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THE BREAKUP OF SOLITON LIKE PULSES ON A NONLINEAR, NONUNIFORM ELECTRICAL LATTICE

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Introduction. In recent years there have been a number of experimental studies on nonlinear electrical transmission lines ([1]–[5]). These studies have treated solitary wave propagation, pulse shaping and collision as well as recurrence phenomena. All this work has been done on uniform transmission lines.

In this paper the break-up of soliton-like pulses on a nonuniform line is studied. This work was motivated by the numerical studies of Madsen and Mei [6] and the theoretical analysis of Tappert and Zabusky [7], see also [8]. These authors treated the so-called gradient induced fission of Korteveg-deVries solitons. Although the experimental configuration used for this work did not support KdV solitons, qualitative features of the analytic treatment were confirmed. This point is worth noting since it is evidence that the analytic treatment of solitons which has been so elegantly elaborated recently [9] provides insight into the behavior of physical systems that are close (in some sense) to systems which possess exact soliton solutions. Apparently soliton phenomena are much more stable under perturbations than the analytical apparatus used to describe them. The verification of this statement is one of the more pressing issues confronting theorists who wish to apply soliton theory to realistic physical situations.

Experimental Configuration and Results. The transmission line constructed for this study consists of 460 sections as shown in Figure 1. Matched 1N4001 reverse biased silicon diodes provide the nonlinear capacitance. Two diodes for each section were selectively paralleled to give a more nearly uniform network. The dispersive capacitors C_s in the homogeneous region are 1% tolerance mica capacitors. In the inhomogeneous region mica trimmer capacitors are used. The inductance is a continuous coil tapped at uniform intervals. Total losses in the system are negligible with the major loss induced by the reverse bias leakage current of the diodes. The diodes were chosen so that leakage current losses are no greater than those induced by a *50* megaohm resistor in parallel with each diode.

A homogeneous line was first constructed and soliton-like propagation was confirmed. Examination of the amplitude versus width relationship of the pulses reveals that the system is not modelled by the *KdV* or modified *KdV* equations. However, the essential properties of evolution of soliton-like pulses from a variety of driving pulses, the stability of waveshape and amplitude dependent propagation velocity are confirmed.

With all other network parameters held fixed the dispersive capacitance C_s was set as indicated in the generic Figure 2. The inhomogeneity is thus induced in the dispersion, leaving the small signal propagation velocity and the line impedence uniform.

The system was excited so as to form a single soliton before the pulse reached position 1. The leftmost traces in Figures 3–6 are traces of the pulse at 1. The adjacent trace is the pulse at 2, the third at 3, and the

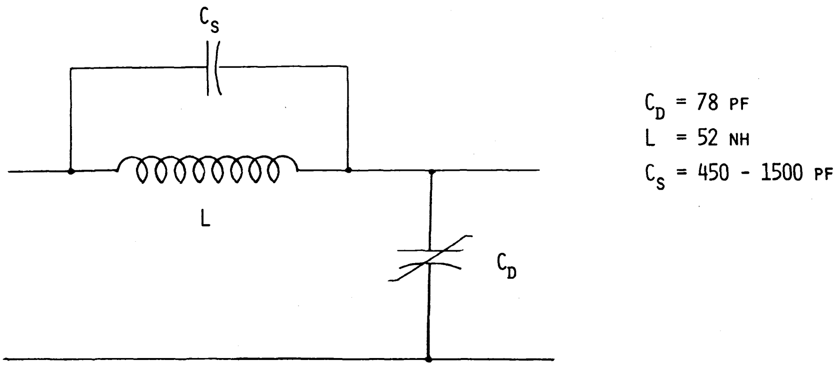

Figure 1. Typical Section

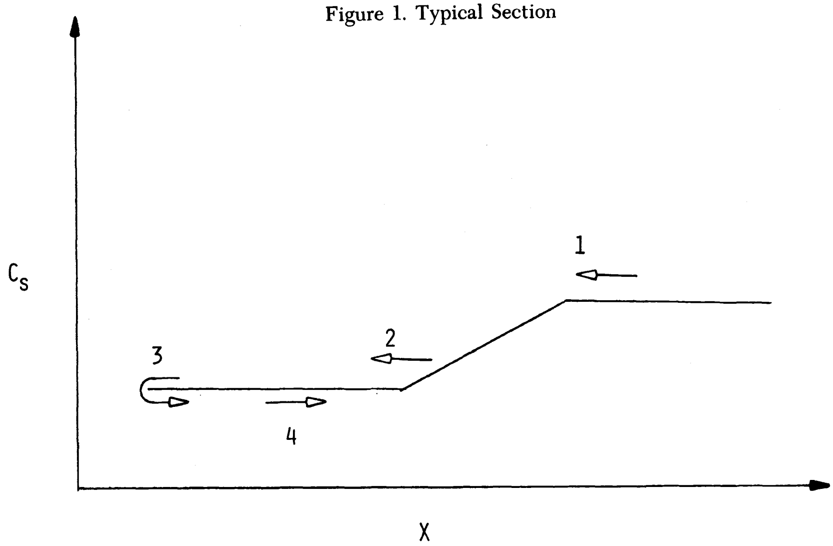

Figure 2. Nonuniformity in C_s

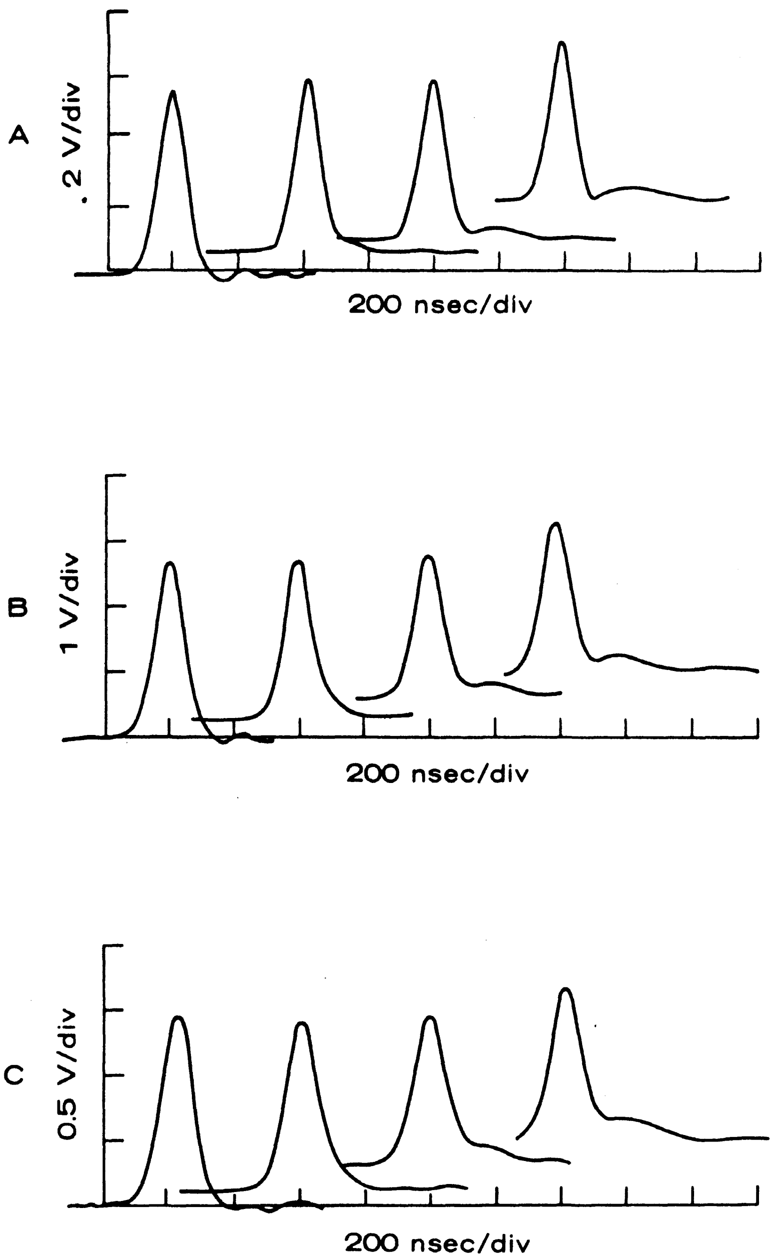

Figure 3. Fission Evolution, $\delta = 10/9$, $L = 1$

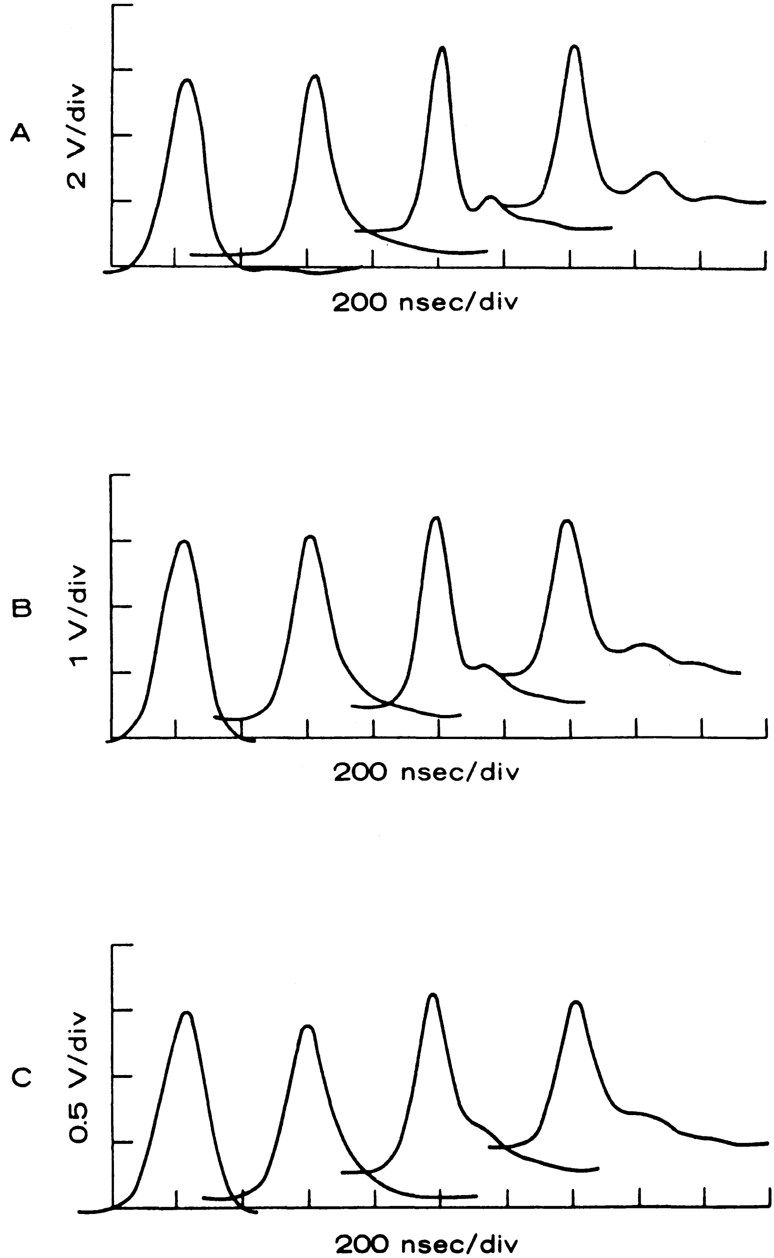

Figure 4. Fission Evolution, $\delta = 20/9$, $L = 1$

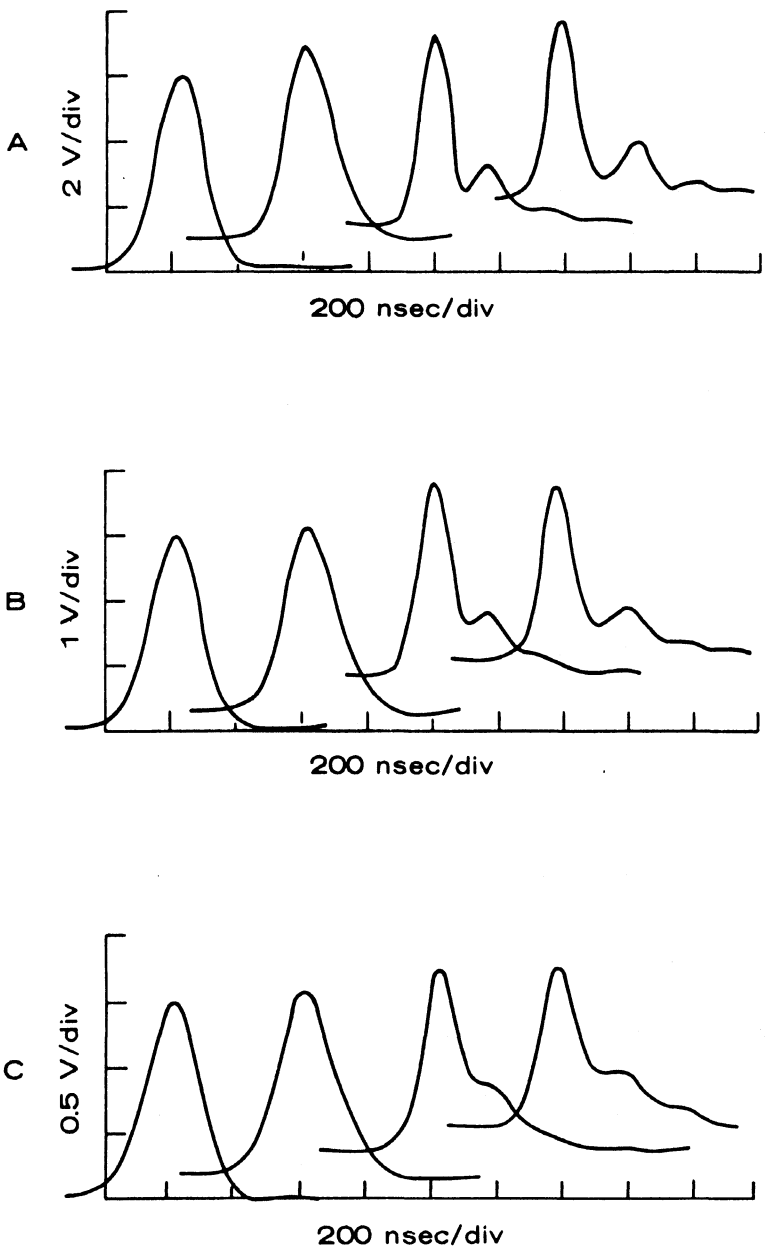

Figure 5. Fission Evolution, $\delta = 10/3$, $L = 1$

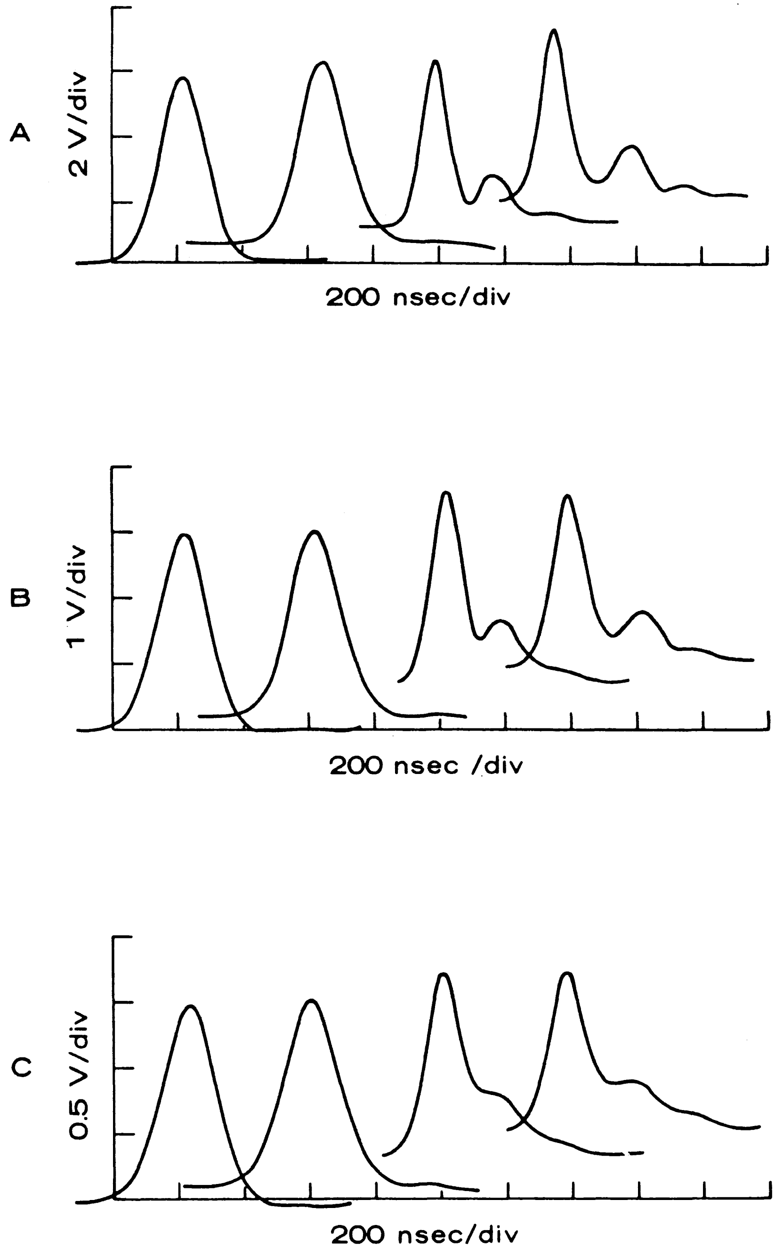

Figure 6. Fission Evolution, $\delta = 10/3$, $L = 0$

fourth is the trace at position 4 after reflection from the open circuit at position 3. The trace at position 3 is scaled by 1/2. The direction of propagation in Figures 3–6 is from right to left.

The most notable feature of the traces is that in each case the single soliton fissions into two or more solitons of unequal amplitudes with small residual oscillations. In Figure 3 where $\delta = C_{s1}/C_{s2}$ is approximately 10/9, two solitons are formed. In Figure 4 where δ is approximately 20/9, three solitons are formed. In Figures 5 and 6, δ is approximately 10/3 and at least three solitons are formed. The presence of a fourth soliton is not certain since its amplitude is of the same order as the residual oscillations. The possible existence of a fourth soliton is somewhat more evident in the original photographs of the oscilloscope traces which were copied to produce Figures 3–6. Furthermore, actual amplitude measurements and visual observations were made using an oscilloscope with greater vertical resolution than the one equipped for photography.

The graphs in Figures 7 and 8 plot the ratio of the amplitudes at position 4 of the first soliton (U1) to the second (U2) and the third (U3) as a function of the initial soliton amplitude (U0) at position 1. The best quality data is the amplitudes of the first and second solitons when δ was 20/9 or 10/3. In these cases the ratio of the amplitudes was relatively insensitive to the amplitude of the initial excitation. In general, the quality of the data when δ was 10/9 is inferior to that taken from the other configurations as is suggested by inspection of Figure 3. However, this data as well as the measurements of the third soliton's amplitude (when it appears) suggest that the leading soliton becomes relatively larger as the initial excitation amplitude increases.

Figures 9 and 10 show the dependence of the ratio of U1/U2 on the amplitude of the initial soliton for various lengths of the gradient region measured in units of half widths of the initial soliton. In nearly all instances, the leading soliton becomes relatively larger as the length of the gradient increases from zero to two. Investigation of longer gradients was not conclusive because of limited line length and difficulty in resolving small amplitude solitons.

The accuracy of these ratios is estimated at 5% for δ greater than two or for U0 greater than four. The remaining accuracies are estimated at 15%.

In all cases reflection from an open circuit at position 3 was used to obtain the greater effective line length needed to observe the complete evolution of the fission process. Although single solitons remain stable under reflection, this fact does not preclude the possibility of reflection affecting the fissioning process.

A plot of the soliton position versus time with the origin defined by the initial pulse's arrival at position 1 indicates that the solitons propagate with constant velocity once they emerge from the initial pulse. However, while linear regression projects the leading soliton back to the origin, projection of the second soliton gives a negative position at

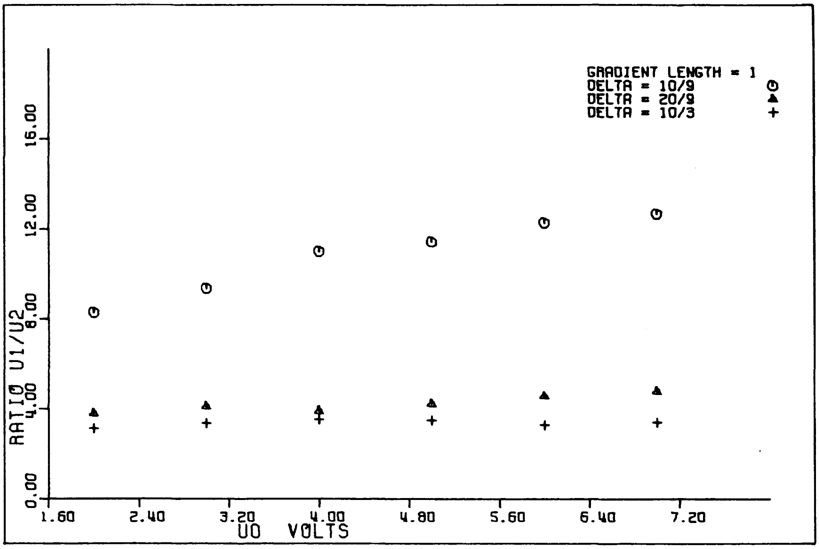

Figure 7. Relative Amplitudes U1/U2

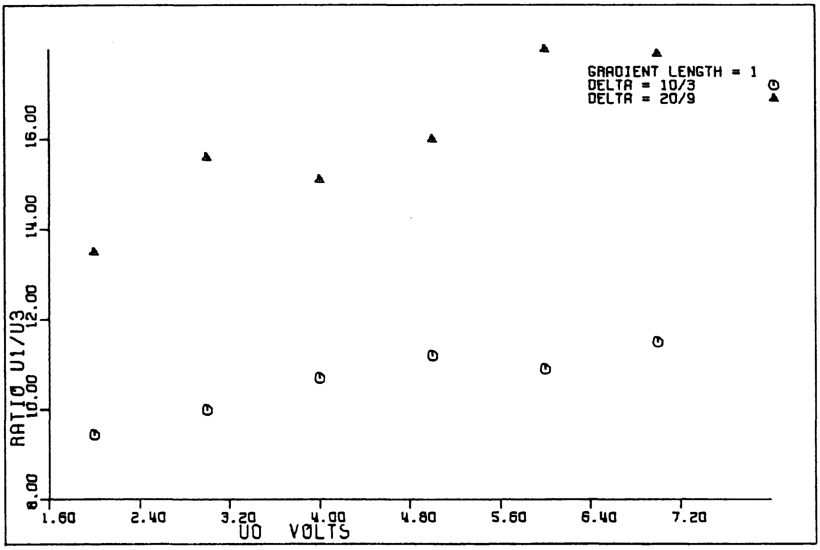

Figure 8. Relative Amplitudes U1/U3

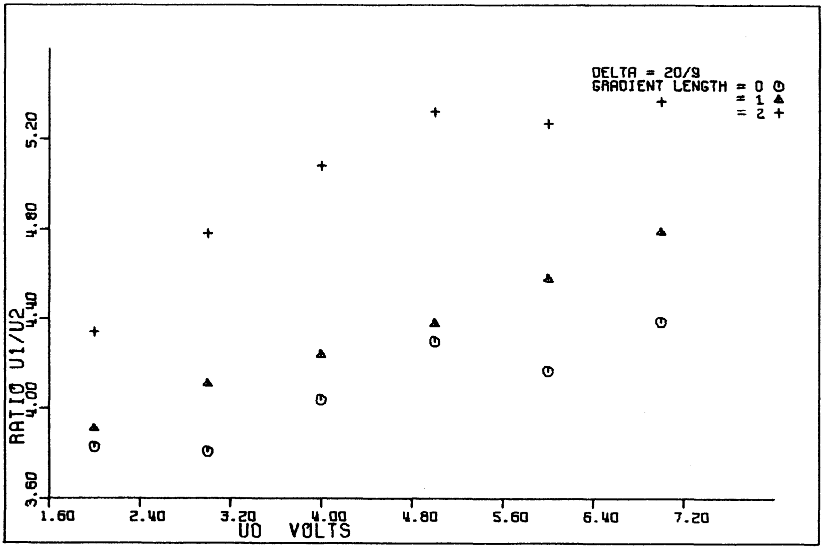

Figure 9. Relative Amplitudes U1/U2

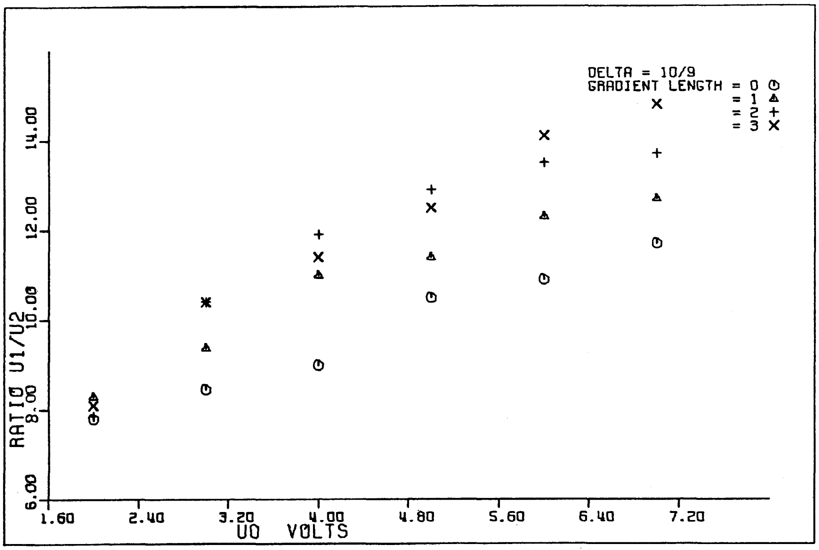

Figure 10. Relative Amplitudes U1/U2

zero time. This displacement from the origin increases from approximately 1/6 half width to 1/2 half width as the gradient length increases from zero to two half widths.

Conclusions. The experiment reported above is one in what could easily be a long series of investigations on nonlinear electrical transmission lines. One very interesting possibility is to physically impose periodic boundary conditions, i.e., make a ring and attempt to observe the recurrence effect in this configuration. The feasibility of such an experiment has been investigated and it appears to be possible. Coupled lines with different propagation characteristics also offer intriguing possibilities. Uniform or mildly nonuniform lines such as the one described above might be further studied in order to establish what has been conjectured here: that the soliton *phenomena* are in fact quite robust. If this is true, it would be possible to consider devices based on the phenomena.

For the theorist, the problem of developing *qualitative* theory of soliton phenomena is suggested by this work. On the time scale of laboratory experiments, how can the fact that nonsoliton systems (that are close to pure soliton systems) qualitatively behave like real solitons be accounted for? How, that is, is it possible to develop a basis for the intuitive understanding of near soliton systems that this, and much other work suggests?

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