CB\mathsf{Un})\,.\mathrm{v}.\,\mathsf{ppp}(BC\mathsf{Un})$ [3; 4; *T140*]

T181 $[B]\therefore B\,\varepsilon\,B\,.\sim(B\,\varepsilon\,\mathsf{Un})\,.\,\mathrm{Cd}\{\mathsf{V}\}>3\ .\supset:\ [\exists DE]:\mathsf{plp}(BDE)\,.\mathrm{v}.\,\mathsf{plp}(DBE)$
[*T179*; *T180*]

T182 $[BDE]:B\,\varepsilon\,B\,.\sim\mathsf{plp}(BDE)\,.\sim\mathsf{plp}(DBE)\,.\,\mathrm{Cd}\{\mathsf{V}\}>3\ .\supset.\ B\,\varepsilon\,\mathsf{Un}$ [*T181*]

T183 $[ABDE]:A \neq B\,.\sim\mathsf{plp}(BDE)\,.\sim\mathsf{plp}(DBE)\ .\supset.\ A\,\varepsilon\,\mathsf{pr}(B)$
[*T181*; *T10*; *D1*]

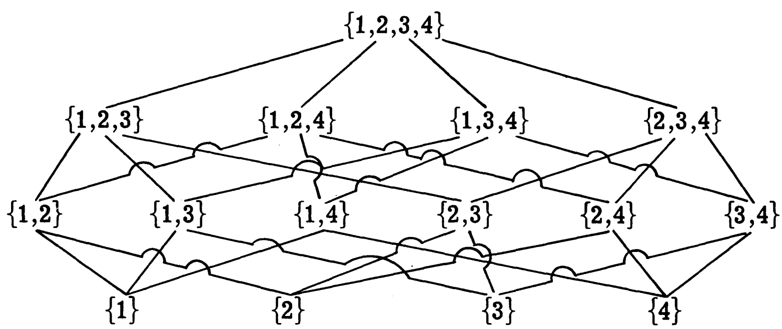
T184 $[AB]::[\exists C].\,\mathsf{plp}(ABC):\mathrm{v}:\ A \neq B\,.\,[DE].\sim(\mathsf{plp}(BDE))\,.\sim\mathsf{plp}(DBE)\ \therefore\supset.$
$A\,\varepsilon\,\mathsf{pr}(B)$ [*T178*; *T182*]

T185 $[AB]::A\,\varepsilon\,\mathsf{pr}(B)\ .\equiv\therefore\ [\exists C].\,\mathsf{plp}(ABC):\mathrm{v}:A \neq B\,.\,[DE].\sim\mathsf{plp}(BDE)\,.$
$\sim\mathsf{plp}(DBE)$ [*T183*; *T177*]

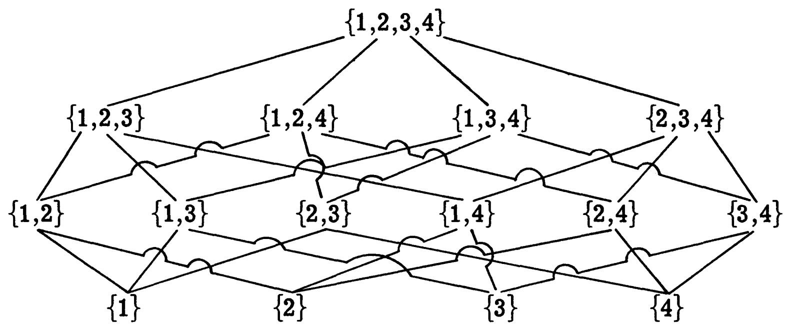
Thus, we have our final collection of primitive ternary terms. Some of these results can be generalized and we will do so in the next chapter, after we have developed more tools involving cardinality of a name.

4 *The non-primitive ternary relations* In this section we shall refer to some of the models of Chapter II and also we shall construct some new models. We use these as before to show certain terms are not primitive. As in the previous section, we also show the equivalence of certain relations and thereby demonstrate the non-primitivity of a number of relations simultaneously.

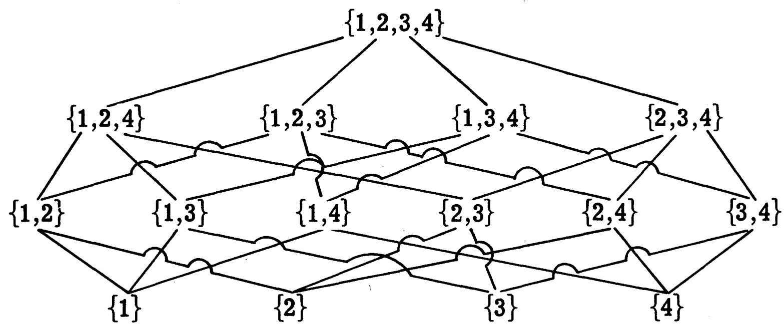
$\mathfrak{G}$



$\mathfrak{H}$



$\mathfrak{L}$



4.1 The Term **ell** is not primitive

C27 $[ABC]:$ **ell** (ABC) $.\equiv.$ $A\,\varepsilon$ **ex**(B) . $B\,\varepsilon$ **lnk**(C) . $A\,\varepsilon$ **lnk**(C)
C39 $[ABC]:$ **lel** (ABC) $.\equiv.$ $A\,\varepsilon$ **lnk**(B) . $B\,\varepsilon$ **ex**(C) . $A\,\varepsilon$ **lnk**(C)
C42 $[ABC]:$ **lle** (ABC) $.\equiv.$ $A\,\varepsilon$ **lnk**(B) . $B\,\varepsilon$ **lnk**(C) . $A\,\varepsilon$ **ex**(C)

T186 $[ABC]:$ **ell** (ABC) $.\equiv.$ **lel** (CAB) [*C27*; *C39*]
T187 $[ABC]:$ **ell** (ABC) $.\equiv.$ **lle** (ACB) [*C27*; *C42*]

Consider models $\mathfrak{G}$ and $\mathfrak{H}$. In each of these we have the following:

ell $(\{1,2\}, \{3,4\}, \{2,3\})$
ell $(\{1,2\}, \{3,4\}, \{1,3\})$
ell $(\{1,2\}, \{3,4\}, \{1,4\})$
ell $(\{1,2\}, \{3,4\}, \{2,4\})$
ell $(\{1,3\}, \{2,4\}, \{1,2\})$
ell $(\{1,3\}, \{2,4\}, \{1,4\})$
ell $(\{1,4\}, \{2,3\}, \{1,2\})$
ell $(\{1,4\}, \{2,3\}, \{1,3\})$
ell $(\{1,4\}, \{2,3\}, \{2,4\})$
ell $(\{1,4\}, \{2,3\}, \{3,4\})$
ell $(\{1,3\}, \{2,4\}, \{2,3\})$
ell $(\{1,3\}, \{2,4\}, \{3,4\})$

These are the only cases where **ell** is defined. But in $\mathfrak{G}$ we have $\{2\}\,\varepsilon$ **pr** $\{2,3\}$ while in $\mathfrak{H}$ $\sim(\{2\}\,\varepsilon$ **pr** $\{2,3\})$, hence, **ell**, **lel**, and **lle** are not primitive.

4.2 The Term **clc** is not primitive

C9 $[ABC]:$ **clc** (ABC) $.\equiv.$ $A\,\varepsilon$ **cntr** (B) . $B\,\varepsilon$ **lnk**(C) . $A\,\varepsilon$ **cntr** (C)
C48 $[ABC]:$ **lpp**(ABC) $.\equiv.$ $A\,\varepsilon$ **lnk**(B) . $B\,\varepsilon$ **pr**(C) . $A\,\varepsilon$ **pr**(C)
C51 $[ABC]:$ **pcl** (ABC) $.\equiv.$ $A\,\varepsilon$ **pr** (B) . $B\,\varepsilon$ **cntr** (C) . $A\,\varepsilon$ **lnk**(C)

T188 $[ABC]:$ **clc** (ABC) $.\equiv.$ **lpp** (BCA) [*C9*; *C48*]
T189 $[ABC]:$ **clc** (ABC) $.\equiv.$ **pcl** (BAC) [*C9*; *C51*]

In models $\mathfrak{C}$ and $\mathfrak{D}$ we have seen that $\{1\}\,\varepsilon\,\mathbf{pr}(\{1,3\})$ in $\mathfrak{C}$ but $\sim(\{1\}\,\varepsilon\,\mathbf{pr}(\{1,3\}))$ in $\mathfrak{D}$. However, in both $\mathfrak{C}$ and $\mathfrak{D}$ we have the following.

$$\mathbf{clc}(\{1,2,3\}\,\{1,2\}\,\{1,3\})$$
$$\mathbf{clc}(\{1,2,3\}\,\{1,2\}\,\{2,3\})$$
$$\mathbf{clc}(\{1,2,3\}\,\{1,3\}\,\{2,3\})$$

These up to symmetry of **lnk** are the only such and hence, **clc**(ABC) and the terms equivalent to it are not primitive.

4.3 The Term **lee** is not primitive

C38 $[ABC]:\mathbf{lee}(ABC)\ .\!\equiv.\ A\,\varepsilon\,\mathbf{lnk}(B)\,.\,B\,\varepsilon\,\mathbf{ex}(C)\,.\,A\,\varepsilon\,\mathbf{ex}(C)$
C26 $[ABC]:\mathbf{ele}(ABC)\ .\!\equiv.\ A\,\varepsilon\,\mathbf{ex}(B)\,.\,B\,\varepsilon\,\mathbf{lnk}(C)\,.\,A\,\varepsilon\,\mathbf{ex}(C)$
C23 $[ABC]:\mathbf{eel}(ABC)\ .\!\equiv.\ A\,\varepsilon\,\mathbf{ex}(B)\,.\,B\,\varepsilon\,\mathbf{ex}(C)\,.\,A\,\varepsilon\,\mathbf{lnk}(C)$

T190 $[ABC]:\mathbf{lee}(ABC)\ .\!\equiv.\ \mathbf{ele}(CAB)$ [*C38*; *C26*]
T191 $[ABC]:\mathbf{lee}(ABC)\ .\!\equiv.\ \mathbf{eel}(ACB)$ [*C38*; *C23*]

Now we consider models $\mathfrak{G}$ and $\mathfrak{L}$ and we see that the following are true.

$\mathbf{lee}(\{1,2\}\,\{1,3\}\,\{4\})$	$\mathbf{lee}(\{1,3\}\,\{3,4\}\,\{2\})$
$\mathbf{lee}(\{1,2\}\,\{1,4\}\,\{3\})$	$\mathbf{lee}(\{1,4\}\,\{2,4\}\,\{3\})$
$\mathbf{lee}(\{1,2\}\,\{2,3\}\,\{4\})$	$\mathbf{lee}(\{1,4\}\,\{3,4\}\,\{2\})$
$\mathbf{lee}(\{1,2\}\,\{2,4\}\,\{3\})$	$\mathbf{lee}(\{2,3\}\,\{2,4\}\,\{1\})$
$\mathbf{lee}(\{1,3\}\,\{1,4\}\,\{2\})$	$\mathbf{lee}(\{2,3\}\,\{3,4\}\,\{1\})$
$\mathbf{lee}(\{1,3\}\,\{2,3\}\,\{4\})$	$\mathbf{lee}(\{2,4\}\,\{3,4\}\,\{1\})$

By the symmetry of **lnk** and **ex** these are all the cases we need consider. Again in model $\mathfrak{G}$, $\{4\}\,\varepsilon\,\mathbf{pr}\,\{1,2,4\}$ while in model $\mathfrak{L}$, $\sim(\{4\}\,\varepsilon\,\mathbf{pr}\,\{1,2,4\})$. Hence, **lee** and those terms equivalent to **lee** are not primitive.

4.4 The Term **pll** is not primitive

C59 $[ABC]:\mathbf{pll}(ABC)\ .\!\equiv.\ A\,\varepsilon\,\mathbf{pr}(B)\,.\,B\,\varepsilon\,\mathbf{lnk}(C)\,.\,A\,\varepsilon\,\mathbf{lnk}(C)$
C11 $[ABC]:\mathbf{cll}(ABC)\ .\!\equiv.\ A\,\varepsilon\,\mathbf{cntr}(B)\,.\,B\,\varepsilon\,\mathbf{lnk}(C)\,.\,A\,\varepsilon\,\mathbf{lnk}(C)$
C35 $[ABC]:\mathbf{lcl}(ABC)\ .\!\equiv.\ A\,\varepsilon\,\mathbf{lnk}(B)\,.\,B\,\varepsilon\,\mathbf{cntr}(C)\,.\,A\,\varepsilon\,\mathbf{lnk}(C)$
C41 $[ABC]:\mathbf{llc}(ABC)\ .\!\equiv.\ A\,\varepsilon\,\mathbf{lnk}(B)\,.\,B\,\varepsilon\,\mathbf{lnk}(C)\,.\,A\,\varepsilon\,\mathbf{cntr}(C)$
C44 $[ABC]:\mathbf{llp}(ABC)\ .\!\equiv.\ A\,\varepsilon\,\mathbf{lnk}(B)\,.\,B\,\varepsilon\,\mathbf{lnk}(C)\,.\,A\,\varepsilon\,\mathbf{pr}(C)$
C47 $[ABC]:\mathbf{lpl}(ABC)\ .\!\equiv.\ A\,\varepsilon\,\mathbf{lnk}(B)\,.\,B\,\varepsilon\,\mathbf{pr}(C)\,.\,A\,\varepsilon\,\mathbf{lnk}(C)$

T192 $[ABC]:\mathbf{pll}(ABC)\ .\!\equiv.\ \mathbf{cll}(BAC)$ [*C59*; *C11*]
T193 $[ABC]:\mathbf{pll}(ABC)\ .\!\equiv.\ \mathbf{lcl}(CBA)$ [*C59*; *C35*]
T194 $[ABC]:\mathbf{pll}(ABC)\ .\!\equiv.\ \mathbf{llc}(BCA)$ [*C59*; *C41*]
T195 $[ABC]:\mathbf{pll}(ABC)\ .\!\equiv.\ \mathbf{llp}(ACB)$ [*C59*; *C44*]
T196 $[ABC]:\mathbf{pll}(ABC)\ .\!\equiv.\ \mathbf{lpl}(CBA)$ [*C59*; *C47*]

In model $\mathfrak{G}$ we have $\{1\}\,\varepsilon\,\mathbf{pr}\,\{1,3\}$ but in $\mathfrak{H}$ we have $\sim(\{1\}\,\varepsilon\,\mathbf{pr}\,\{1,3\})$. The following table shows, however, that **pll** has the same meaning in both models. Hence, **pll** and its equivalents are not primitive.

$$\begin{array}{ll}
\mathbf{pll}(\{1,2\}\ \{1,2,3\}\ \{1,4\}) & \mathbf{pll}(\{1,3\}\ \{1,3,4\}\ \{1,2\}) \\
\mathbf{pll}(\{1,2\}\ \{1,2,3\}\ \{2,4\}) & \mathbf{pll}(\{1,3\}\ \{1,3,4\}\ \{2,3\}) \\
\mathbf{pll}(\{1,3\}\ \{1,2,3\}\ \{1,4\}) & \mathbf{pll}(\{1,4\}\ \{1,3,4\}\ \{1,2\}) \\
\mathbf{pll}(\{1,3\}\ \{1,2,3\}\ \{3,4\}) & \mathbf{pll}(\{1,4\}\ \{1,3,4\}\ \{2,4\}) \\
\mathbf{pll}(\{2,3\}\ \{1,2,3\}\ \{2,4\}) & \mathbf{pll}(\{3,4\}\ \{1,3,4\}\ \{2,3\}) \\
\mathbf{pll}(\{2,3\}\ \{1,2,3\}\ \{3,4\}) & \mathbf{pll}(\{3,4\}\ \{1,3,4\}\ \{2,4\}) \\
\mathbf{pll}(\{1,3\}\ \{1,3,4\}\ \{1,2\}) & \mathbf{pll}(\{2,3\}\ \{2,3,4\}\ \{1,2\}) \\
\mathbf{pll}(\{1,3\}\ \{1,3,4\}\ \{2,3\}) & \mathbf{pll}(\{2,3\}\ \{2,3,4\}\ \{1,3\}) \\
\mathbf{pll}(\{1,4\}\ \{1,3,4\}\ \{1,2\}) & \mathbf{pll}(\{2,4\}\ \{2,3,4\}\ \{1,2\}) \\
\mathbf{pll}(\{1,4\}\ \{1,3,4\}\ \{2,4\}) & \mathbf{pll}(\{2,4\}\ \{2,3,4\}\ \{1,4\}) \\
\mathbf{pll}(\{3,4\}\ \{1,3,4\}\ \{2,3\}) & \mathbf{pll}(\{3,4\}\ \{2,3,4\}\ \{1,3\}) \\
\mathbf{pll}(\{3,4\}\ \{1,3,4\}\ \{2,4\}) & \mathbf{pll}(\{3,4\}\ \{2,3,4\}\ \{1,4\})
\end{array}$$

$$\begin{array}{l}
\mathbf{pll}(\{1,2\}\ \{1,2,3\}\ \{1,3,4\}) \\
\mathbf{pll}(\{1,2\}\ \{1,2,3\}\ \{2,3,4\}) \\
\mathbf{pll}(\{1,2\}\ \{1,2,4\}\ \{1,3,4\}) \\
\mathbf{pll}(\{1,2\}\ \{1,2,4\}\ \{2,3,4\}) \\
\mathbf{pll}(\{1,3\}\ \{1,2,3\}\ \{1,2,4\}) \\
\mathbf{pll}(\{1,3\}\ \{1,2,3\}\ \{2,3,4\}) \\
\mathbf{pll}(\{1,3\}\ \{1,3,4\}\ \{1,2,4\}) \\
\mathbf{pll}(\{1,3\}\ \{1,3,4\}\ \{2,3,4\}) \\
\mathbf{pll}(\{1,4\}\ \{1,2,4\}\ \{1,2,3\}) \\
\mathbf{pll}(\{1,4\}\ \{1,2,4\}\ \{2,3,4\}) \\
\mathbf{pll}(\{1,4\}\ \{1,3,4\}\ \{1,2,3\}) \\
\mathbf{pll}(\{1,4\}\ \{1,3,4\}\ \{2,3,4\})
\end{array}$$

$$\begin{array}{l}
\mathbf{pll}(\{2,3\}\ \{1,2,3\}\ \{1,2,4\}) \\
\mathbf{pll}(\{2,3\}\ \{1,2,3\}\ \{1,3,4\}) \\
\mathbf{pll}(\{2,3\}\ \{2,3,4\}\ \{1,2,4\}) \\
\mathbf{pll}(\{2,3\}\ \{2,3,4\}\ \{1,3,4\}) \\
\mathbf{pll}(\{2,4\}\ \{1,2,4\}\ \{1,2,3\}) \\
\mathbf{pll}(\{2,4\}\ \{1,2,4\}\ \{1,3,4\}) \\
\mathbf{pll}(\{2,4\}\ \{2,3,4\}\ \{1,2,3\}) \\
\mathbf{pll}(\{2,4\}\ \{2,3,4\}\ \{1,3,4\}) \\
\mathbf{pll}(\{3,4\}\ \{1,3,4\}\ \{1,2,3\}) \\
\mathbf{pll}(\{3,4\}\ \{1,3,4\}\ \{1,2,4\}) \\
\mathbf{pll}(\{3,4\}\ \{2,3,4\}\ \{1,2,3\}) \\
\mathbf{pll}(\{3,4\}\ \{2,3,4\}\ \{1,2,4\})
\end{array}$$

4.5 The Term eee is not primitive

C22 $[ABC]: \mathbf{eee}(ABC) .\equiv. A\, \varepsilon\, \mathbf{ex}(B) . B\, \varepsilon\, \mathbf{ex}(C) . A\, \varepsilon\, \mathbf{ex}(C)$

Here we may consider models $\mathfrak{C}$ and $\mathfrak{D}$ from Chapter II. In both we have $\mathbf{eee}(\{1\}, \{2\}, \{3\})$ only, but $\{1\}\ \varepsilon\ \mathbf{pr}\ \{1,3\}$ in $\mathfrak{C}$ and $\sim(\{1\}\ \varepsilon\ \mathbf{pr}(\{1,3\}))$ in $\mathfrak{D}$. Hence, **eee** is not primitive and it has no equivalents.

4.6 The Term lll is not primitive

C43 $[ABC]: \mathbf{lll}(ABC) .\equiv. A\, \varepsilon\, \mathbf{lnk}(B) . B\, \varepsilon\, \mathbf{lnk}(C) . A\, \varepsilon\, \mathbf{lnk}(C)$

Here again we consider models $\mathfrak{C}$ and $\mathfrak{D}$ from Chapter II. In both we have only **lll**$(\{1,2\}, \{1,3\}, \{2,3\})$ but $\{1\}\ \varepsilon$ **pr** $\{1,3\}$ in $\mathfrak{C}$ while $\sim(\{1\}\ \varepsilon$ **pr** $\{1,3\})$ in $\mathfrak{D}$. Hence, **lll** is not primitive. Like **eee**, **lll** has no equivalents. This ends the section on non-primitive ternary terms and the chapter on ternary relations.

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To be continued

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