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Facts and the Choice of Logical Foundations

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In [5] Routley proposes that mathematical theories of choice should be applied to the choice of logical foundations. Since this idea is rather programmatic so far, it is not possible to evaluate its adequacy. But his opinions concerning certain fundamental aspects of logic call for comments and criticism—especially since the article seems to bring forth rather "empiricist" views on logic. More importantly, many questions considered in the article are of great significance in the philosophy of logic, and they have often caused confusion. In this paper, I shall first comment on Routley's appraisal of the weights of different factors in such a choice. As soon as we speak about a logic in a sufficiently precise and abstract sense, for instance as a formal theory, it is not quite obvious that the external factors, concerning applicability, etc., should be considered more important than the internal ones, as he seems to suggest.

Secondly, I shall say something general about Routley's notion of logic, especially about the distinction between intensional and extensional logics, and, furthermore, about the far-reaching consequences his program would ultimately lead to if carried out.

Thirdly, I shall touch on the notion of inconsistency as it is used in Routley's paper.

1 Logic of logics Since Routley's discussion about the choice of logical foundations belongs to the area that could be called "the logic of logics", let me first relate it to the current research done by mathematical logicians in that area. Routley does not mention that research, and on the basis of what mathematical logicians have been doing recently, I cannot completely agree with some of his claims about the topic.

According to Routley ([5], p. 78), a choice of a logical theory should not be an arbitrary matter. The question of such a choice ought to be approached in a logical and systematical way. This much can be taken for granted. Perhaps the question could even be approached by using a theory of rational choice, although it is not clear how it would work in practice.

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However, I cannot agree with his claim that logicians have not approached the question of logical foundations (the choice of a logic) in a logical fashion, that their choice of systems has been extremely unsystematic. It is enough to look at the work done in mathematical logic during the last fifteen years. Mathematical logicians long ago realized that what is often called classical logic is not ideal in every respect. In particular, first-order logic (elementary logic) has appeared to be too weak in its expressive power for certain metamathematical purposes, whereas higher-order logics, type theories, etc., are so strong that their properties either are not known very well or they have not been considered very well-behaved logics. These observations have, in fact, created a new " logic of logics", commonly called "abstract logic", studied in a set-theoretic framework. In such a framework it is possible to state a general definition of a logic and to study properties and relations of logics.

Since logicians often know which properties would be desirable in general and which for certain specific purposes and how to define those properties in an exact set-theoretic way, I would consider at least a part of the study in abstract logic highly systematic. It is sufficient here to refer to certain works of Barwise where such questions are systematically dealt with (see, e.g., [1]).

Now, it is very important to notice here that although a part of the systematic work in abstract logic is concerned with a search for straightforward generalizations of elementary logic which would be more powerful but still have a workable model theory, this new concept of a logic is general enough to furnish us with a (classical) framework for a study of logics which are not at all classical, even of logics that are connected with intensional and inexistential discourse, as, e.g., in natural language.

It certainly is true that one motivation for new logics has been their applicability to other fields of mathematics. But the factors which have been regarded important for a new logic are not only pragmatic, connected with applicability, scope, accountability to the data, correspondence, and so on. For a logic is after all a mathematical theory, or at least it is dealt with by mathematical methods. So there are certain intrinsic, perhaps somewhat vague, standards or factors related to mathematical elegance and fruitfulness that are to be taken into account (as in mathematics in general). Thus a large part of the work in abstract logic has been concentrated in a search for logics whose mathematical fertility, or elegance, and applicability would be in balance.

So it is not very easy to accept Routley's opinion ([5], p. 88) that the factors which are related to those intrinsic mathematical standards are wholly lightweight factors. As soon as we think of a logic as a mathematical discipline, and such it certainly is as soon as it is made exact, those factors cannot be lightweight ones. Mathematics, as well as logic, is an art as well as a tool. Moreover, as we know from the applicability of many of the mathematical theories, even from theories of physics, mathematical elegance and applicability are coexistent properties; rather than being independent of one another, they are mutually supportive.

Thus I am not quite convinced of whether one should look for a new logic with the sole purpose that it would act as a framework for something, without also putting a heavy weight on its internal properties. I am not saying that a mere formal framework couldn't be useful in clarifying and deepening vague ideas in any philosophical area. Philosophical logic, even in such a poor sense of "logic", appears to be useful to some extent. But even if applications of philosophical logic were our main concern, couldn't we try to take a lesson from mathematical logic, or even mathematical physics, where internal logical or mathematical properties are often utilized in applications?

2 Intensional versus extensional foundations If we are looking for a new logic or if we want to use a mathematical theory of choice in search of "the best logic", there are two questions to be answered first: What is a logic?, and How are different features of a logic to be taken into account?. The second question is discussed by Routley, and I have made some vague remarks concerning it as well. So I turn to the first question, which is a very basic one.

It is a little disconcerting that this question is not answered by Routley, at least not in the paper at hand. For is it possible to use a mathematical theory of choice for selecting a logic if we don't know what a logic is as a mathematical entity, if we don't know what we are choosing? Or must we think that if we can somehow define the relevant factors and constraints of a logic without defining a logic, those factors and constraints fully determine a logic, whatever this word means? There are some results in abstract logic indicating that in some cases various properties determine a logic, but there the starting point is a definition of a logic.[1]

There is no similar definition, nor are there similar results, available in philosophical logic. Perhaps the definition of a logic as it is stated in the framework of abstract logic could be applicable, or could be extended so as to become applicable, to what Routley might mean by a logic. Some work has been done, anyway, to capture (in that classical, extensional framework) certain nonclassical and philosophical logics by means of abstract logic (see, e.g., [3] and [4]). Of course, an answer to the first question could be different if a framework in which it is given were nonclassical.

Instead of further dealing with this question, which seems to be somewhat open so far, I would like to deal with another, closely connected problem discussed by Routley.

It is common to distinguish between extensional and intensional logics, i.e., logics which provide explications of some extensional modes of reasoning connected with natural language, mathematics, empirical sciences, etc., and logics providing explications of some kinds of intensional reasoning. The way in which Routley discusses that distinction is, however, somewhat confusing. While he correctly remarks that "...every intensional language has an extensional semantics, so that whatever can be said intensionally can be given extensional re-expression (of a sort)", he says also that "...mathematics is intensional and not essentially extensional..." (see [5], pp. 83–84).

The latter remark seems to suggest that the term "intensionality" contains more than non-truth-functionality, nonsubstitutivity of equivalents, relevant reasoning, or any other technical meaning. For how can ordinary (informal) mathematics, as ordinary mathematicians themselves understand mathematics, be intensional? As far as ordinary mathematics is intensional, this feature seems to come from context-dependency, rather than from non-truth-functionality, etc.

(if context-dependency can be counted as an intensional feature). However, according to the usual view, the hard core (so to speak) of mathematics, or of some branch of mathematics, is essentially extensional. This is even supposed to be a distinctive feature of mathematics. And it is that hard core that can be formalized in an extensional logic, more or less satisfactorily. So instead of regarding pure mathematics as essentially intensional, as Routley does, I would rather turn that claim around and say that whatever can be defined in strictly mathematical terms, in the current sense of ''mathematical'', is essentially extensional. Although a logic used in ordinary mathematical reasoning is intuitive (and intuitive logic in general might be considered intensional to some extent), the hard core of mathematics is sufficiently exact, abstract, context-independent, truth-functional, dependent on modus ponens, or whatever we need to characterize extensionality.

Moreover, if mathematics were essentially nonextensional, how could there be any formal logic which would be extensional? For can we define a formal logic, its syntax and semantics, if we do not use exact mathematical, especially set-theoretical, terms? If the metatheory of a logic, being a mathematical theory, were essentially intensional, so would that logic be, too.

So it seems to me that by saying that mathematics is essentially nonextensional, Routley is contradicting his claim that every intensional language has an extensional semantics. It is easy to agree with this latter claim as long as by "semantics" we mean set-theoretic semantics (which is often called "formal semantics") of a formal language. I would even say that every logic or language, whether basically extensional or intensional, becomes extensional as soon as it is made formal—independently of any questions of truth-functionality, etc.

Even though the claim that every intensional language has an extensional semantics thus seems correct to some extent, it is not quite clear, however, what it ultimately means. It may be somewhat misleading, or at least seems to need some elaboration. In the light of what I just said, the claim appears to imply that many informal languages have a formal semantics. This follows if we agree that intensional semantics is basically informal and that an extensional semantics is basically set-theoretical (in the current sense). But as far as an informal language is to some extent vague, how could it have a formal semantics before it is first translated into a formal language? So I would rather say that an intensional language has an extensional semantics after this language has been made extensional. But even this is not quite correct. For, strictly speaking, the semantics obtained is not a semantics of the original language but of the translated one.

By claiming that current mathematics and set theory are extensional, thus implying that any formal logic is extensional, I don't want to claim that there is no need for intensional mathematics, especially set theory, to act as a metatheory of intensional logics. It also seems that what Routley ultimately wants is an intensional metatheory. But it is obvious that a change from an extensional mathematics or set theory to an intensional one would mean a radical change of the mathematical paradigm, so to speak. That is, it would require that mathematicians and set-theoretists themselves, or a major part of them, change their opinion about what mathematics is. The attempts to clarify the foundations of current extensional set theory has so far taken a century. So couldn't we expect that even if the paradigm changed, it might take at least another century to make a new paradigm even as well established, even as free from perspicuous paradoxes, as the old one is now? Moreover, as to our intuitions about sets, aren't we used to thinking of them as extensional entities in some sense? So what we would need to begin with is a deeper intuition of sets and other mathematical entities as intensional objects. If we agree that intuition is a basis for all theories, we can also agree that this could be the most difficult step in the enterprise. For even if we manage to create an intensional set theory as an exact discipline (and there exist recent attempts to that effect), how can we guarantee that our intuitive reasoning, underlying this attempt, is not based on extensional thinking (as is the case in those recent attempts)?

More generally, for Routley the best choice of a logic is ultimately an ultralogic (intensional, inexistential, ultramodal, paraconsistent, and relevant) whose metatheory is also based on an ultralogic (see [5], p. 97). But it is very difficult to get an intuitive grasp of such a logic. As soon as such an ultralogic is formalized, its metatheory must be a sufficiently exact mathematical theory, presumably a set-theory, in the framework of which the basic principles of that logic can be precisely expressed. So what I would like to see is how the rules of inference, the rules of satisfaction, etc., could be stated so that we really know what we are allowed to do, and what not. Although Routley himself acknowledges that difficulty, it would be very instructive if this matter could be further elaborated (for instance, by a simple example), for it appears rather disconcerting. For the time being, however, we may adopt a reasonable principle of expediency, as Routley suggests, or perhaps the Carnapian principle of tolerance. For it is very well-known to relevance logicians themselves that it can be very hard to know whether even certain set-theoretically simple metalogical arguments are completely in accordance with relevance logic itself, that is, that they are not based on extensional mathematical reasoning. Consider, e.g., the discussion about the acceptability of disjunctive syllogism. How difficult could it be to know, then, the same thing about a full-blown relevant model theory?

3 Inconsistency Another thing that is somewhat misleading is Routley's discussion of inconsistency. Although his paper is not detailed enough to bring out what is really involved in his notion of inconsistency, let me make some general remarks on the basis of what I am able to extract from it.

It seems to me that the terms 'inconsistent' and 'consistent' can be properly applied only to language, or rather to logic, and possibly to propositional attitudes, etc. But if we are allowed to make any distinction between mental and physical it is not very easy to understand what it means to say that *the world* is consistent or inconsistent (see [5], p. 89), as far as physical phenomena or situations are concerned. If, for instance, one gets the impression that a motion of a body is inconsistent, does it mean that the motion really is inconsistent in some sense? Doesn't it rather mean that one is not able to explicate the motion consistently by theoretical means?

It does not sound intelligible to me to say that the world is inconsistent (even if I knew what "the world" exactly is) unless it is interpreted as meaning, for instance, that there are inconsistent statements *in* the world or *about* the world.

However, even this cannot be quite correct without a qualification. For there is no absolute inconsistency. If, to be more definite, we turn to logical inconsistencies, a sentence having a certain syntactic form can be inconsistent in one logic but consistent in another. As Routley does it (although it is not quite clear whether he means here only classical logic), a reasonable way to define inconsistency in a logical framework is to say that a sentence or a set of sentences is inconsistent in a logic if every sentence of the corresponding language can be inferred from it in that logic. Then what is classically inconsistent need not be inconsistent, e.g., in relevance logic. (Thus we can perhaps say that relevance logicians are studying classical inconsistencies from the standpoint of relevance logic. But in so doing, they are just changing the meaning of the formulas of certain syntactic forms.)

Now, if the consistency of a logical system means that not every sentence can be inferred, or not everything is true, then it is somewhat difficult to see why "...the consistency requirement is especially objectionable" ([5], p. 89). As Routley puts it, that requirement "...legislates *a priori* against paraconsistent and dialectical theories; that is, it makes the very large assumption that the world is simply consistent...". For if a paraconsistent or dialectical theory is inconsistent in the sense indicated, then it is trivial. Perhaps what is meant here, however, is that a theory can contain sentences that are of the form of classically inconsistent sentences.

But in any case the main point in Routley's discussion seems to be that a logical theory must be (in some relevant sense) *in conformity with* the world, and the world can be inconsistent. Let us suppose, then, to avoid trivial theories, that inconsistency in connection with a paraconsistent theory means the classical inconsistency of some of its sentences and that such a theory describes the world "correctly", that is, it is "true" in the framework of the paraconsistent logic in question, so that the conformity constraint is satisifed. Does this somehow strengthen our intuitions about what it means that the world itself is inconsistent? It cannot be inconsistent in the sense that a classically inconsistent sentence would be classically true in it. Neither can it be inconsistent in the sense that an ultralogically inconsistent sentence would be ultralogically true in it. It must mean, then, that a sentence of the syntactical form of a classically inconsistent sentence is ultralogically true in the world. But I still cannot see why the world itself would deserve to be branded as inconsistent. In particular, we can take, for instance, a sentence of the form $\phi \wedge \neg\phi$ and have a semantics such that $\phi \wedge \neg\phi$ is true in a model $\mathfrak{M}$, say. But does that entitle us to say that $\mathfrak{M}$ is inconsistent? Can't we just say that our semantics is different from the classical one, which implies only that the meaning of the sentence $\phi \wedge \neg\phi$ is, by definition, different from its classical meaning? We could also say that in this case the "basic structure" of the world is thought to be different from that of the classicists' world. I have discussed the question of inconsistency at great length because Routley's article is too concise to make his point clear. Furthermore, the notion of inconsistency seems to produce similar difficulties in related areas, as, for instance, in dialectics.

On the other hand, it is quite clear that a nonclassical formal semantics is needed such that it makes classically inconsistent formulas satisfiable. But as I see it, such is not needed because the physical world is inconsistent, but rather because our attitudes toward the world can be (classically) inconsistent or expressed in (classically) inconsistent terms.

NOTE

1. The first of such results is contained in [2].

REFERENCES

[1] Barwise, J., "Axioms for abstract model theory," *Annals of Mathematical Logic*, vol. 7 (1974), pp. 221-265.

[2] Lindström, P., "On extensions of elementary logic," *Theoria*, vol. 35 (1969), pp. 1-11.

[3] Manders, K., "First-order logical systems and set-theoretic definability," forthcoming.

[4] Paulos, J., "A model-theoretic semantics for modal logic," *Notre Dame Journal of Formal Logic*, vol. 17 (1976), pp. 465-468.

[5] Routley, R., "The choice of logical foundations: Non-classical choices and the ultralogical choice," *Studia Logica*, vol. 39 (1980), pp. 77-98.

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