

Formatives

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Abstract An answer to the question of ‘sentential unity’ (What makes a sentence a single linguistic unit rather than just a string of words?) is one of the goals of any theory of logical syntax. A ‘Fregean’ theory claims that a sentence is a function (unsaturated expression, containing gaps) whose gaps are filled with either arguments (saturated, gap-less) or other functions which have already been saturated. A ‘Leibnizian’ theory construes a sentence as a syntactically complex subject (quantified term) plus a syntactically complex predicate (qualified term). Subjects and predicates just naturally fit one another to form sentences. An ‘Aristotelian’ theory takes a sentence to consist of a pair of terms connected by a binary formative expression (functor), whose only role is to connect terms to form more complex expressions (e.g., sentences). After an examination of the formal nature of such functors, it is argued that this third sort of theory not only answers better the question of sentential unity, but it also provides a better account of the nature of logical constants in general.

I want to survey here three distinct theories of logical syntax. For reasons that should become apparent, I shall label them Fregean, Leibnizian, and Aristotelian. In spite of this atavistic order, there is more here than simple reverse chronology; among other things, this order represents a scale of increasing clarity about the ultimate nature of logical constants, or formatives.

I It is an essential first task for any formal logic to make a distinction between those elements that determine the logical form of a sentence¹ and those that do not. For the modern mathematical logician logical formatives are functions on expressions. The most basic kind of function, however, is not logical. To see this we need to look at Frege’s answer to the question of “sentential unity”. The question is: What makes a sentence more than just a list of terms? What makes the terms of a sentence combine to form a single logical unit? Frege’s solution was to distinguish between two kinds of expressions—those

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which are *complete*, in need of no other expression, and those which are *incomplete*, in need of one or more other expressions in order to be completed. One difference between complete and incomplete expressions is this: only the former make reference to objects; the latter are semantically tied to concepts.² An incomplete expression has, in effect, one or more gaps; such an expression is completed by having its gaps filled by complete expressions. There are of course basically complete expressions, ones that never had gaps to begin with, as opposed to those that were incomplete but have been rendered complete by gap-filling. The basically complete expressions are singulars, viz., names, pronouns, and definite descriptions. These kinds of expressions are complete *au fond*. The basic kind of incomplete expressions consists of those that are completed by having their gaps filled by singulars; they are often called “predicates”. Filling the gaps of such a predicate with singulars (paradigmatically, names) results in a complete expression of a higher syntactic order—a sentence. As a sentence is itself a complete expression, it can in turn serve to fill a gap in a still higher predicate (or “function”). Such higher predicates are propositional functions (e.g., negation, conjunction, etc.) and quantifiers. These expressions are also incomplete, but are completed not by names but by entire sentences. So the problem of sentential unity is solved by taking sentences to result from the filling of gaps in incomplete expressions with complete (or completed) expressions. Unity is completeness, empty holes filled with pegs.

Historically the Fregean solution to this problem in logical syntax was both brilliant and original. By taking all predicates, whether monadic or relational, as simply incomplete expressions in need of completion by singulars it permitted relational sentences to be treated on a logical par with old-fashioned categoricals, accounting in one stroke for much of the vastly increased power of today’s standard system of logic. Yet in doing this a new problem was raised: Granted that there are higher predicates (functions such as propositional functions and quantifiers), how do we know which expressions are functions and which are not? What is the basis for drawing the all-important distinction between, on the one hand, singulars and basic predicates, and, on the other, functions? Quite simply, what is the basis for distinguishing logical constants from logical variables, formatives from nonformatives?

It is important to notice that on the Fregean theory there are really two kinds of sentences (from the point of view of logical syntax). There are sentences that result from the completion of a predicate by an appropriate number of singulars, and there are sentences that result from the completion of a function by an appropriate number of sentences. The first kind contain no higher function, no formative at all. They are syntactically primitive. They exhibit minimal syntactic complexity. They are “atomic”. All other sentences consist of one or more sentences plus appropriate logical constants. And these constituent sentences are themselves either atomic or not. Any sentence that has at least one formative will exhibit some syntactic complexity greater than that of its atomic constituent(s). Such sentences are, naturally, “molecular”.

It should be clear that, whatever logical formatives are, sentences need not be constructed by applying logical formatives to logical variables. The existence of atomic sentences would prove this, since they contain no logical formatives.

So, again we have to ask, what is to count as a logical constant, or forma-

tive? Well, for starters, universal and particular quantifiers are to be taken as constant. So is negation (always construed as applying to sentences and not predicates – and certainly never singulars). The copula, e.g. ‘is’, is to count as a constant – but only when it has the sense of identity (which it has whenever it is flanked by two singulars). And certain sentential connectives, like ‘and’ and ‘only if’, are to be counted as constants. But expressions such as ‘most’, ‘three’, ‘because’, and ‘John said that’, which play grammatical roles, at least, just like the accepted quantifiers and connectives, are not to count. A survey of any of the standard texts used today to teach mathematical logic is of little help in revealing the criteria for what is to count as a logical constant; all one gets is a list. We are introduced first to the propositional calculus, being told that ‘not’ applies to sentences one at a time, and ‘and’, ‘or’, ‘only if’, and ‘if and only if’ apply to sentences two at a time. We later learn that some of these could be defined in terms of others, and that all could be defined in terms of, say, ‘not both . . . and . . .’. But no hint is given as to why we pick just these and not any other expressions to be formatives. The same holds when we are introduced to the first-order predicate calculus; we are simply told that the quantifiers are ‘all’ and ‘some’. These may very well be the only logical quantifiers, but if so, we still want to know why. Finally, identity theory is introduced by the simple assertion that ‘is’ (taken as identity) is a logical constant. It appears that what counts as a logical formative for the Fregean is whatever appears on the standard list of logical constants. Of course, the question of what a logical formative is rarely gets asked by contemporary logicians. The standard logical constants are too familiar, too useful, too obvious to warrant real scrutiny.

2 Leibniz’s logical studies represent in many ways the culmination of high scholastic logical theorizing. This entailed, naturally, a respect for Aristotelian theory in general (though not in all of its particulars), with more immediate influence from Hobbes and a sympathetic connection with the Port Royalists.

The Leibnizian theory of logical syntax,³ like any such theory, must be able to account for the unity of a sentence. Where the Fregean sees sentences as the result of completing an incomplete expression by the use of other complete (or completed) expressions (filling holes with pegs), the Leibnizian simply takes a sentence to be a concatenation of two expressions which are syntactically distinct and mutually fit for partnership with one another in sentence formation. The two kinds of expressions are subjects and predicates; every sentence is logically construed as consisting of a subject and a predicate. There is no complete/incomplete distinction here. (In a sense, both subjects and predicates are “incomplete” in that each requires the other.) Subjects and predicates are themselves syntactically complex expressions. Each consists of two elements – a syncategorematic, or formative, expression and a categorematic, or material, expression. In turn, categoremata may themselves be syntactically simple or complex. Categoremata (from now on simply called *terms*) are homogeneous from the point of view of logical syntax. Any term can be equally well-fit for duty either in a subject or a predicate. So whatever distinguishes subjects and predicates must not be material; it must depend on the formatives involved.

There are three kinds of formatives on this theory. One applies to terms

to yield new terms. Another applies to terms to yield subjects. And the third applies to terms to yield predicates. No formative applies to a pair consisting of a subject and a predicate to yield a sentence. An important point: for the Leibnizian every sentence can itself be treated as a term. Negation is a term-to-term formative; thus, if ' P ' is a term so is 'not P '. Quantifiers are term-to-subject formatives; they are either universal or particular. Every logical subject, then, consists of a quantifier and a term. We can say that every logical subject is a quantified term. Term-to-predicate formatives are qualifiers, or copulae. Qualifiers are either affirmative, e.g., 'is', 'are', or negative, e.g., 'isn't', 'aren't'. Every logical predicate, then, is a qualified term.

On this view, an elementary sentence is construed as a concatenation of a quantified term and a qualified term. Where the Fregean achieved sentential unity by putting pegs in holes, the Leibnizian achieves sentential unity by merely juxtaposing pairs of expressions that are mutually attractive by their very (syntactic) nature—like two magnets.

The power of the Leibnizian theory of logical syntax is seen in the syllogistic. Classical syllogistic works because each sentence contains a pair of terms, each of which can occur in another sentence either as a subject-term or a predicate-term. In other words, any term can be either quantified or qualified; and this is the fundamental requirement for syllogistic. However, by construing all sentences as categorical (subject plus predicate), the Leibnizian theory has had great difficulty in accounting for relational sentences and compound sentences. And singular sentences were not always handled with clarity. It was these failures that led to the demise of Leibnizian syntax (and with it classical syllogistic) and the rise of mathematical logic (with its Fregean syntax).⁴

The Fregean, as we have seen, has been unable to offer an account of the nature of logical constants. He or she cannot say why some expressions are formatives and others not; the Fregean can provide no common feature or features shared by all and only logical formatives. What of the Leibnizian? What, if anything, distinguishes syncategoremata from categoremata? The scholastics gave a semantic answer: Categoremata signify on their own; syncategoremata only signify in construction *with (syn)* categoremata. But the question at hand is one of logical syntax. Answers in semantic terms are unlikely to satisfy, and most certainly introduce a host of further questions inherited from the semantic theory itself. But an answer to the question (framed in the form: What feature is shared by all and only logical constants?) was suggested to Leibniz by Hobbes, and its consequences were fully worked out by Sommers.⁵ The answer is that sentences are compositional, the result of expressions being added or subtracted from one another. Consequently, logical constants can be seen as analogous to the algebraic operators of addition and subtraction. In other words, all logical constants can be seen as signs of opposition, coming in positive/negative pairs. Consider the logical constants found by a Leibnizian theory of logical syntax. The two qualifiers, 'is' and 'isn't', say, are *prima facie* oppositional; we could easily construe them as plus and minus. Thus, 'Some man is wise' becomes 'Some man + wise'; 'Some man isn't wise' becomes 'Some man – wise'. 'Not' is surely a sign of negative opposition, construable as '–'. And just as terms may be negated, they may be unnegated as well. In algebra unnegated terms are tacitly positive; the same holds for terms in a linguistic expression. We can take unmarked terms

as tacitly positive, so that ‘Some man is wise’ and ‘Every logician is unreasonable’ are paraphrasable as ‘Some (+man)+(+wise)’ and ‘Every (+logician)+(-reasonable)’ respectively. That quantifiers are also construable as an oppositional pair can be seen by examining various logical equivalences. For example, ‘Some *A* is *B*’ is equivalent to ‘Some *B* is *A*’. Taking ‘is’ as positive, equivalence is algebraically justified here only when ‘some’ is taken to be positive as well, i.e.,

$$+(+A)+(+B) = +(+B)+(+A)$$

but

$$-(+A)+(+B) \neq -(+B)+(+A).$$

This suggests that the universal quantifier be taken as a negative formative. That this is justified can be seen by looking at the equivalence between such sentences as ‘Every passenger was hurt’, ‘Not a passenger was unhurt’ and ‘All who were unhurt were nonpassengers’. Algebraic equivalence here is maintained only by treating the universal quantifier as ‘-’. Thus,

$$-(+P)+(+H) = -(+(+P)+(-H)) = -(-H)+(-P).$$

Remember that on this theory sentences are themselves terms. Every term is negated or unnegated (i.e., negative or positive), so every sentence is therefore either negated or unnegated, and consists of a subject and a predicate. Every subject is a quantified (universal -; particular, +) term. Every predicate is affirmative, +, or negative, -. The *general* logical form of any elementary sentence, then, is

$$+/- (+/- (+/-S)+/- (+/-P)).$$

We can read this as: It is/isn’t the case that some/every *S*/non-*S* is/isn’t *P*/non-*P*. As in algebra, many of the plus signs are suppressed in natural language sentences. On the Leibnizian theory of logical syntax, any expression that can be treated as a member of a positive/negative oppositional pair, or that can be defined in terms of such expressions (e.g., ‘none’, ‘only’, etc.) is a logical constant, or formative.

3 In the *Prior Analytics* Aristotle tended to reformulate sentences such as ‘Every man is mortal’ and ‘Some log is white’ as ‘Mortal belongs to every man’ and ‘White belongs to some log’. Such reformulated sentences exhibit two features. First, the material elements, the *terms*, are placed at each end (*terminus*) of the sentence. Second, the formative element is a single (though grammatically complex) expression standing between the two terms. It literally *connects* the two terms (it is a *copula* in the genuine sense). An Aristotelian theory of logical syntax, then, logically reconstructs each sentence as a pair of terms connected by a formative (from now on, a *term functor*).⁶

We saw that the Fregean accounts for sentential unity in terms of filled gaps (pegs in holes), and the Leibnizian does so in terms of mutual fit (the magnet theory). For the Aristotelian every sentence is the result of two terms being connected by a third element, an element whose only role is literally to connect them (think of this as the glue theory). On the first view, logical constants are

always things with holes, and those holes are to be filled with (sentential) pegs. The ultimate pegs, the ones that are complete in themselves, those not in need of completion by something else, fill the holes of incomplete things which are not logical constants. Such a completion results in an atomic sentence. On the second view, logical constants always apply to terms (simple or complex), in order to fit them for the role of subject or predicate. No further constants are required to put subjects and predicates together to form sentences. Quantified and qualified terms are now ready to form sentences by simply being juxtaposed (like rabbits in a hutch, no introductions or intermediaries are required). Consider the elementary sentence 'Socrates is wise'. The Fregean construes this as a predicate containing a gap ('. . . is wise') and a name ('Socrates') containing no gap itself which fills the gap in the predicate. The sentence is seen as consisting of just two elements; there is no formative. The Leibnizian construes this sentence as a subject (for most scholastics: 'Every Socrates'; for Leibniz: 'Some or every Socrates'; for Sommers: 'Some Socrates')⁷ and a predicate ('is wise'). Again, the sentence is seen as consisting of two elements (albeit, each element itself is seen as a syntactically complex expression).⁸ On the third view, every sentence consists of three elements: two terms and a term functor. A sentence is more than just a list of terms simply because it contains an expression whose very function is to combine those terms to form a sentence, 'Socrates is wise' is parsed as 'Wise belongs to Socrates', a term-pair ('wise' and 'Socrates') connected by a functor ('belongs to').

We have seen that the Aristotelian accounts for sentential unity in terms of the glue theory: every sentence is a pair of terms connected by a term functor. Term functors form sentences from pairs of terms. Which expressions can serve as term functors, sentence formers? We can think of a language, natural or constructed, as a set of terms (the lexicon) and a set of grammatical functors, which are combined in specified ways (the grammar) with lexical items to form sentences. From a logical point of view, only some of the ways in which terms are combined are of interest in the attempt to account for formal inferences. Since formal inference depends essentially on certain formal features of the sentences involved, and since the formal features of a sentence are fully determined by the formative expressions of that sentence, it follows that the logician will be interested in only those grammatical functors that have certain formal features. As it turns out, a close inspection of standard systems of formal logic (systems designed to account for inferences in natural language), viz., syllogistic and first-order predicate calculus with identity, reveals that formal inference can be fully accounted for in terms of a small number of formal features. These features are inversion, symmetry, associativity, transitivity, and reflexivity. Only those grammatical functors in the language which, among themselves, display these formal features will be necessary in accounting for formal inference, and thus be of interest to the logician.

For the Aristotelian there are essentially three kinds of logical functors: a unary functor, which applies to any term to form its inverse; a binary term functor, which is symmetric and associative; and a binary term functor which is transitive and reflexive. Any grammatical functor that has one of these three sets of formal features will be a *logical* functor, a formative. Any expression that can be defined in terms of logical functors will also be a logical functor.

Negation is an example of a unary logical functor forming the inverse of any term to which it applies. Both terms and sentences can be negated (a sentence is just a complex term). In English ‘not’, ‘non’, ‘un’, ‘less’, ‘it is not the case that’ are examples of negators. English examples of logical functors which are symmetric and associative are ‘belongs to some’, ‘and’, ‘both . . . and’. Examples of logical functors which are transitive and reflexive are ‘belongs to every’, ‘only if’, ‘if . . . then’. English examples of logical functors definable in terms of undefined functors are ‘only’, ‘no’, ‘or’, ‘either . . . or’, ‘unless’, ‘is (identical to)’.

In seeing that logical form can be accounted for in terms of term functors, and that the arsenal of term functors must exhibit a specific set of formal features, the Aristotelian can offer a rational account of what constitutes a logical formative in terms of just those formal features. And this is surely a better way of shedding light on the nature of logical constants than a mere listing of natural language expressions which function as logical constants—even if the list turns out to be the right one.

NOTES

1. For our purposes, “sentence” is restricted to statement-making sentences, i.e., truth-bearers.
2. Geach has held that the object/concept distinction is foundational for the complete/incomplete distinction (see [7], pp. 215ff). Dummett, on the other hand, argues convincingly that the linguistic distinction is prior to the ontological one for Frege (see [2], pp. 57ff; also see [5]).
3. For an account of Leibniz’s theory see [1] and [3].
4. In spite of this it is possible to formulate a logic based on Leibnizian syntax which rivals the standard logic of today in terms of simplicity, naturalness, and power. Indeed, the possibility of such a “new syllogistic” is evidence that the Fregean revolution was too hasty. For more on this see especially [18] and [6].
5. Hobbes’ suggestion is in *De Corpore*, I, i, 2. Leibniz accepts it in *Of the Art of Combination* (see [9], p. 3). Sommers develops the suggestion fully in [17] and [18]. Also see [16], reprinted in [6].
6. For a fuller account of term functors see [19]. The logic of term functors closely resembles Quine’s logic of predicate functors (see especially [10], [11], [13], and also [8]). Predicate functor algebra, in turn, draws inspiration from Schönfinkel’s early combinatory logic (see [14]).
7. On the quantification of singulars see [15], Chapters 1, 2, and 6 of [18], and [4].
8. That is why there is no atomic/molecular distinction for the Leibnizian. See Chapter 1 of [18].

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