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# HOW TO STOP TALKING TO TORTOISES

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Lewis Carroll in his splendid paper [1] describes the conversation which Achilles had with the Tortoise when he finally caught it, a conversation as instructive as the footrace which preceded it. Briefly, the Tortoise would admit 'p', and ' $(p \supset q)$ ', and ' $((p \& (p \supset q)) \supset q)$ ' and so on, but would not concede 'q'. The aim of the present paper\* is to provide Achilles with a reply with which to end this conversation. First, a development of C. L. Hamblin's theory of dialogue in [3] is described. This development is more explicit in its account of commitment, in the generation of locutions, and in the specification of immediate logical relations. Secondly, a dialectical system **DT** is defined within the theory. It is then argued that **DT** escapes a fatal defect in the modeling of argument common to all Hamblin's systems in [2] and [3]. Fourthly, it is shown that **DT** enables Achilles finally to call a halt to his conversation with the Tortoise. Finally, an extension of **DT** is made to enable field linguists to use Quinean techniques when investigating dialogues with Tortoises.

In considering dialogues, it is clear that we require the notions of a *participant* and a *locution*. The participants in dialogues may include not only people and tortoises, but fictional characters, organizations such as corporations and governments, and perhaps even machines; they form a set P. The locutions are grammatically complete utterances, types rather than tokens, forming a set L. Following Hamblin, I shall mean by a *locution act* a member of the set  $P \times L$  of participant-locution pairs. By a *dialogue of length* n, I shall mean a member of the set  $(P \times L)^n$  of sequences of n locution acts, and by a *dialogue*  $d \in D$ , a dialogue of length n for some n. Each member of a dialogue is of the form  $\langle n, \langle p, l \rangle$ ,  $n \in N$ ,  $p \in P$ ,  $l \in L$ , but is identified with the triple  $\langle n, p, l \rangle$ . The set  $E = N \times P \times L$  of such triples is

<sup>\*</sup>Parts of this paper formed part of my 'Bizarre Dialectic', read to the Australasian Association for Logic Conference, August 1975. I wish to express my gratitude to Prof. C. L. Hamblin for his helpful criticisms of that paper, without wishing to imply that he would approve of the particular maneuvers here employed to escape those criticisms.

the set of *locution events*. By a *dialectical system*, I shall mean a triple  $\langle P, L, R \rangle$ , where R is a set of *rules* which define a set K of *legal dialogues*.<sup>1</sup>

I shall speak freely of a locution event's first member as its *stage*, of its second member as its *speaker*, and of its third member as its *locution*. For the formulation of some of the later rules of dialogue, it is necessary to limit consideration to dialogues with only two participants; I shall do this from the beginning, which enables me also to speak of the *hearer* of a locution event, namely the participant who is not its speaker. The *previous* and *next* locution events to  $\langle n, p, l \rangle$  in a dialogue are those whose first members are n - 1, n + 1 respectively.

The first important divergence from Hamblin [3] is the treatment of the set L of locutions. Instead of taking L to be the union of various otherwise unspecified sets, it is generated from a given set S of statements (eternal declarative sentences). Statements are the only kind of locution normally considered by logicians, and a suitable S would be that found in the propositional calculus, consisting of sentence letters together with statements produced from them by truth-functional connectives, though a more complex set could be used if desired. I assume that the members of S can be alphabetically ordered, and that the following are all members of S:

- (i) The *negation*, N's, of any statement  $s \in S$ .
- (ii) The conditional, C'(s, t), of any ordered pair of statements s,  $t \in S$ .
- (iii) The (alphabetically ordered, left-associating) conjunction, K'T, of any non-empty set of statements  $T \subseteq S$ . Where  $T = \{s\}, K'T = s$ .

I shall also on occasion use the letters 'p' and 'q' as schematic letters holding place for statements. Quotation marks around expressions containing schematic letters are to be understood as quasi-quotation marks in the sense of Quine [4], pp. 33f. In particular, I shall write '-p' for the negation of 'p', ' $(p \supset q)$ ' for the conditional of  $\langle p', q' \rangle$ , and '(p & q)' for the conjunction of  $\{p', q'\}$ . By the *denial*, D's, of  $s \in S$ , I shall mean N's unless  $(\exists t \in S) (s = N't)$ , in which case D's = t. The negation of '-p' is '--p', but the denial of '-p' is 'p'.

We extend the language by the use of locution modifiers. These were in effect used by Hamblin [2] without explanation. By a *locution modifier* in **DT** I shall mean an expression which, with a statement, forms a locution other than a statement. (More generally, a k-adic locution modifier with klocutions forms a locution.) The locution modifiers here introduced cannot grammatically be used except with statements, and hence no problem of their iteration or interaction arises.

The method of constructing locutions by means of locution modifiers has the following advantages. It allows us to produce a set L for use in dialogue very simply from the ordinary formation rules of a logical calculus, and it connects each locution nearly with a statement, providing a ready-made one-one function to the set of statements from each other class of locutions. If other locutions, not produced from statements, were desired, they could of course be added. It should be remembered that locutions other than statements cannot be grammatically combined by use of truth-functional or other statement connectives, which may be applied, of course, only to statements; though the statement to which a locution modifier is applied may itself be formed from simpler statements by the use of statement connectives.

# Specification of the set L of locutions for System **DT**

Name	Form	Reading	Function to S
Statements S	' <b>p</b> '	'p'	Ι
Withdrawals	"*p'	'I'm not sure that $p$ '	W
Questions	'?p'	'Is it the case that $p$ ?'	Q
<b>Resolution demands</b>	".p'	'Resolve whether $p$ '	R
$L =_{df} S \cup \{l: (\exists s \in S)(lWs \lor lQs \lor lRs)\}$			

Before we can introduce the rules R of a dialectical system, notions needed in formulating the rules must be defined. The first of these is the distinctive feature of dialectic, commitment; the others are various useful syntactic relationships between locutions or sets of locutions, and syntactically defined properties of sets of locutions.

The commitment of a participant may be visualised as a slate on which tokens of locutions are written and from which they may be erased. Formally, there is a commitment function from  $N \times P$  to the power set of L, which assigns a set of locutions as the commitment  $C_n(p)$  of each participant p at each stage n of each dialogue  $d \in K$ . The commitments so assigned are used in stating the rules R. The commitment function is defined inductively, by specifying the initial commitment of each participant and the effect of each kind of locution event on its speaker's commitment and on its hearer's commitment. The effect of a locution event on the commitment of its speaker and of its hearer depends, in the System **DT**, only on its locution. Where a locution l is included in the commitment C of a participant p at a stage n, I shall also say that p is committed to l at n, or that l is one of p's commitments.

The commitment of a participant at any stage of any dialogue  $d \in K$  is finite and public, and can be ascertained by anybody who knows the commitment rules under which the dialogue is being conducted and what locution events have occurred in it up to that stage. Hamblin's treatment [3] of commitment is less formal, and deliberately leaves unspecific whether commitment is to linguistic entities or their denotata.

Intuitively, commitment is something like belief, but it is not belief. Beliefs, whatever they are, may be kept private. Commitment, since dialectic is an empirical science, must be public. Indeed, participants need not commit themselves to their beliefs, nor believe their commitments. The morally desirable characteristic of *sincerity* is presumably some sort of congruence between private beliefs and public commitments, and it is notorious that there is no dialectical test of sincerity. Further, the participants in dialogues need not be able to believe, for they need not be people. It is possible to ascribe commitments to Mr. Pickwick, Exxon Inc., the Peruvian government or a robot, even though we may be reluctant to admit they have beliefs.

The first syntactic relation we shall need is that of an *immediate* consequence, Imc. Where  $T, U \subseteq S, T Imc U'$  is to be read 'Every member of U is an immediate consequence of the set T'. We define Imc with the help of a list V of preferred valid argument schemata. The schemata to be included in this list will be discussed later. The shorter V is, the less logically adept will the participants appear. At least the schema for modus ponens should be included, thus:

The letters 'p', 'q' in V are schematic, holding place for statements, and the quotation marks in V are quasi-quotation.

The relation of immediate consequence is specified in terms of V:  $T \operatorname{Imc} U$  iff for each  $u \in U$ , the expression obtained by writing the quotation names of all the members of T in some order, followed by a slash, followed by the quotation name of u is an instance of a schema of V. For T, U,  $Z \subseteq S$ , it may be that  $T \operatorname{Imc} U$  but not  $T \cup Z \operatorname{Imc} U$ . This allows for the fact that the members of T can be so 'buried' among others that the relation is not immediate.

The relation lmc is not, it should be noticed, transitive. Only instances of the schemata in V, and not those they entail, form sets between which lmc holds. This is realistic when we consider complex but truth-functionally valid argument schemata. Even students of logic sometimes fail to recognise these as valid, as their teachers well know. It is convenient further to define, for s,  $t \in S$ ,

$$s \operatorname{imc} t =_{df} \{s\} \operatorname{Imc} \{t\}.$$

The connection of Imc to more familiar notions can be seen by introducing an inference rule. Where a set of statements exemplifies an argument schema of V, the rule permits the inference of the post-slash statement from the pre-slash statements. Suppose that the schemata of Vtogether with this inference rule are sufficient for the deduction of all valid statements of L. If we then define

$$K^{\bullet}U \operatorname{csq} s =_{df} (\exists T \subseteq U)(K^{\bullet}T \operatorname{Imc} s)$$

then the familiar relation of *logical consequence* is the ancestral of this, \*csq.

The relation lmc is used to define a set  $\lambda$  of *logicians' conditionals*. For  $t, u \in S$ , the conditional  $C'\langle t, u \rangle \in \lambda$  iff there is a non-empty set  $T \subseteq S$  and  $t = K'T, u \in U$ , and T Imc U. That is to say, a logicians' conditional has as antecedent the result of alphabetically ordering and left-conjoining a set of statements, and as consequent a statement which is an immediate consequence of that set. In particular, for  $t, u \in S$ ,  $t \text{ Imc } u \supset C'\langle t, u \rangle \in \lambda$ .

*Immediate inconsistency* is to be understood as a syntactic property of

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sets of statements; for  $T \subseteq S$ , 'Imn T' means that it is obvious that not all members of T are true. Imn must, like Imc, be specified syntactically. A convenient way to do this is to make Imn depend on Imc, and thus ultimately on the schemata of V. This is achieved by invoking the *denial* function D. For  $t \in S$ , I shall say

$$\operatorname{Imn} T \text{ iff } (\exists U \subseteq S)(U \operatorname{Imc} \{t\} \& T = U \cup \{D^{\prime}t\}).$$

Adding statements to T does not necessarily preserve Imn, for if enough are added the inconsistency may cease to be obvious. Again, it is convenient to define a relation between individual statements:

$$s \operatorname{imn} t =_{df} \operatorname{Imn} \{s, t\}.$$

Hamblin [3] introduces Imc and Imn, treating the latter as a relation. He does not say how they are to be specified, though he hints that it should be syntactically. He prefers in [2] to add a sufficient set of axioms 'inerasably' in each participant's commitment. The present procedure has the usual advantages of using schemata rather than axioms. It also seems to me more realistic, in that we first come to understand implication as a relation holding between statements, and when we wish to demonstrate either that this relation holds or that it does not, we cite other pairs of statements which exhibit the same syntactic relation as the pair in which we are interested. Hence the daring original conjecture of formal logic: that every instance of implication can be explained purely in terms of syntactical (formal) relationships. This procedure is more naturally accounted for by providing a syntactical specification of the relation Imc than by supposing that everybody's commitment contains a built-in set of statements sufficient with substitution for the deduction of all valid sentences. It is perhaps not going too far to see in the move from built-in axioms to syntactic relations a move from a view of logic as ultimately a question about our powers of thought, a matter of psychology, to a view of logic as a question about the public institution of discussion, a matter of sociology.

The third syntactic relation is that of being an answer to a question. System **DT** has only a much oversimplified account of questions, as we shall see. Where  $s \in S$ , I shall mean by the *allowable answers* to Q's a member of the set  $\{s, D's, W's\}$ ; that is, a statement, its withdrawal, and its denial bear the relation ons to the question of the statement.

The rules R of the dialectical system **DT** are formulated in terms of commitment and the syntactic characteristics already defined. The rules should enable us to decide, given a legal dialogue  $d_n \epsilon$  K of length *n*, whether the addition of a particular event  $\langle n, A, l \rangle$  renders the dialogue  $d_{n+1}$  illegal, considering only the events at n - 2, n - 1, the commitment  $C_n(A)$  at *n* of the speaker *A* of the event, and the commitment  $C_n(B)$  at *n* of its hearer *B*. The same rules apply to both participants: 'A' may refer to either participant at any stage, since either may be the speaker of the next event except where this is excluded by the rules in terms of commitments and the preceding two events.<sup>2</sup>

Though the rules are stated as applying to dialogues in the formal language L, it is intended that they should add to our understanding of the unstated conventions which govern ordinary conversation. To indicate these suggestions in an intuitive way, I shall cite after each rule a typical point of order or objection in ordinary language used to protest against violations of the convention which the rule is intended to illuminate.

The rules governing the different kinds of locution will be discussed in turn; we can now state the first rule:

 $\mathbf{R}_{\text{Gram}}$ : No legal dialogue contains an event  $\langle n, A, l \rangle$  unless  $l \in L$ .

Objections to breaches of  $R_{Gram}$  include 'Could you repeat that?', 'I didn't understand that', 'I beg your pardon', and 'What?'. The initial commitment rule provides that the initial commitment of both participants is null: for all  $p \in P$ ,

 $CR_0: C_0(p) = \Lambda.$ 

Statements are the most familiar kind of locution. The only rule restricting their use in **DT** is:

R<sub>Repstat</sub>: No legal dialogue contains an event  $\langle n, A, s \rangle$ ,  $s \in S$ , such that  $\{s\} \subseteq C_n(A) \cap C_n(B)$ .

This rule forbids any locution event whose stage is n and whose locution is a statement if both participants are committed at n to that statement. Breaches of this rule in actual discussion are signalled by such points of order as 'We've already agreed that that is so', 'That's been conceded', 'Yes, dear, I know'. The commitment rule for statements is:

 $CR_{S}: \text{ After } \langle n, A, s \rangle, s \in S,$   $C_{n+1}(A) = C_{n}(A) \cup \{s\}$  $C_{n+1}(B) = C_{n}(B) \cup \{s\}.$ 

The commitment rule  $CR_s$  means that after A says 'p', the commitments are adjusted so that

- (i) 'p' is included in A's commitment
- (ii) 'p' is included in B's commitment.

Clause (i) needs no comment. Clause (ii) may strike some as surprising, and indeed it is not observed in many kinds of dialogues. It is, however, observed in most kinds of two-person dialogues about impersonal matters, as is indicated by such protests as 'Well, if you didn't agree why didn't you say so?'. Nor is (ii) so illiberal as it sounds, because B can immediately withdraw the statement if he does not want to be committed to it; (ii) is, in fact, a formalisation of the convention that *silence means assent*.

Though A can add statements to B's commitment, he cannot take them out. The only way a partipipant can be rid of an unwanted statement commitment is to remove it himself, by withdrawing the statement. This immediately gives us the commitment rule for withdrawals:  $CR_{W}: \text{ After } \langle n, A, W's \rangle,$   $C_{n+1}(A) = C_{n}(A) - \{s\}$  $C_{n+1}(B) = C_{n}(B).$ 

In other words, after a participant says the withdrawal of a statement, that statement is not included in his commitment; the commitment of the hearer of a withdrawal is unchanged. It should be noted that one need not be committed to a statement before withdrawing it.

There is, however, a restriction on the withdrawals which can occur:

**R**<sub>Imcon</sub>: No legal dialogue contains an event  $\langle n, A, W's \rangle$  where  $s \in \lambda$ .

This means intuitively that a participant cannot say 'I'm not sure that if p then q' where 'q' is an immediate consequence of (the conjuncts of) 'p'. In actual conversation, attempts to withdraw conditionals corresponding to *modus ponens*, *modus tollens*, simplification, and other familiar valid argument schemata provoke objections like 'But don't you see that it would *have* to be?' and responses like those cited for breaches of R<sub>Gram</sub>.

Since the only way of removing a statement from one's commitment is by withdrawing it, once a participant has become committed to a logicians' conditional he cannot cease to be so in that dialogue.  $R_{Imcon}$ , and the notion of a logicians' conditional, have no analogues in Hamblin's systems, though the 'inerasable' character of the axioms in the system in [2] achieves a similar result.

The treatment of questions in system **DT** is frankly rudimentary. Only bipolar questions of the form  $(?p' \\earrow L$ . To each contradictory pair of statements there are two such questions, (?p') and (?-p'), which have different answer sets  $((!!p') \\ons only (?p'), (!!p') \\ons only (?-p')$ , though it is possible nothing would be lost by conflating these pairs of questions. No attempt is made to solve the fallacy of many questions, which arises in the system if for example 'Clarence has stopped beating his wife' e S; Hamblin gives a dialectical solution of the fallacy in [2], p. 269.

R<sub>Quest</sub>: No legal dialogue contains an event  $\langle n, A, Q's \rangle$  unless it also contains an event  $\langle n + 1, B, l \rangle$  and l ans Q's.

In other words, after a question there must be a next event whose locution answers the question, and whose speaker is the hearer of the question.

The second rule restricting questions is:

 $\mathbb{R}_{\text{Repquest}}$  No legal dialogue contains an event  $\langle n, A, Q's \rangle$  such that  $(\exists l, l' \in L)(\{l\} \subseteq \mathbb{C}_n(A) \& l \text{ ans } Q's \& \{l'\} \subseteq \mathbb{C}_n(B) \& l' \text{ ans } Q's).$ 

That is, a question cannot be asked if each participant is committed to an answer to it. This rule does not, of course, mean that both participants need to be committed to the *same* answer to render a question illegal. If at n, A is committed to 'p' and B to '-p', since each of these statements ans both '?p' and '?-p', neither of those questions can occur at n. Offences against  $R_{Repquest}$  are indicated by such points of order as 'But I've already told you . . .', and offences against  $R_{Quest}$  by 'Answer the question', 'Let me

answer your first question before you ask me another'. Questions do not affect commitment:

CR<sub>Q</sub>: After 
$$\langle n, A, Q's \rangle$$
,  
 $C_{n+1}(A) = C_n(A)$   
 $C_{n+1}(B) = C_n(B)$ .

The treatment of resolution demands constitutes the most important difference between DT and the systems described by Hamblin. A resolution demand does not itself affect commitment:

CR<sub>R</sub>: After 
$$\langle n, A, R's \rangle$$
,  
 $C_{n+1}(A) = C_n(A)$   
 $C_{n+1}(B) = C_n(B)$ .

The first rule connected with the use of resolution demands is:

R<sub>Resolve</sub>: No legal dialogue contains an event 
$$\langle n, A, R's \rangle$$
 unless either:

(i)  $(\exists T \subseteq S)(s = K'T \& T \subseteq C_n(B) \& \text{Imn } T); \text{ or }$ 

(ii)  $(\exists u \in S)(\exists T \subseteq S)(s = C'\langle K'T, u \rangle \& T | \text{Inc} \{u\} \& T \subseteq C_n(B) \& \langle n - 1, B, W'u \rangle \in d \}.$ 

In explaining this rule, I shall suppose for ease of exposition that the participants differ in gender.<sup>3</sup> Ms A can issue a resolution demand only (i) when Mr B has a set of immediately inconsistent statements among his commitments, or (ii) when the previous event was his withdrawal of a statement which is an immediate consequence of (some of) his commitments. Thus if she has got him to admit statements of which s is an immediate consequence, she may then ask him the question of s. He may admit it; or deny it, rendering his commitments immediately inconsistent and leaving himself open to a resolution demand under clause (i) of  $R_{Resolve}$ ; or he may withdraw it, and leave himself open to a resolution demand under clause (ii). If the relation and is elaborated to include other locutions, it is desirable that the elaboration be done in such a way as to preserve this function of questions as admission elicitations.

Resolution demands are themselves rather like points of order, in that they raise objections to the way in which a participant has conducted his part in the dialogue. The rules governing them constitute a further analysis of a rule<sup>4</sup> which merely prohibits participants from committing themselves to inconsistent statements and from withdrawing immediate consequences of their commitments. A participant who commits himself to inconsistencies, or withdraws consequences of his commitments, does not thereby break  $R_{Resolve}$ . The rule  $R_{Resolve}$  is broken only when a participant Aissues a resolution demand *improperly*. I call an utterance of the resolution demand of a conjunction improper if the conjuncts are not immediately inconsistent ('But that's possible' is a protest against this sort of impropriety), and also if its hearer B is not committed to some of them ('But I haven't conceded . . .'). The resolution demand of a conditional may be improper in the latter way too, if B is not committed to all the statements whose conjunction forms its antecedent, but it is also improper if its consequent is not an immediate consequence of its antecedent ('But that doesn't follow'), or if the previous event was not B's withdrawal of the consequent ('Of course it follows, but so what?').

Resolution demands are to be answered according to

R<sub>Resolution</sub>: No legal dialogue contains an event  $\langle n, A, R's \rangle$  unless it also contains an event  $\langle n + 1, B, l \rangle$ , where l is either:

- (i) the withdrawal of one of the conjuncts of s; or
- (ii) the withdrawal of the antecedent, or of one of the conjuncts of the antecedent, of s; or
- (iii) the consequent of s.

This rule is, in effect, the strong arm of deductive logic, which forces a participant to back down when it is pointed out to him that some of his commitments are immediately inconsistent, or that he has refused to admit something which is immediately derivable from what he has said or conceded. When it is breached, A usually protests against B's failure to observe it by appealing to logic as if it were an authority; or by simply repeating the resolution demand.

The rules of DT have now been stated. A dialogue which intuitively 'breaks' a rule is formally a member of that rule. If the statement of a rule is of the form

No legal dialogue contains an event  $\langle n, A, l \rangle$  such that  $\phi$ 

the rule itself is the set

 $r = \{ d \in \mathbb{D} : (\exists e \in E) (e \in d \& e = \langle n, A, l \rangle \text{ such that } \phi) \}.$ 

The set R of rules is

$$R = Ur$$

The set K of legal dialogues of the System **DT** is

$$\mathbf{K} = \mathbf{D} - \mathbf{R}.$$

In Hamblin's 'Why-Because with Questions' system in [2], resolution demands are formed from statements, which need not be conjunctions or conditionals, and must be followed by the withdrawal of the statement whose resolution was demanded or by the withdrawal of its negation. There is no requirement that the other participant be committed to an inconsistent set of statements when resolution is demanded. Indeed, in Hamblin's system there is nothing to prevent a dialogue from beginning with a resolution demand except the good manners of the participants. It is instructive to compare resolution demands in his system with bipolar questions, which in his system are merely one of the sorts of permissible questions, but are the only sort for which **DT** provides. In his system,<sup>5</sup> a question '?(p, -p)' may be answered by the statement 'p', by the statement '-p', or by the withdrawal '(p, -p)'. The other two allowable answers, '(p & -p)' and '(p & -p)' will presumably be avoided by astute participants, though Hamblin gives no way of objecting to the latter. Thus after a bipolar question, Hamblin allows a participant to commit himself to either of the contradictories or to express his lack of commitment on the matter. After a resolution demand "p, Hamblin requires the answerer to say either "p, or "-p, in neither case committing himself to anything, though possibly removing a commitment. For the purpose of eliciting an admission, therefore, the bipolar question is the more appropriate locution in Hamblin's system. Indeed, there seems to be no function fulfilled by the resolution demand in Hamblin's system which is not better fulfilled by the bipolar question, except that of forcing a participant to withdraw one of immediately inconsistent commitments; the formulation of the rules would be more specific if a rule were incorporated preventing resolution demands from occurring except in that way. The same remarks apply to **DT**, and so the situations in which resolution demands are permitted have been restricted.

In System 7 of [3], there are locutions called 'retraction demands', which must be followed by 'retractions'. The latter are like withdrawals except that they can only be used to withdraw statements to which the speaker is actually committed. Retraction demands cannot be issued unless at least one participant is committed to the statement whose retraction is demanded. But since in System 7, participants are forbidden to incur immediately inconsistent commitments, it is not clear what the justification for retraction demands is.

A far more serious criticism of Hamblin, however, is that in none of his systems in [2] and [3] does he provide machinery to deal with a participant who refuses to admit immediate consequences of his commitments, corresponding to clause (ii) of  $R_{Resolve}$  in **DT**.<sup>6</sup> Without such machinery, the very notion of an immediate consequence (or of axioms) lacks dialectical force. In Hamblin's systems, a participant may behave like the Tortoise in Lewis Carroll's charming fable [1]: he may be committed to 'p' and to ' $(p \supset q)$ ', and still refuse to admit 'q'. As a matter of fact about dialogues, this simply does not occur, as indeed was part of Carroll's point in his fable.

Suppose that the Tortoise is committed to statements of the form 'p' and ' $(p \supset q)$ ', and that Achilles asks the question '?q'. The Tortoise can escape admitting 'q' only by saying '-q' or '"q', according to the minimum relation ans and R<sub>Quest</sub>. In the first case, its reply renders its commitment immediately inconsistent, and thus becomes liable to a resolution demand under clause (i) of R<sub>Resolve</sub>. Similar provisions are made by Hamblin. But if the Tortoise takes the second course and replies '"q', Hamblin provides Achilles with no redress. Let the Tortoise look to its shell! The system **DT** does provide a way, with clause (ii) of R<sub>Resolve</sub>. If the relation ans is elaborated beyond the minimum, clause (ii) can be adjusted accordingly.

The second clause of  $R_{Resolve}$  thus gives effect to Achilles' protest that Logic would take the Tortoise by the throat and force it to admit the consequences of its commitment. It is not logic in the sense of the theory of deduction to which he is appealing, for an a priori theory can do no more than describe relations between abstract objects. It cannot force a person (or even a reptile) to do anything. But the rules of dialogue as actually observed do force the Tortoise to admit 'q' (or to withdraw one of its other commitments). Achilles should demand:

$$!(((p \supset q) \& p) \supset q).$$

It is the need for the resolution demand locution modifier ".", and especially for the second clause of the rule  $R_{Resolve}$  neglected by Hamblin, that is the moral of the fable. The system **DT** satisfies these needs. None of Hamblin's systems provides any reply for Achilles; and this I think casts doubt on the extent to which 'the concept of argument is realised' in the system in [2], as he claims (p. 265). That system is inadequate for arguments with Tortoises.

The puzzle which the Tortoise presents is an extremely important one for dialectic, and one moreover which logic cannot solve without invoking dialectical considerations. The twin ideas that one 'cannot say' things which would render one's commitments immediately inconsistent, and that one 'must admit' to immediate consequences of one's commitments, and the presupposition of these ideas that anyone who does not do so is breaking some sort of rule, are the fundamental observations or intuitions which motivate the studies of logic and dialectic. The lack of machinery for dealing with the second of these errors is therefore a serious defect in a dialectical system. Any account of dialogue must provide a method of dealing with Tortoises.

Though **DT** provides a reply for Achilles the list V of preferred valid argument schemata may contain no more than *modus ponens*. Additional schemata can be added by noticing that the resolution demand of a conditional can occur only if the conditional  $\epsilon \lambda$ . A field linguist investigating a language, however, does so most effectively by participating in dialogues in it. Since *ex hypothesi* he does not know the rules, he is sure to make mistakes. To accommodate his mistakes, it is necessary to extend **DT**.

The idea behind this extension is to introduce points of order which can be raised to protest against breaches of the rules of **DT**; then to amend those rules to accommodate the points of order. We then add a rule,  $R_J$ , which provides that if an event occurs which renders the dialogue illegal in the original sense, the dialogue is *legal in an extended sense*, or *legal*<sup>+</sup>, provided that at the next stage some participant raises the appropriate point of order. In this extension, points of order cannot be debated; nor can they legally<sup>+</sup> occur unless justified.

We begin the extension by introducing an additional set  $L_R$  of order locutions:

 $L_{\rm R} =_{df} \{ L_{\rm Gram}, L_{\rm Repstat}, L_{\rm Imcon}, L_{\rm Quest}, L_{\rm Repquest}, L_{\rm Resolve} \}.$ 

An order locution for  $R_{Resolution}$  would require extra rules to give it the effect of the preceding resolution demand, and is omitted from this extension. The commitment effect of order locutions is:

 $CR_{J}: \text{ After } \langle n, A, l \rangle, \ l \in L_{R}, \\ C_{n+1}(p) = C_{n-2}(p).$ 

In other words, after a point of order, the commitment of each participant p reverts to what it was before the illegal event which occasioned the point of order occurred.

The set  $L^+$  of locutions of the extended system is:

 $L^+ = L \cup L_R$ .

But we must make some amendments to permit order locutions to occur; first, the grammatical rule:

 $\mathbf{R}_{\text{Gram}}^+$ : No legal<sup>+</sup> dialogue contains an event  $\langle n, A, l \rangle$  unless  $l \in L^+$ .

The relation ans is extended so that  $(\forall s \in S)(L_{\text{Repquest}} \text{ ans } Q's)$ . This extension of ans does not affect the function of questions as admission elicitations, since it can occur only after illegal questions.  $L_{\text{Quest}}$  occurs rather after illegal answers, so it is not itself an answer.

 $R^+_{Resolution}$ : (Like  $R_{Resolution}$ , but with the additional clause:) or (iv)  $L_{Resolve}$ .

We next define the set R' of amended rules:

$$\mathbf{R'} =_{df} \mathbf{R} \cup (\mathbf{R}_{\text{Gram}}^+ \cup \mathbf{R}_{\text{Resolution}}^+) - (\mathbf{R}_{\text{Gram}} \cup \mathbf{R}_{\text{Resolution}}).$$

There is a one-one function J from each order locution  $l \in L_R$  to the corresponding amended rule  $r \in R'$ .  $L_{\text{Gram}} J R_{\text{Gram}}^+$  but no locution  $J R_{\text{Resolution}}^+$ . With the help of this function, we may state the extension rule:

**R**<sub>J</sub>: No legal<sup>+</sup> dialogue contains an event  $\langle n, A, l \rangle$  such that any dialogue containing that event  $\epsilon r \epsilon \mathbf{R}'$  unless it also contains an event  $\langle n + 1, p, l' \rangle$  and l'Jr.

The rules  $R^+$  of the extended system are given by:

 $\mathbf{R}^+ =_{df} \mathbf{R}' \cup \mathbf{R}_J.$ 

The extended system  $DT^+$  is the triple  $\langle P, L^+, R^+ \rangle$ ; its legal<sup>+</sup> dialogues are just the members of  $K^+ = D - R^+$ .

Field linguists who know that the language they are studying is used only for dialogues in  $K^+$  should use the following features in drawing up a list V of valid argument schemata preferred by native speakers.

- (i) If (n, A, L<sub>Imcon</sub>) ε d, then it may be inferred that the locution l of the previous event (n 1, p, l) is W's: s ε λ.
- (ii) From the events  $\langle n, A, R'C'(s, t) \rangle$ ,  $\langle n + 1, p, l \rangle \epsilon d$ , where  $l \neq L_{\text{Resolve}}$ , it may be inferred that  $C'(s, t) \epsilon \lambda$ .
- (iii) From the events  $\langle n, A, R'K'T \rangle$ ,  $\langle n + 1, p, l \rangle \in d$ , where  $l \neq L_{\text{Resolve}}$ , the linguist should form conditionals by alphabetically ordering and left-conjoining all but one of the conjuncts of K'T as antecedent, and the denial of the remaining conjunct of K'T as consequent, and withdraw these in the hope of evoking  $L_{\text{Imcon}}$  from his interlocutor.
- (iv) From the knowledge that a conditional  $s \in \lambda$ , it may be inferred that the expression formed by writing the quotation names of the conjuncts of the antecedent of s, followed by a slash, followed by the quotation name of the consequent of s, exhibits a schema of V.

The role of  $L_{\rm Imcon}$  as signalling conditionals which exemplify valid argument schemata preferred by the natives suggests that utterance of  $L_{\rm Imcon}$  is one of the 'bizarreness reactions' to which W. V. Quine refers, [5], p. 53. Without understanding its role, the linguist would be unable to investigate the languages of those who can not only talk to Tortoises as Achilles did but can also, unlike that warrior, cease to do so.

### NOTES

- 1. Hamblin in [3] took a system to be rather  $\langle P, L, K \rangle$ .
- 2. This feature is a necessary but not sufficient condition for the legal dialogues of a dialectical system to be *undistorted* in the sense of Wellmer [6], p. 47.
- 3. No reflection is thereby intended upon the logical aptitude of people of either sex.
- 4. Hamblin's System 7 in [3] has a rule with the effect of the first part of the one discussed. In general, replacing a rule by a locution which protests against breaches of that rule, and finding in turn rules to govern the new locution is a useful strategy in dialectic.
- 5. Hamblin in [2] used locution modifiers forming questions and withdrawals which apply to sets of statements enclosed in parentheses. I have preserved that feature, while adapting his notation to resemble mine typographically.
- 6. At one point Hamblin mentions this difficulty: 'There must be rules, that is, requiring him to concede consequences of his various admissions. Again this is difficult to formalize realistically.' [2], p. 278. Hamblin was at this point considering answers to questions in dialogues of the Greek Game, exemplified in passages of Plato's dialogues. Special aspects of dialogues of this kind, such as the use of arguments from the authority of the poets, obscure the general issue of conceding consequences in them. Nevertheless, the incorporation of resolution demands and both clauses of R<sub>Resolve</sub> seems to me to go a long way towards the formalization of this requirement in the Greek Game.

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