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THREE THEORIES OF DIALECTIC

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Formal theories of dialectic are lacunae in the literature of logic. This is unfortunate but understandable, since their intuitive counterparts are often not formulated clearly or extensively. Usually some fragment of an intuitive theory is presupposed rather than developed by an author, the classic example being Marx. Given this sort of situation, questions arise not only over the exact nature of these theories but also over their very consistency. In this regard little has changed since Herr Dühring first attacked dialectical materialism. To eliminate some of these, I will in this paper develop and demonstrate the consistency of three formal systems of dialectic. The formal theories will be called TAS_1, TAS_2, and TAS_3, respectively. These theories form a hierarchy, the second being a consistent extension of the first, and the third being a consistent extension of the second. All have as their domain the set of events, both mental and physical. Moreover, all three are finitely axiomatized first-order theories with identity.

The first formal system, TAS_1, is simply a theory of triads. The nonlogical vocabulary of TAS_1 consists of three predicate letters. The standard interpretation of these is as follows, where lower case Greek letters are used as placemarkers for individual constants and variables.

$\ulcorner T\alpha \urcorner$: α is a thesis
$\ulcorner A\alpha\beta \urcorner$: α is the antithesis of β
$\ulcorner S\alpha\beta\gamma \urcorner$: α is the synthesis of β and γ.

The axioms of TAS_1 fall into two classes: the existential and the relational.

Axioms of Existence:

E1 $\exists x Tx$
E2 $\forall x(Tx \rightarrow \exists! yAyx)$
E3 $\forall x \forall y(Ayx \rightarrow \exists! zSzxy)$.

Axioms of Relation:

R1 $\forall x \forall y(Ayx \rightarrow (Tx \mathbin{\&} \neg Axy))$
R2 $\forall x \forall y \forall z(Szxy \rightarrow (Tz \mathbin{\&} (Axy \vee Ayx) \mathbin{\&} \neg(Sxzy \vee Syxz)))$.

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The addition and succession of natural numbers other than zero provides a model for TAS_1. If the predicate letters are reinterpreted as follows, the axioms are all true under this arithmetic interpretation. Since there exists a model of the theory, it is consistent.

$\ulcorner T\alpha \urcorner$: α is an odd number
$\ulcorner A\alpha\beta \urcorner$: α is the even number which succeeds β
$\ulcorner S\alpha\beta\gamma \urcorner$: α is the sum of β and γ and either β succeeds γ or γ succeeds β.

The second theory, TAS_2, postulates the existence of nodal points in addition to triads. A nodal point is a synthesis of two events (a thesis and its antithesis) which is qualitatively different from each of these. TAS_2 is generated from TAS_1 by the addition of a new predicate letter, a definition, and two new axioms of existence.

$\ulcorner D\alpha\beta \urcorner$: α is qualitatively different from β
$\ulcorner N\alpha \urcorner$: defined as $\ulcorner \exists\beta\exists\gamma(S\alpha\beta\gamma \;\&\; D\alpha\beta \;\&\; D\alpha\gamma)\urcorner$
E4 $\exists x Nx$
E5 $\forall x(Nx \rightarrow \exists y(Ny \;\&\; y \neq x))$.

This theory also has an arithmetic model, namely, in the addition, division and succession of natural numbers other than zero. The previous numerical reinterpretations of thesis, antithesis, and synthesis are retained, while qualitative difference is reinterpreted in the following fashion:

$\ulcorner D\alpha\beta \urcorner$: α is not divisible by β without remainder.

The third system, TAS_3, incorporates a minimal theory of time in addition to those of triads and nodal points. It has one additional predicate letter for a temporal relation of precedence. The axiomatic formulation of TAS_3 is a hybrid of those of its predecessors.

$\ulcorner P\alpha\beta \urcorner$: α precedes β.

Axioms of Existence:

E1-E4 of TAS_2
E5 $\forall x(Nx \rightarrow \exists y(Ny \;\&\; Pxy))$
E6 $\forall x \exists y Pxy$.

Axioms of Relation:

R1 $\forall x \forall y(Ayx \rightarrow (Tx \;\&\; Pxy))$
R2 $\forall x \forall y \forall z(Szxy \rightarrow (Tz \;\&\; (Axy \vee Ayx) \;\&\; Szyx \;\&\; Pxz \;\&\; Pyz))$
R3 $\forall x \forall y(Pxy \rightarrow \neg Pyx)$
R4 $\forall x \forall y \forall z((Pxy \;\&\; Pyz) \rightarrow Pxz)$.

Again we can provide an arithmetic model for this theory by merely extending the model of TAS_2, reinterpreting P as the less than relation. In a similar fashion other consistent theories of dialectic can be generated from TAS_3 simply by adding further temporal axioms which more narrowly determine the notion of time involved in the triadic and nodal occurrences. However, none of these theories is complete since neither $\forall x(Tx \vee \exists y Axy)$

nor its negation is a consequence of these systems. Such incompleteness might indeed be a feature of any system of dialectic which is reasonably faithful to the intuitive theory of Engels.

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