

## A Diachronic Semantics for Inexact Reference

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We have evidence, in the form of paradoxes like Eubulides' paradox of the heap, that the semantical relations between ordinary languages and the world are inexact. This lack of fit has seemed to be unproblematic, and perhaps even appropriate, given our physical limitations and the methods we use for getting information. We can correctly apply our concepts even though we measure inaccurately. Until we assume that small changes in the world cause no differences in the applicability of the concepts, and reiterate applications, we encounter no problem. But Eubulides and many others since his time have claimed that this sort of reiteration yields contradictory assertions.

Although inexact reference is ubiquitous and exact reference is rare, only for the latter do we have fully satisfactory semantical theories. I wish to suggest a framework for a semantical theory for inexact reference which departs in several important respects from the usual ones. My central pre-supposition, that reference is a diachronic phenomenon,<sup>1</sup> can take two forms. The weaker is that the reference of a token of a singular term (for example, 'Fred') is not determined merely by facts about the time and situation in which it is actually used, but also depends, in an essential manner, on other, usually earlier, uses of 'Fred' by speakers appropriately connected to the speaker at the given time. At least this much has been supposed in the historical pictures of reference which have been in development since the early papers of Kripke, Kaplan, Putnam, and others. But a supporter of this form of diachronic semantics might say further that at time *t* the reference of 'Fred' is *already determined* and, moreover, that one might be in a position, given all the

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appropriate information about earlier uses of expressions, to describe a semantics for the language at that time. If this were true, programs like the Montague grammar program (see, e.g., [12]), if taken as an attempt to provide a "time-slice" description of reference, would still be viable, at least as far as this point is concerned. In principle, a researcher could provide a complete description of the language at  $t$  based on data prior to  $t$ ; and a series of such semantics could allegedly completely picture the language.

My (stronger) presupposition is that such time-slice descriptions of semantical relations are too truncated to be sufficiently accurate. My first point is a minor one: The reference of 'Fred' at  $t$  may be influenced by future as well as past uses of the expression (and others which are coreferential). A second, connected, but somewhat controversial point is that the reference of 'Fred' at  $t$  may be influenced by what we *deem* to be the reference of 'Fred' at  $t$  and at other past and future times. Thus, I claim that in using an expression with the intent that it refer in the normal way, the speaker makes a hypothesis which may or may not turn out to be correct. One thing this means is that "initial baptisms", such as those described in Kripke's work are more than the mere oversimplification which he intended, they are misrepresentations.

A third point: The "time-slice" models of reference have no hope whatsoever of accounting for *change* of reference. If they were accurate, these models might correctly give the reference for expressions in a language at  $t$ . But this result would be a mystery. Some changes are abrupt; some less so. But what mechanisms are operating? To answer such a question a description of reference needs "outlets" into which one can plug some sort of explanatory theory. My belief is that this is best achieved by a two-tiered theory. As an abstract system, the model I give serves for either of these tiers. It can model "epistemic reference"—the hypotheses which underlie our everyday use of language—or reference itself—the truth of the matter, insofar as there is one.

A *social theory of language determination* is what is required to show how reference is determined by use and the reverse: how the facts about reference affect the ways in which people use expressions. Since developing such a theory is not an appropriate aim for a paper such as this, I offer only a coarse generality or two in the appropriate places to give some indication of how epistemic reference and reference are related. Putnam's Principle of the Division of Linguistic Labor—that some of us do succeed in referring to things about which we know very little, as long as some of us have more information (see, e.g., [14])—is the sort of principle on which such a theory would rely. The theory would, for example, enable one to determine whose usage is more heavily weighted in determining reference, or what we must believe about what we take ourselves to be referring to, or what we must believe about those with whom we are trying to communicate.

A fourth point: The models I give are geometric. It is a well-known fact that a Kripke semantics, for example, for propositional S4,<sup>2</sup> can be understood as a "translation" of propositional intuitionistic logic and that these are equivalent, in an appropriate sense, to both a Heyting Algebra representation and a topological representation. (For the latter, see below.) It has not been the practice, since Tarski's theory of truth, to represent semantical relations geometrically.<sup>3</sup> Thus some defense of this approach is needed.

Most transparent is the simple defense that a model should be one that allows and can potentially explain changing reference. *Location in a context* becomes an important feature. In doing a traditional semantics, whether or not it is thought to be a mere "time-slice" model, the parameters of time, place, and possible situation are frozen. In a geometrical model we allow these to vary and are able to look at the referents of our expressions as five-dimensional entities. (for visualizability, I usually omit all spatial dimensions, or all except one.) Thus Fred, throughout his life and with all of his possibilities, is the referent of 'Fred'—or at least can be.

A perhaps more obscure defense is one that I will merely mention, since I believe the previous paragraph justifies my position to anyone who thinks semantics is diachronic. In brief, I believe that any attempt to get away from the mystique of the syntax and semantics of formal logic will be helpful to the philosophy of language. We ought to realize that a logic is just one kind of formal system and that other systems, even if they are merely isomorphic variants of logics, may give more perspicuous representations of the phenomena we want to understand. This is especially true since, to a significant degree, a model is helpful because of the metaphors it evokes. The preservation of some aspects of the structure of the phenomenon being modeled is an insufficient criterion of adequacy.

A final remark to those readers who believe that what follows is just an unnecessary complication of a good and useful formal representation of reference. To draw an analogy, let me ask: What good would a classical mechanics be that considered the mass of a particle to be distributed throughout the particle rather than concentrated at a point? The mathematics would be much more complicated and there would be no increase in either understanding or in application. Similarly, why should we look at variation through time and possible situation as part of *semantics*? Should the ideal be to represent everything that happens in language use in a model of reference? How about blindness, nervousness, lying?

Of course not. But I don't think this is a trivial dispute about where to draw the line. Some psychological and social features of language use are indeed part of pragmatics, as I use this expression. But a semantical theory should do more than describe the ultimate reference relations: It should also show where they come from and how they are based on other facts in the world. Some of the more important of these facts are the speaker's beliefs (usually implicit) about reference—at least as far as these beliefs are exhibited by usage. The facts about what a speaker thinks an expression refers to are essential for determining (insofar as it is determinable) what the expression does refer to. And we know, since Kripke and Putnam have pointed it out, that this determination cannot be merely a local one. For these reasons I think that the complications of a diachronic semantics are necessary for the backbone of a theory of reference.

Further, since reference is, in general, inexact, we are sometimes uncertain about the application of predicates in some kinds of contexts. If we took this uncertainty at face value, reference would appear to be much more indeterminate than it is. Using a diachronic semantics we can get a truer picture by looking at beliefs about the application of predicates in a variety of contexts.

This paper proceeds as follows: In Part 1, I describe the picture of epistemic reference—how it varies from local to larger contexts of use. I also briefly indicate how the variation of epistemic reference through larger contexts may determine a global picture of the actual reference of an expression. In Part 2, I outline a formal representation of global reference which can either be interpreted epistemically or metaphysically. In Part 3, I offer some criticisms of the two major kinds of models of inexact reference which have heretofore been suggested.

*I* In order to arrive at a working definition of ‘inexact reference’ we need some preliminaries. For a given predicate *P* of a given language (which we will consider to be one language even though its details change), we can partially order possible contexts of use with respect to similarity. (This problematic ordering is itself “context dependent”. Thus it will vary for different speakers and for different predicates.) Letting this spectrum of possibilities function as a dimension which takes on values along with the dimensions of space-time, we can consider a set of points indexed by three spatial dimensions (of which, for clearer illustration I will ignore two or sometimes three), a temporal dimension, and a dimension of possibility. In other words, each point is a place-time in a possible situation. Note that because the ordering of possibilities can be at best partial, distinct points may be equally distant from a given point and in the same “direction”. Space-time will be taken to have whatever metric it has. Although I take it that one can speak of possibilities as being more or less distant, it seems quite inappropriate to assign quantities in this mere ordering.

Any connected set of such points will be called a *context*. Thus a context may involve spatio-temporal portions of several distinct possible worlds. By narrowing the context in the “possibility” dimension we have a more and more complete specification of the situation we are in. Either we can think of a set of points or a context as a set of beliefs, which we refine by getting more precise knowledge as to which of a number of similar possible situations we are in; or, metaphysically, we can think of a context as a set of possible facts some of which are more or less similar to, but different from, others in the set. (For example, at one point there may be brown, at another, red-brown.) As we narrow the context we rule out some of these facts. At the limit of a single point we have a single consistent set of facts.

Just as contexts are extended by including other possible contexts from which they are not distinguishable, objects can also be so extended. These inflated objects, which are in truth many possibilities of objects, are useful because they mirror more exactly what is important about the way we use language: We do not usually know or even *believe* enough about an individual to fix absolutely and descriptively the possible situation it is in. Further, metaphysically, not all the facts about an individual are relevant to facts about reference—whether a singular term or a predicate is in question. An example or two is in order. Consider a context which, because of its limited extent in space-time, is almost filled with Tess. Now, some facts are irrelevant to the question of whether ‘the tallest person in Philosophy 201’ denotes Tess or not. For example, whether Tess has a missing thumb nail, has grey hair, is the best scholar in the class, and so forth are among the innumerable irrelevant possible

features of Tess. We can specify the context we want merely by saying, "Consider the states of affairs in which Tess is the tallest person in Philosophy 201." We do not have and need not have any more information about this context. It contains many distinct possible situations, but we may talk about it without distinguishing them.

Epistemically, the usefulness of considering individuals in an "ambiguous" context is even clearer. What would someone have to know in order to correctly apply the predicate 'is the tallest person in Philosophy 201'? Certainly one would not need to know facts about the situation which are irrelevant to that application.

In general, we specify a context by saying what is true in it and what exists there. Other things will be true in *some* of it and other things will exist in *part* of it. But in order to say this we have to narrow the context to where only these things are true and these things exist.

If  $P$  is a predicate, I will use  $\lceil H(P) \rceil$  to denote the comprehension of  $P$ , that is, the set of individuals to which  $P$  may or may not apply. ('May' alludes to what is psychologically possible. This sense of the possible, which I will not explain further here, is neither included in nor excludes what is metaphysically, logically, or epistemically possible. Roughly, we limit ourselves to those things to which it is psychologically possible to apply a predicate (or to what it is possible for a singular term to denote) in order to rule out reference failures caused by presuppositions not being satisfied (is this idea green or not?).) I shall say  $P$  *refers inexactly in a context*  $C$  if  $C$  is an appropriate context for evaluating  $P$ , i.e., one in which facts relevant to the application of  $P$  do not vary significantly, and there is an object  $O$  in  $H(P)$  in  $C$ , and  $P$  does not apply to  $O$ , and the negation of  $P$  does not apply to  $O$ . Such objects are in the *boundary* or border of  $P$  in  $C$ .  $P$  *refers inexactly* if there is an appropriate context in which  $P$  refers inexactly.  $P$  *refers exactly in*  $C$  if its boundary is empty in  $C$  and  $P$  *refers exactly* if it refers exactly in all appropriate contexts. In [18] I discuss this phenomenon more fully.

For simplicity of exposition I will limit my discussion to one-place predicates. Frequently in a given use a many-place predicate can be construed as a one-place predicate. For example, 'is taller than' can be replaced in some contexts by several one-place predicates containing names, such as 'is taller than Alice,' 'is taller than David,' etc. This simple device will not accommodate occurrences of many-place predicates in which more than one position is governed by a quantifier. Thus, what I say here applies at best to a fragment of the language. Nevertheless, I have no reason to think this picture of reference is not generalizable.

The singular term  $a$  can be treated analogously by construing the "negation" of a singular term to be that predicate which applies to all things in  $H(a)$  which  $a$  does not denote and which are not on the border of being  $a$ . Because of this formal similarity I will frequently use the neutral 'e' or the neutral 'refers to,' as ambiguous between respectively a singular term and a predicate or 'applies to' and 'denotes'.

I note here that the model below leaves open the question of whether there are terms that refer directly (in the sense of David Kaplan), that is, terms whose reference is not determined by objects satisfying relevant descrip-

tions. Below I do appeal to a notion of “relatively direct reference” that can, however, be omitted without any significant damage to the picture. If there is such a thing as direct reference, the question remains as to whether facts about direct reference are irreducible facts about the world or whether they depend, to a greater or lesser extent, on epistemic reference. As well, the question of whether beliefs of the speaker determine reference is left open—whether, for example, I have a paradigm of a tiger (which we might call, in a sense, the *meaning* of ‘tiger’ for me) and whether this paradigm functions at all to determine reference. (Although I do believe that for many important classes of expressions, the paradigms associated with them will have little immediate effect on the reference of the expression, a paradigm will have some effect on how an expression is used—at least if the world is much as we believe it to be. In turn, use will help to determine reference globally.)

Let us now look at judgments about reference—uses of an expression which are purported to refer to a particular object. Suppose that any preconditions for asserting (whatever these are) are satisfied, so that *if* reference is successful, an assertion would be made. In this case, when is the attempt to refer successful? (Of course, we are speaking of semantical reference: The *speaker* may succeed in referring even if the expressions she uses do not apply to what *she* speaks of.<sup>4</sup>) Superficially, reference is successful when the use of the expression is part of a Kripkean communication network of speakers and the expressions *these* speakers use do refer to the object. Unfortunately, I cannot here even begin to address the question of when a particular use of an expression is a part of such a network. But I think it will be helpful to note the following components of the connection between *deeming* that *e* refers to *O* and its being the case that *e* refers to *O*:

1. There is a fact of the matter as to whether *e* refers to *O* in a context *C* (barring borderline cases and presupposition failures).

2. There are some attempts at reference that we might call *primitive*. In these cases reference is as direct as possible and semantic reference coincides with speaker reference. Frequently, a singular term is used anaphorically with the relatively directly referential expression or a predicate is deemed to apply to the thing directly referred to. Then these particular deemings of the reference of the term or predicate will be more important than others in determining its actual referent. For example, suppose the speaker gestures and says, “Look at this”. Suppose she manages to pick out a single object. And suppose further she says either, “This and other things like it are persons” or “This is Tess”—or uses some similar expression of the simpler sort that introduces an object or its kind. Such cases are cases of primitive reference. They are related to Kripke’s initial baptisms but are more general and necessarily involve the use of an expression rather than its mention. These uses are primitive, because the object is accessible by (relatively) direct reference and because they play a central role in determining what the referent is in suitably connected cases where the object is not immediately accessible, or where it is immediately accessible, but a directly referring expression is not used. A minimal condition for *secondary* reference being correct is that it be presupposed that the use in

question of the expression refers to what the linguistic community refers to by it.

3. In any use of an expression contained in an utterance which satisfies the presuppositions for making a meaningful veridical assertion, we will say that the speaker deems that (an expression used in making this assertion) *e* refers to (some object) *O*.

4. For contexts *C* and speakers *S* we have a partial function which will assign objects to  $\langle e, C, S \rangle$  as *e*, *C*, and *S* vary. We say that *O* is a value of this function for  $\langle e, C, S \rangle$  just in case *S* deems that *e* refers to *O* in *C*. We can turn this around as well. For each object and expression there is a partial deeming function which for speakers in contexts gives values *yes* or *no* according as the speaker in that context deems that the expression or its negation refers to the object.

5. To completely describe the epistemic reference of a predicate *P* we look at the values of this function in all appropriate contexts for *P* and over all objects in  $H(P)$ . (What counts as the comprehension of *e* is determined in the same way: We start with local (narrow) contexts and get a partial function which tells for each relevant local context and each appropriate speaker *S* whether or not *S* deems that the object is in the comprehension of *e* in *C*. In fact, the metalanguage notions I am using here, like reference, epistemic reference, and appropriate context, get *their* reference in the way I am describing.)

6. This partial function, which has to do with epistemic reference, may help determine a distinct partial function which is the reference function itself. This function will assign to an object and an expression in a context the value *yes* or *no* (or no value) according as it is in the reference of the expression or its negation (or is in its boundary). Some of the myriad factors which are part of this determination are the psychological state of *S* in *C*, whether or not *S* is an expert in the field of beliefs which are relevant to “*e*-things,” the evidence used for making the judgments, the closeness of *S* to the linguistic community, the nature of *C* (whether it is peripheral or not for uses of *e*), and so forth. These things and others determine how particular uses are weighted. If clear trends result from the values of the function in the most heavily weighted contexts, in other words if there is agreement about what is being talked about in the situations that matter most, then there will be a determinate reference function. In some cases there may be a mere tower of Babel.

7. Note that the actual reference function, not the epistemic reference function, is the one that underlies normal talk about reference. Similarly, it is the actual function about which, according to our deemings about reference, we will be right or wrong.

8. All of this is consistent with, but by no means presupposes, there being factors independent of our psychology and social conventions which participate in the determination of reference. For example, certain kinds of realists may be committed to the view that there is a nonreducible fact of the matter

as to whether an object is in the extension of a predicate in certain contexts. Or it may be that certain facts about our biological history help determine the kinds of things we refer to.

9. I have spoken of *trends*. It may be that a predicate will have roughly the same extension in all appropriate contexts. (Of course, the major difference between the metaphysical and the epistemic function is that (if communication is successful) the former will vary less over different contexts. The lightly weighted contexts are more likely to be ones where mistakes are made—where the deeming function has a different value from that in other contexts and from that of the reference function.) Or there may be local uniformities which suffer gradual or abrupt change as a result of the social determination of language. The following can occur: At these points of abrupt change, or after some gradual change has leveled off, we may deem either that reference has changed so that, properly speaking, we have a new expression type that is a homonym of the old one; or we may deem that we have radically different beliefs about *e*-things and thus make different judgments about objects that are in the extension of *e*, or even about objects that are in its comprehension. Many philosophers would claim that there is a false dichotomy here, or, perhaps, that *any* change is a mixture of change in belief and change in reference and that it is, in general, indeterminate, how much there is of each. (This is the view of Putnam, Quine, and others.) A slightly more correct description is the following: We are looking at some sort of change in the use of predicates or singular terms which falls within the comprehension of both 'change in the reference of *e*' and 'change in beliefs about the sorts of things to which *e* refers'. In the cases of interest—scientific change for the most part, or cases where a nonscientific expression has its use modified to a greater or lesser extent by becoming part of technical usage, change is not by means of mere redefinition since scientific expressions do not, in general, have definitions which pick out the reference. Rather, the possibility of making scientific beliefs more accurate is maximized if at least the borders of concepts are left vague. Although a paradigm or core of beliefs which is tied to central cases of the phenomenon is necessary to prevent scientific parlance from being vague (in the pejorative sense), even paradigms do not determine reference. Change in usage occurs both by slight shifts of paradigm and shifts in the borders of the concept.<sup>5</sup> There are changes through time. But, in addition, as argued by Kitcher ([8], p. 538), different tokens of a single expression may at any particular time refer to different kinds of entities or phenomena at least partly by being tied to different paradigms.

In most cases, when we have an interest in determining whether change of reference has actually occurred, there is evidence about usage which, at least in principle, could give us a justifiable empirical hypothesis.<sup>6</sup> Moreover, in practice, the scientists themselves make judgments—whether justified or not. For these reasons I suggest that 'change in reference of *e*' and 'change in belief about the sorts of things to which *e* refers' are both merely predicates that refer inexactly. At the present time it may *seem* that their borders are more extensive than they in fact are. More philosophical research in developing



theories of meaning change and belief change, as well as closer studies of actual usage by scientists should enable us to make better hypotheses. What will seem to happen is perhaps that our usage becomes more refined. But, as I have noted earlier, this is quite consistent with there being a fact of the matter *now* with respect to many instances of scientific change.

2 The deemed reference of an expression which refers inexactly is a variable set, one that can change from context to context. The actual reference is also a variable set; but in many cases where communication is reasonably successful, it will vary less than the epistemic referent because many of the contents of the latter are errors. Before seeing how these notions can be modeled for a fragment of a natural language describable as a first-order language, I will look at a model for a propositional language whose truth predicate may have borderline cases.

I have argued elsewhere [18] that the predicate ‘is true’ refers inexactly, but that there is no such thing as degrees of truth. Thus, if an assertion is made at a given situation and a meaningful proposition is expressed, the assertion may be neither true nor false. To represent this in the formal model, we start with a set of points  $X$ , where each point is a particular place-time possibility. We will let  $\{X, T\}$  be a topological space.  $T$ , the open sets, will be interpreted as truth values of propositions. That is, for a proposition,  $p$ , let

$$\llbracket p \rrbracket = \{x \in X: p \text{ is true at } x\}.$$

This means that, in the structure, the truth value of a proposition is expressed by the set of points (places or situations) at which  $p$  would be true. We designate two special truth values: of the necessary proposition which is everywhere true and of the contradictory proposition which is everywhere false. These are, respectively:

$$\begin{aligned}\llbracket \text{true} \rrbracket &= X \\ \llbracket \text{false} \rrbracket &= \emptyset.\end{aligned}$$

Keeping in mind that truth values are open sets in  $X$ , we define truth values for logical compositions intuitionistically, as follows:

$$\begin{aligned}\llbracket p \wedge q \rrbracket &= \llbracket p \rrbracket \cap \llbracket q \rrbracket \\ \llbracket p \vee q \rrbracket &= \llbracket p \rrbracket \cup \llbracket q \rrbracket \\ \llbracket p \rightarrow q \rrbracket &= \cup \{u: u \cap \llbracket p \rrbracket \subseteq \llbracket q \rrbracket\}.\end{aligned}$$

The last requires some explanation. We want to include in  $\llbracket p \rightarrow q \rrbracket$  exactly those points where, if  $p$  is true, then  $q$  is. Thus a situation will be in the truth set of  $p \rightarrow q$  just in case either  $p$  is not true or  $p$  is true and  $q$  is as well.

To provide for the possibility of truth value gaps, we define a strong negation.

$$\llbracket \neg p \rrbracket =_{df} \llbracket p \rightarrow \text{false} \rrbracket.$$

That is to say, the denial of  $p$  is true exactly where  $p$  implies a contradiction. Thus we have

$$\llbracket \neg p \rrbracket = \cup \{u: u \cap \llbracket p \rrbracket = \emptyset\}.$$

This turns out to be the interior of the complement of  $p$  since truth values are open sets. Thus, there may be a boundary where  $p$  is neither true nor false.

The following stability condition will hold. (' $A$ ' is a variable ranging over propositions.)

If  $u \subseteq v$  and  $v \subseteq \llbracket A \rrbracket$  then  $u \subseteq \llbracket A \rrbracket$ .

Thus, in this model it will follow automatically that truth (falsity as well) will be preserved as we refine our viewpoint by specifying a narrower context.

Note how this simple structure correctly mirrors both inexact reference and change of reference for this limited language. Imagine we are ourselves located in this space of points in some context. This context, or point of view, would, practically speaking, be larger than a single point. If we have in mind an epistemic interpretation of this structure, the context will be contained in the point set which represents a belief. The truth value of a complex proposition, if it has one, will depend on its internal structure. In this simpler model, the structure is represented only coarsely; the model given below will also represent the structure of the atomic propositions. The epistemic reference relation is to be represented, in either model, context by context. Even though the structure can tell us nothing explicitly about how this epistemic reference relation comes about, the spatial representation of the proposition (the places where it is true) should show regularities which prevail over several contexts, if there are any. Each member of a community of speakers would have such a representation. An appropriate social theory of language determination would make use of such representations, each of which would be suitably weighted, to obtain a new spatial representation that would be of reference itself—in this simpler model, just the places where the proposition is indeed true, as opposed to those places where it is merely deemed true. Further, having only a comparison between the representation of reference and the various representations of epistemic reference, we could, in principle, discern something of what social principles carry the latter into the former.

For most propositions, there would exist points at which they are neither true nor false. Unless the proposition  $A$  is the necessary or the contradictory proposition,  $\llbracket A \rrbracket$  will be neither the whole space nor the empty set. Even if we fix the possibility parameter, the truth value of a proposition may change through time. We may find that the boundary of  $\llbracket A \rrbracket$  is nonempty: There may be points that are neither in  $\llbracket A \rrbracket$  nor in  $\llbracket \neg A \rrbracket$  since  $\llbracket \neg A \rrbracket$  is another open set, the interior of the complement of  $\llbracket A \rrbracket$ . This quality is just what we want for inexact reference. But the former quality, that the truth value changes through time, may be the most suitable way to represent the common mixture of change of reference and inexact reference. A proposition's having contexts in which it is neither true nor false may facilitate its having truth values changing through time.

Only the total structure of the space can give an accurate picture of reference. If we want to know if someone in a given (relatively narrow) context is giving an expression the correct reference, that is, if we want to know how this person's judgment coincides with the truth of the matter, we must look at a much larger context in the space that represents reference itself. Locally the speaker may be making a correct judgment, one that even agrees with those

of many other speakers; but on the whole this may not be the case. These can be departures in usage that are unimportant in the long run. But this coin has another side. If we want to know the truth about reference at a given point, we can only determine this by looking at epistemic reference throughout a much larger context.

We can extend this structure in a natural way to accommodate propositions with a first-order structure. In the simplest case, we can imagine that we are only concerned with the singular term 'Tess' and the predicate 'is tall'. Let our space have just two dimensions and let us partially order the neighborhoods of the points of this space.<sup>7</sup> One dimension is time, those points later in time being greater in this ordering. The other dimension is possibility. We can take the actual *now* as base point and those points that are less similar to the actual (this being relativized to the predicate in question as noted in Section 1) will be greater in this ordering than those that are more similar. The time dimension is amenable to a metric, but, as noted earlier, a metric seems inappropriate when speaking of possibilities.

We could plot this space, using a (three-dimensional) cylinder. Its axis would be time, with *now* somewhere in the midst of this axis and with the axis extending however far time does extend in the past and future. In layers around this axis we could arrange the varying possibilities. Each layer would consist of nonactual points (at the given time) that were at a level of similarity with the actual. (Their being on a level does not necessarily mean that they are equally similar to the actual, just that they come at analogous places in some chain in the order. Visualize a branching tree structure. The base is the actual. Then take those points such that there is no point more similar to the actual than these. This is the first layer. Take any point on the first layer. Those points than which there is no point more similar to this point than these will be part of the second layer. To get all of the second layer, we produce all of the parts of it gotten in this way from points of the first layer. And so on. Points at a layer may not be comparable with each other to any greater extent than this: They are an equal number of layers removed from the base.)

Again, we are interested in sets of points and the partial ordering indicated above imposes, in a somewhat less than deterministic manner, an ordering on sets of points. The open sets which are truth values will be determined by open sets which, for the moment, we consider to be "occupied" by individuals and parts of individuals.

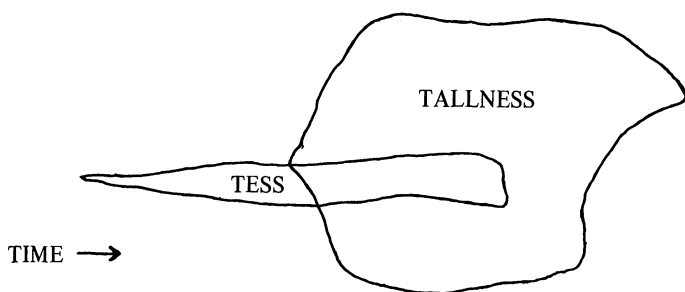
$$\begin{aligned} \llbracket \text{'Tess'} \rrbracket &= \{x: \text{Tess is at } x\} \\ \llbracket \text{'is tall'} \rrbracket &= \{x: \text{something at } x \text{ is tall}\} \\ \llbracket \text{'Tess is tall'} \rrbracket &= \{x: x \in \llbracket \text{'Tess'} \rrbracket \cap \llbracket \text{'is tall'} \rrbracket\}. \end{aligned}$$

We can stretch the notion of "occupation" and also think of tallness as "occupying" those regions where there are (parts of) tall things. In a fit of metaphysical confusion and thirst for picturability, we can identify Tess with the points she "occupies" and tallness with the points it "occupies". In that case, things do become clear (if slightly incorrect): The intersection of those two entities which are tallness and Tess is the region where Tess is tall.

The open sets corresponding to the negations of expressions are defined in the obvious way. For example:

$$\llbracket \text{'is not Tess'} \rrbracket = \cup \{u : u \cap \llbracket \text{'Tess'} \rrbracket = \emptyset\}.$$

If Tess is tall somewhere and short somewhere, there will be a region, the boundary, where she is neither tall nor short. Presumably Tess will grow taller, for the most part, as time increases. In a richer language, which contained such predicates as 'is somewhat tall', 'is very short', and so forth, we could describe Tess's heights to as precise an extent as we needed. But, unless we appeal to centimeters or inches, reference will remain inexact. There will be boundaries also around the extensions of these more precise predicates.



Let us now consider a larger first-order language (and try to avoid misrepresentations of metaphysical matters). We want an entity in the model to correspond to each expression that has a referent. We should not think of predicates themselves as being any sort of region, but rather as applying to parts of the various sorts of regions which are individuals. A nice way to picture an individual's having a property is to think of its being painted. Thus we might describe predicates as having referents that are splashes of colors. We do want to keep the original space-time-possibility points distinct from other individuals, however. As before, we will just consider the dimensions of time and possibility. In fact, I believe that even if I were to develop and refine this model so that it could represent more of the subtleties of the problem we are interested in, it would not be of much use to consider spatial dimensions. (On the other hand, if one were to give a model that could help explain the *basic* reference relations, such as primitive epistemic reference, one might find these dimensions useful. Pointing seems the most basic sort of referring and pointing necessarily involves some specification of direction in space.) We think of an object or individual as a two-dimensional "leaf" distinct from the space of time-possibility points we had before. We can project this leaf into the space. The projection is the place where the individual exists. For an individual called *a*, we let  $\llbracket a \rrbracket$  be the projection of its leaf. (Recall that an individual is considered to consist of all its possibilities.) Each predicate will have associated with it a partial function that colors a part of this leaf or fails to, depending, respectively, on whether or not the predicate applies to that part or not. We

can also project these “dyed” bits into the point space and call the projection of the extension of predicate  $P$ , ‘ $\llbracket P \rrbracket$ ’.

The projections of the sheaves of variously dyed leaves determine the open sets of the space. We can then define  $\llbracket A \rrbracket$ , for  $A$  atomic (taking the normal syntax for a first-order language), in the obvious way. For example, if  $A$  is  $Pa$ ,

$$\llbracket Pa \rrbracket = \{x: x \in \llbracket a \rrbracket \cap \llbracket P \rrbracket\}.$$

If  $A$  and  $B$  are propositions of the language, define  $\llbracket A \vee B \rrbracket$ ,  $\llbracket A \wedge B \rrbracket$ ,  $\llbracket A \rightarrow B \rrbracket$ ,  $\llbracket \neg A \rrbracket$  as before. Let  $A^{x/b}$  be the result of substituting every occurrence of  $x$  in  $A$  by  $b$ . Then

$$\llbracket \exists x A \rrbracket = \{x: \text{there is a } \llbracket b \rrbracket \text{ such that } x \in \llbracket A^{x/b} \rrbracket\}$$

$$\llbracket \forall x A \rrbracket = \cup \{u: \text{for all } \llbracket a \rrbracket, \text{ for all } v \subseteq u, \llbracket a \rrbracket \cap v \subseteq \llbracket A^{x/a} \rrbracket\}.$$

(I state these somewhat incorrectly for perspicuity. The definition assumes that every object has a name in the language. Since this is not really the case, the correct definition would speak of alternative assignments to names in the familiar way. The interested reader can correct the definitions as an exercise.)

These quantifiers are the intuitionistic ones and  $\forall$  is not definable as  $\neg \exists \neg$ . The reader might question how this stipulation for  $\forall$  squares with my earlier remark that stability is to be guaranteed. First, note that indeed stability is already guaranteed for universal statements. If a point set is in  $\llbracket \forall x A \rrbracket$ , then all of its subsets are as well. Secondly, the motivation behind the definition of the truth set of a universal statement is the following: As I noted earlier, when we describe a possible world nontrivially, what we say is, in general, true of an infinity of them. (It follows that we can never completely (non-trivially) describe the *actual* world.) Consider the statement:

All trees have pink leaves.

This may *seem* to be true in a certain context  $u$ , which is rather wide because it is not asserted that there are individuals that are trees in  $u$ . Note, this is not equivalent to saying that there are *no* individuals that are trees in  $u$ . When we specified  $u$ , what we said did not decide the matter either way. Thus  $u$  is a context which is neutral with respect to having trees. As we narrow the context to (say)  $v$ , we can see that the statement may be false. In specifying  $v$ , we may say:

Tom exists. Tom is a tree. Tom has (all) yellow leaves.

$v \subseteq u$ . Consider again the universal statement, put in a more perspicuous form:

$\forall x (x \text{ is a tree} \rightarrow x \text{ has pink leaves}).$

Now Tom is in  $v$ . Thus

(Tom is a tree  $\rightarrow$  Tom has pink leaves)

does not have  $v$  as a part of its truth set. Thus the universal sentence does not have  $u$  (which contains  $v$ ) as part of its truth set.

What we have done is to make our definitions comply with our desire

that, in whichever context we are, getting more information (narrowing the context) cannot change the truth value of sentences that already are true or false.

We can now say about any object and property something analogous to what we said about Tess and tallness. That is, if the object is in the extension of a predicate at some time and fails to be in it at another, then there may be times when it is neither the case that it is in the extension nor in the extension of the negation of the predicate. Let us now note some more facts about the model which would indicate that, described in the above way, it forms a useful idealization in which to study change in reference.

1. Singular terms refer inexactly in the following way: Consider any point in our space. An object may exist at that point or fail to exist at that point or neither of these things may be true. In this way, we can accommodate the times when an object is coming into being or going out of being—neither of which usually happens at an instant of time. The reader might think that at least the uncertainty we feel in such borderline cases is more fuzzed than this. Shouldn't an epistemic model provide for this? I have argued elsewhere [18] that the structure of the concepts we apply is simply that of extension, extension of the negation, and boundary. But the dimension of possibility lets us see more graphically why this is the case. Consider a point "near" to the actual situation which is not itself actual. At such a point the object in question may also exist, fail to exist, or may neither exist nor fail to exist. When we are at actual points in time which are on the boundary of the object's "places" of existence, nearby points of possibility may be either points where the object exists or points where it does not. This is analogous to our being able to vary the actual situation as little as we please and its still being the case that, depending on how we vary it (not how much) we may get one result or another. That is, points of possibility which are equally close in the ordering to the actual boundary point will be such that the object exists at some, fails to exist at others, and neither exists nor fails to exist at others.

When we put this together with the fact noted above, that we almost never know exactly where we are located in the realm of possibility (or, metaphysically, that it may be irrelevant exactly where we are), it becomes apparent why boundaries *seem* fuzzed even though our judgments are not and the extensions of our predicates are not. It is merely our sense that things *could* be a number of different ways combined with our sense that we do not know exactly which way we are a part of.

This structure of a topological space with a partial ordering on its points can also represent the fact that some uncertainty results from our considering aspects of an object. For example, Dale may be alive, in the sense that her heart is beating (because it is connected to a machine), but dead in the sense that her brain waves have ceased. In the possible alternative situation in which heartbeat is the proper signal of life, we get a different verdict than in the situation in which brain waves are the proper sign. Each of these situations might be equally close to the actual one.

2. Analogous remarks can be made for predicates. A predicate  $P$  refers epistemically inexactly in a context if the context is appropriate for  $P$ , contains

an object  $O$  in  $H(P)$  and we deem that  $O$  is a borderline case of  $P$ . The structure, by including possible, nonactual points of time, enables us to explain why epistemic inexactness might seem to have degrees even though it does not, just as it did in the case of a singular term. As well, since predicates referring inexactly usually do not have a single criterion of application, but at most a cluster of criteria and perhaps only a cluster of features which constitute a paradigm, we can represent this in the model by ordering the merely possible contexts accordingly. For example, if genetic structure is a more important part of the paradigm of a human cell than shape is, then those possible situations where shapes of cells vary are likely to be nearer than those where the genetic structure varies.

3. The extension of a predicate can vary through time. Consider the places in the space where an individual exists in all its possibilities. Part of the predicate's extension may lie there. If, at some times during its actual existence, the individual is in the extension of the predicate, and, at some points it is not, either this could be because the predicate denotes a property that an individual can have at some points of its existence and fail to have at others (for example, being tall); or it could be because there is a shift in the language and the predicate is changing its (timeless) reference. The structure could not tell us which explicitly, of course, but given a structure that represents enough of the language, we should, in principle, be able to conclude which of these things is occurring.

Let me give an example. The word 'fish' has changed reference in some idiolects. Let us look at the individuals that are fishes. Since this is a property an individual has through all its existence if it has it at all, the fact that an individual has a property at some time and not at another shows that the word changes reference. This change might be abrupt, or, it may have a boundary and be such that, close by, nonactual possibilities give conflicting results. But in either case, the following can serve as a criterion for recognizing the difference between a predicate that changes reference and one that, in general, may have just parts of individuals in its extension. We look at regions of change. For each individual, we consider the base space of points. The predicate has changed extension within a given region as a result of a change in language if the following is true: There are at least two individuals that have the property within that region (or, two that have its negation) and eventually cease to have it (or, respectively, gain it) within the region, there is some set of properties ( $P_1, \dots, P_n$ ) these individuals share, that does not change within the region and that is an important subset of the cluster of properties associated with the original predicate.

3 In this section, I will sketch and offer some criticisms of two classes of models of inexact reference: those similar to the supervaluation account of Kit Fine [4] and those similar to the fuzzy set logic of Goguen [6], Zadeh [19], and others. In the case of the latter, at least, there is some disagreement among the proponents as to how the details would be filled in, in particular how the truth values of complex sentences should depend on the truth values or satisfaction conditions of their parts. For the most part, my criticisms are

not at all affected by these details. Thus I merely sketch and do not consider all alternatives.

A somewhat oversimplified supervaluation model works in the following way: For any interpretation of the language in question, one assigns a family of complete specifications or precisifications. A pair of sets will count as a precisification of a predicate which refers inexactly just in case the first is a reasonable (though, in general, arbitrary) candidate for the extension of the predicate and the second member is the complement of the first set in the given domain. Then an atomic sentence will be true (false) under the interpretation if it is true (false) under all precisifications.

A general criticism of this view is given by Sanford ([17], p. 206), a proponent of the fuzzy set semantics. "Grant then that a certain statement is true if its predicates are made completely precise in any appropriate way. Why should the statement thereby be true if its predicates are not made precise in any of these appropriate ways?" To make what I think is the same point slightly differently: If we simply refer to things and if reference is in some sense vague, why should we look at many different precise reference relations to evaluate the sentences we use?

Fine himself gives some justification for the appeal to precisifications. He claims that how an extension can be made precise is already part of the meaning of the predicate ([4], p. 277). This does seem to be true. And, moreover, we frequently make use of one of the (arbitrary) precisifications of a predicate for legal purposes. Thus, 'adult' has now (relative to the locale one is in) an exact reference—even though there may be epistemic borderline cases. He argues further that we tend to precisify reference in science. This is certainly the case when we quantify the phenomena we deal with. For example, to cease speaking of colors of light and to begin to speak of the wavelengths of light is a way of replacing predicates that refer inexactly by those that refer exactly. But note that there are very few instances in which we can agree with Fine that "language retains identity upon precisification", though not "upon arbitrary change of meaning" ([4], p. 275).

I would say that any change of reference is a change in language (though naturally we still have numerically the same language: Many changes have to occur before, say, Latin becomes French). Thus, to evaluate a sentence by looking at *one* of its precisifications would be to evaluate a different sentence. Can Fine justify looking at all precisifications? Can he say that by thus capturing all the *potentials* of the sentence, he has thereby evaluated *it*? Again, I would say, "No." Fine could go in two directions here: He could say that in fact the sentence *will* be precisified in one of the potential ways; or, he can say that this need not be the case, but that in some sense the sentence already has all of these precise referents. The first alternative is clearly wrong. It is almost never the case that ordinary concepts are relieved of their borderline cases.<sup>8</sup> (Yet, sometimes they are made more precise, in the sense of being divided into finer grained concepts. In this way redness can become orange-red, red, and blue-red.) We have both scientific and legal expressions that have exact reference, but these are not in common use: "Out of the office" people tend to use expressions like 'adult' or 'fruit' with their old inexact reference. Moreover, actual changes may go in all directions. That is, we may broaden con-



cepts, as in the case of the expression 'goal-oriented behavior' among artificial intelligence researchers. Or reference may simply slip, as in the history of the many-roled word 'atom'.

If Fine opts for the second alternative, he is claiming that precisifications exist in whatever sense we once thought referents existed. We could take him to be postulating them as theoretical entities. If this is the case, a minimum criterion for accepting them is that they save the phenomenon in at least as natural a way as alternatives. But though they do solve the puzzles we have had with inexact reference—exemplified most graphically by the sorites paradox—they do so at the cost of putting a structure on semantics that seems directly orthogonal to the structure it has. Reference varies through contexts. It is not various at a given context.

I would like to note some objections to the details of Fine's account. The first stems from an example in a paper of Dummett's,<sup>9</sup> although Dummett did not present the example as a criticism. Consider the sentence, 'It is either red or orange'. In Fine's account this sentence is going to be true not only of red things and orange things, but also of all borderline cases of red, on the orange side. But clearly this is incorrect. Of a border case, it is correct to say 'It is *neither* red nor orange (but something between)'.

For a similar reason, I would dispute the desirability of the result that the law of the excluded middle is valid in Fine's system. 'It is raining or it is not raining' is sometimes true, sometimes false, and sometimes (with a drizzly fog) neither true or false. Again, reference is varying through contexts and truth value follows this variation. It is not the case that in all contexts we have the value *true* just because we have the value *true* for all the various exact references that are compatible with the usage in that context. To talk of exact reference is, as noted above, to change the subject.

The logic of our discourse is not classical logic. On the one hand, classical logic cannot capture variable sets, which are essential to complete accounts of denotation and satisfaction. On the other hand, there are classically valid sentence forms which have ordinary language instances that will have no truth value.

Let us now turn to fuzzy sets. Those who espouse this semantics for sentences with predicates with inexact reference claim that such sentences can have, in addition to the "standard" truth values, any of  $n$  truth values. Usually,  $n$  can take real values between zero and one because, it is argued, there should be no abrupt changes in the truth value of a sentence such as 'This is a table' as we gradually deprive the object in question of its tableness. In other words, if we had finitely many steps of truth values, this would seem arbitrary and also more counterintuitive than having one exact cut-off. Roughly, these many truth values are computed in a way analogous to Fine's method of precisifications: A set of possible (specific) interpretations is assigned to each sentence for each valuation. The important difference is that in fuzzy semantics, the final truth value, rather than being a supervaluation, is a continuous function from possible interpretations to the interval  $[0,1]$ . If the function which assigns truth values is not continuous, so that, for example, there are ten truth values ranging from (say) *very true* to *very false*, the choice of this *particular* number also seems arbitrary. Why ten truth values and not eleven? There is not

even a psychological basis for such an absolute figure. Perhaps for some particular determinables (take hue for an example) there may be a normal number of discriminations we can make. But if this varied from individual to individual should we say each had her own semantics? More importantly, such a justification would make truth depend on epistemic features in an inappropriate way.

Yet, even a many-valued interpretation which uses a continuous truth function fails in this way. Psychologists have found that we tend to rate examples of instances of such simple properties as redness as closer to or farther from "paradigm redness" (see [15]). Many properties are, for this reason, gradable. But this is no excuse for saying that sentences attributing redness to things have gradable truth values. A chip that is less red than the paradigm may still be red.

Fine offers an interesting criticism of the multivalued approach. (See [4], pp. 269-270, 278-279.) He makes note of what he calls *penumbral connections* among predicates. These are meaning connections that make it, for example, impossible for a single object to exhibit two contrary properties (all over). For example, he claims that 'This is pink and red' is always false. In his logic, this is true. But, in a multivalued logic, if the truth value of 'This is red' and 'This is pink' are intermediate, then the truth value of 'This is pink and red' also is intermediate.

I am not sure that this criticism is a valid one. In the first place, my intuitions about contraries among border cases diverge somewhat from Fine's. Not only is it not clear to me that the sentence in question should be false of some object on the border between pink and red, I have a slight tendency to say that it is true. I think this might be a result of taking the connective 'and' in a way that deviates from that intended in a first-order logic. But this should awaken us to the fact that inexact reference may have the effect of distorting parts of the language for which we thought we had an exact semantics. Thus Fine's claim that "all vagueness can be blamed on constituents as opposed to constructions" ([4], p. 275) seems unwarranted.

But this notion of penumbral connection suffers from a different problem. One must not confuse the enterprise of constructing a logical theory with that of constructing a theory of language. The latter will certainly take into account the meanings of expressions involved. But we should not expect a purely logical theory to distinguish between 'This is red and pink' and 'This is red and empty', but only among these and 'This is red and not red'. I am not trying to make a case for drawing a particular line between truth for logical reasons and truth for reasons of meaning connections, but there are differences in the degrees of abstractness of kinds of truth.

I am not criticising Fine for mixing in meanings when he is allegedly looking for a logic that works for these predicates. I think this is appropriate for the kind of theory he constructs. Thus, in order to find which precisifications of (for example) 'is red' are appropriate, he must depend on our (non-logical) beliefs about red things. For this reason, his criticism of Zadeh is correct when taken as the assertion that the semantics of these predicates cannot be accurately given unless the meanings of these predicates are considered. But there are other ways of doing this than looking at admissible specifications. The "meanings" of these predicates depend, in the complex

ways I have listed above, on their use. The facts about contraries should come out of this. Further, logical combinations of sentences involving contrary predicates should either get their semantics in the standard (intuitionistic) way or, if this is to be a matter of meaning and not just a matter of logic, a sentence like ‘This is red and pink’ must be checked through all the appropriate contexts as though it were *not* just a logical conjunction.

My tendency is to say that the sentence is neither true nor false of a borderline case of red/pink. For, if an object is neither red nor not-red, we cannot apply the meaning postulate

If something is red, it is not pink

to it. For this reason, it is a mistake to think we are dealing with classical connectives with these predicates.

Criticisms similar to those I have given against Fine’s model can be given against the many-valued approach. Are these many ‘truth values’ to be understood as theoretical entities? If so, do they save the phenomena in an intuitively plausible way? As theoretical entities, they save some of the phenomena, but the real existence of continuum-many truth values seems implausible. A claim to such an existence would have to be justified by a demonstration that we could, in principle, apply them all. But clearly we cannot; nor do we have the slightest tendency to apply them. It makes no sense to try to decide whether the truth value of ‘Sam is bald’ is 0.75000 . . . or 0.750010 . . . or . . . (In [18], I give more detailed arguments as to why there are and *should* be only two truth values.)

In a recent paper, King [7] puts a slight twist onto the many-valued approach. Although he ultimately comes back to a Zadeh-like picture, he toys with the idea that statements containing predicates that refer inexactly are not the *sort* of things that are true or false. Rather, they are more or less *accurate*. Accuracy, like Zadeh’s “truth values”, varies in the interval  $[0, 1]$ . My response to this is that accuracy, if it is not to be conflated with truth, is to be understood as a measure of distance from a paradigm. Thus, the structure of theories for this measure would not imply very much about truth. Further, it is very unclear how one would compare predicates involving different paradigms, as in ‘Paul is tall and red-headed’. On the other hand, if accuracy is conflated with truth, the criticisms given above will apply.

As to a claim (which King rightly eventually drops) that truth and falsity do not apply at all when inexact reference is involved, it conflicts with the data of many thousand years. We have taken sentences with inexactly referring expressions to be in the comprehension of ‘is true’, and have been successful with this hypothesis in all but a few isolated instances found in the minds of philosophers.

Some authors have suggested adding a modal operator like ‘Definitely’ or ‘Determinately’ to formal languages with predicates that refer inexactly. ([4] and [16] are the two most interesting attempts of this.) The basic semantics for sentences not containing these operators can be either that of the super-valuation or many-valued accounts. The justification for adding such an operator is the claim that “vagueness ramifies”. Since I have argued elsewhere

[18] that such operators, if they have a use, should be understood epistemically as a measure of certainty, I will not repeat that argument here.

I think that a general point can be made about the accounts I have discussed. In all cases, there is an underlying exactness in the model, whether it is Fine's precisifications or Zadeh's possible interpretations. This exactness is then (exactly!) manipulated to mimic the inexactness of our language. But this underlying exactness does not exist. The connection between language and the world is inexact even at the deepest theoretical levels.

**Conclusion** My basic criticism of the prevailing semantical accounts of predicates that refer inexactly is that they fail to be diachronic and thereby lose the essential structure of reference. I have not offered a fully-developed alternative, but rather an alternative basis on which a model can be built. Still, it is interesting to note that the prevailing accounts, as well as my sketch, have a common *formal* core.

I leave the details for the interested reader,<sup>10</sup> but it may be evident that this core is a Kripke semantics such as that given in [9]. In all these systems, we evaluate expressions at a point. In the case of a diachronic semantics, what we find is the epistemic reference of an expression at a given place and possible situation. In the multivalued and supervaluation semantics, the evaluation is a partial one—either a possible interpretation or a precisification, respectively. The fuzzy set model determines reference by putting all these partial evaluations together and assigning a value between zero and one to a statement that an individual has a property depending on the “extent” to which it has it. This so-called “extent” is a measure on the set consisting of the partial interpretations in which the individual has the property. The supervaluation model determines truth value, also, by considering these partial interpretations. But, in this case, there is only one middle ground. Sentences will be either true or false, or, if the precisifications conflict, will have no truth value.

My basis is more complex. I take the “evaluations” at the points in my space to be real (not theoretical), and dependent upon each other. They are, however, unimportant considered singly, or even in clumps corresponding to some psychologically distinguishable moment of time. But, on the other hand, one should not only speak of some super, global reference, it is relevant to semantics that the complete structure of metaphysical reference, as well as the structure of epistemic reference that, together with a social theory of language determination, determines it, should be kept in the description. Notice that this description, in fact, the whole of the language in which we do semantics, has expressions which refer inexactly. But the point is that one can talk about any part of the language, about reasoning in it, about changes in it, without the enforced contradictions of synchronic logics. And this is because the assertions are made in the same spirit in which they are normally made. That is, we speak of what is local to us, want this to apply to regions “farther off”, and make efforts to gather evidence that will enable us to do so accurately. But we are not surprised if language, the world, and ourselves change gradually so that what we say ceases to be appropriate.

## NOTES

1. In [1], Boyd presents a dynamic view of reference which is in some respects similar to the view I present here. Unfortunately, I obtained his paper well after this paper had taken shape. A comparison would be helpful in refining both views.
2. See [10]. For the “translation” of propositional intuitionistic logic and the Heyting Algebra representation, see [5].
3. Dummett, in the 1977 *William James Lectures* (unpublished), reveals this tendency very well. He remarks that no one would think of calling the interpretation of S4 using sets of real numbers under the usual topology a semantic theory because we don’t know how to relate operations on subsets of the real line to the meanings of these constants. Part of my intention is to challenge this view.
4. Naturally this alludes to the distinction made by Donnellan in [2].
5. Kitcher [8] describes the phenomenon of scientific conceptual change in a way that seems compatible with the sketches I give here.
6. Parsons [13] gives an empirical procedure (which does not always give a unique answer) for determining if there has been reference change from theory to theory.
7.  $u$  in  $(X, T)$  is a neighborhood of a point  $x$  if  $u$  contains some open set which has  $x$  as a member.
8. Dummett [3] gives some good arguments as to why this cannot happen for a large variety of predicates.
9. See page 20 of the third of his *William James Lectures*.
10. An attempt by Dana Scott to state the common core of fuzzy propositional logic and modal logics is described in [11].

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