

A NOTE ON THE TRUTH-TABLE FOR 'IF p THEN q 'LEE C. ARCHIE, B. G. HURDLE, Jr., and
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The argument presented here purports to be a defense of material implication. The authors do not think that the argument is correct because, among other things, material implication does not seem to be in general equivalent to other types of ordinary-language implications. Yet if the following argument is sound, the truth conditions of 'If p then q ' are identical to those of ' $p \supset q$ '.

Most logicians agree on a sufficiency condition for the falsity of 'If p then q '; namely, 'If p then q ' is false when p is true and q is false.¹ In order to compose the truth-table for 'If p then q ', besides the uncontroversial operations for conjunction and negation, only four more assumptions are needed. First, a statement of the form 'If p then q , and if not- p then q ' is interchangeable with ' q whether or not p '. These are equally plausible ways of asserting q and denying that the truth or falsity of p is relevant to the truth of q . Second, ' q whether or not p ' asserts ' q ', as does 'If p then q , and if not- p then q '. For instance, "Smith resigns whether or not he loses his rook" is an alternative way of saying "If Smith loses his rook then he resigns, and if Smith does not lose his rook then he resigns" or, more simply, just "Smith resigns". Our third additional assumption is that 'If p then q , but not if not- p then q ' is not self-contradictory. For example, a statement of this form is sometimes used by a person who points out that p implies q but not- p does not imply q . Fourth, 'If not- p then q , but not if p then q ' is not self-contradictory either. That is, this statement might be adopted when one asserts that not- p implies q but p does not imply q . With these assumptions, the question marks on a truth-table for 'If p then q ' wither away.²

1. A. J. Dale, "A defense of material implication," *Analysis*, vol. 34 (1974), p. 91.

2. Truth-tables for 'If p then q ' according to the sufficiency condition are given in Thomas S. Vernon and Lowell A. Nissen, *Reflective Thinking: The Fundamentals of Logic*, Wadsworth, Belmont, California (1968), p. 77.

An initial composition of a truth-table on this information for 'If p then q ' as well as the other statement forms mentioned yields Table 1.

TABLE 1

	A	B	C	D	E	F	G	H
	p	q	Not- p	If p then q	If not- p then q	q whether or not p	If p then q , but not if not- p then q	If not- p then q , but not if p then q
1	T	T	F	?	?	T	?	?
2	T	F	F	F	?	F	F	?
3	F	T	T	?	?	T	?	?
4	F	F	T	?	F	F	?	F

The false truth values in columns **D** and **E** are assigned in accordance with the sufficiency condition. The truth values in column **F** are given in column **B**, for ' q whether or not p ' is equivalent to ' q '.³ In general, ' q whether or not p ' may be phrased as ' q regardless of p '. Finally the false truth values in columns **G** and **H** result from one of the respective conjuncts being false in columns **D** and **E**.

Since ' q whether or not p ' (column **F**) is true on lines 1 and 3, we may conclude that both 'If p then q ' (column **D**) and 'If not- p then q ' (column **E**) will be true on lines 1 and 3 also (since the conjunction of the latter two statement forms is interchangeable with the former one). In addition, since the second conjunct of column **G** ('not if not- p then q ') is the denial of column **E** ('If not- p then q '), we may conclude that **G** will be false on lines 1 and 3 because **E** is true on those lines. Also, since the second conjunct of column **H** ('not if p then q ') is the denial of column **D** ('If p then q '), we may conclude that **H** is false on lines 1 and 3 because column **D** has been shown to be true on those lines. This information is tabulated in Table 2.

TABLE 2

	D	E	F	G	H
	If p then q	If not- p then q	q whether or not p	If p then q , but not if not- p then q	If not- p then q , but not if p then q
1	T	T	T	F	F
2	F	?	F	F	?
3	T	T	T	F	F
4	?	F	F	?	F

3. A similar interpretation of ' q whether or not p ' is given in Richard Purtill, *Logic for Philosophers*, Harper and Row, New York (1971), p. 11.

As pointed out above, the statement forms in columns **G** and **H** are not self-contradictory; consequently, **G** is true on line 4 and **H** is true on line 2. This result yields enough information to fill in the remaining question marks on Table 2.

Let us consider column **G**: 'If p then q , but not if not- p then q ' is true on line 4, so both conjuncts are true. Consequently, 'If p then q ' (column **D**) is true on line 4. Last, let us consider column **H**: 'If not- p then q , but not if p then q ' is true on line 2, so both conjuncts are true. Consequently, 'If not- p then q ' (column **E**) is also true on line 2. All question marks on Table 2 have disappeared, and clearly this leaves identical truth conditions for 'If p then q ' and ' $p \supset q$ '.

Our purported defense of material implication seems adequate even if we admit that specific substitution instances for p and q adversely affect the senses of 'if' and 'then' in our original formulation, for although the truth-table would not work for *those* instances, it will work for others which meet the sufficiency condition and the other assumptions. If it is thought that the horseshoe is more or less presupposed in the selected arrangements of 'whether or not' and complex 'if'-expressions, that is in fact what the argument does claim.

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