ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 14, Number 1, Winter 1984

SURFACES

Preface. A symposium on surfaces was held as part of the Society for Industrial and Applied Mathematics meeting at Stanford University in July, 1982. The need for such a surfaces symposium arises for various reasons including the following:

1. the problems involved in constructing truly multi-dimensional surfaces are now becoming well-defined, and
2. solutions for some of these problems are being found.

This maturing of the field is exemplified by other conferences, e.g., the April, 1982 Oberwolfach conference published as *Surfäces in Computer Aided Geometric Design*, edited by R. E. Barnhill and W. Boehm, North-Holland Publishing Company.

The SIAM surfaces symposium was encouraged by Gene Golub (partly by his earlier surfaces day at Standford). From the outset, we thought that the value of the sessions would be enhanced by the publication of refereed proceedings. Tom Sherman encouraged this effort and the result is the proceedings which follow. We have received a good deal of help with the manuscripts, both from the authors and the referees. In addition, Bill Scott, the Managing Editor of the Rocky Mountain Journal of Mathematics, has been very helpful. Sylvia Morris expertly typed about half the volume. We are indebted to everyone mentioned above, as well as to our colleagues who have helped us with friendly interactions.

Introduction to surfaces. Surfaces arise in many parts of science and engineering. In most cases surface data available to the scientist are arbitrarily located so that tensor product methods appropriate for simpler data with rectilinear grids are not applicable. The papers in this volume address this more robust problem of arbitrarily spaced information with truly multivariate surfaces.

This symposium, like most dealing with complex applied problems, includes a wide variety of topics. This is a large field and all topics despite their importance are not equally represented. Most of the papers deal with three-dimensional surfaces, but several concern new extensions to four-dimensional surfaces. In this field, as in all science there are a number of recurring dualities or alternative ways of looking at the same topic. Phil Davis has pointed out the complementarity of interpolation and approximation. A similar complementarity may be seen in representation and design of surfaces. Interpolation methods are frequently used for approximation methods as, for example, in the Finite Element Method.

There are two broad categories of three-dimensional surface interpolation methods: (1) triangular interpolants and (2) distance-weighted interpolants. Triangular interpolants require two preprocessing steps: (a) a triangulation and (b) the estimation of gradients. The papers by Barnhill and Little, Alfeld and Barnhill, Nielson and Franke, and Renka and Cline consider triangular interpolants. (The first two papers are best read in the order indicated.) Triangulations are considered by several authors and the paper by Cline and Renka is concerned exclusively with this preprocessing step. The estimation of gradients is required for the construction of smooth surfaces and is considered be several authors and the papers by Akima and by Stead are exclusively on this preprocessing step. Interpolation over a sphere is addressed by Lawson, who generalizes methods in Barnhill and Little.

Distance-weighted interpolants and multistage methods are considered for three-dimensional surfaces by Foley and for four-dimensional surfaces by Barnhill and Stead. Multistage methods combine the evaluation of a surface with its approximation; that is, the rendering of the surface is linked with the method of approximation.

The design of free-form or sculptured surfaces is addressed by Jensen and by Kochevar. Jensen considers geometric modelling and Kochevar uses bivariate B-splines to approximate surfaces.

Approximation aspects of surface fitting are considered in Wahba's paper. The problem of "overshoot" or "ringing" for curves is considered by Salkauskas.

Frequently one wants to find the surface area, volume, or mass properties of a surface, for which numerical integration rules are needed. Numerical "cubatures" are considered by Barnhill and Little (paper #5).

The preferred form of local approximation of surfaces is by means of piecewise polynomials. A necessary initial step to the construction of spaces of piecewise polynomials is to determine their dimension, which is considered in Schumaker's paper.

Recently there has developed a demand for four-dimensional "surfaces." For example, temperature considered as a function of three spatial variables defines a four-dimensional surface. The construction of four-dimensional surface approximations can be carried out either by means of (1) tetrahedral interpolants or (2) distance-weighted interpolants. Tetrahedral interpolants are considered by Barnhill and Little and by Alfeld and distance-weighted interpolants by Barnhill and Stead.

An outline of where each of the seventeen papers in this volume fits into the subject of surfaces follows. This outline's purpose is to help the reader move through this volume in a (fairly) linear fashion. The numbers correspond to the order of the papers in the volume.

OUTLINE OF PAPERS

I. THREE-DIMENSIONAL SURFACES

A. Triangular Interpolants
- #6 Barnhill and Little
- #3 Alfeld and Barnhill
- #13 Nielson and Franke
- #14 Renka and Cline

(1) Preprocessors for Triangular Interpolants
- (a) Triangulation
- #8 Cline and Renka
- (b) Gradients
- #4 Akima
- #17 Stead

(2) Interpolation over a Sphere
- #12 Lawson

B. Distance-weighted Interpolants
- #9 Foley

C. Design
- #10 Jensen
- #11 Kochevar

D. Approximation over a Sphere
- #18 Wahba

E. Spline-Blended Approximations
- #15 Salkauskas

F. Cubatures
- #5 Barnhill and Little

G. Dimensions of Spaces of Piecewise Polynomials
- #16 Schumaker

II. FOUR-DIMENSIONAL SURFACES

A. Tetrahedral Interpolants
- #6 Barnhill and Little
- #2 Alfeld

B. Distance-weighted Interpolants
- #7 Barnhill and Stead

All papers received by the editors by September 30, 1982.

R. E. Barnhill
Department of Mathematics
University of Utah
Salt Lake City, UT 84112

G. M. Nielson
Department of Mathematics
Arizona State University
Tempe, AZ 85287