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INTRODUCTION TO DISCRETE SYSTEMS

Certain discrete systems (i.e., coupled systems of ordinary differential equations) also possess soliton solutions and can be analyzed mathematically as isospectral flows. Indeed, the modern history of the soliton begins with the computer studies of Fermi, Pasta, and Ulam on a chain of masses coupled to their nearest neighbors by nonlinear springs and constrained to vibrate longitudinally. Before this work, it was generally believed that the nonlinearities would cause the linear Fourier modes to share energy so that as time increased, the system would approach a state in which all Fourier modes were nearly of equal amplitude. Such a state would look very chaotic, nearly random. However, the computer studies showed that the Fermi-Pasta-Ulam lattice did not approach such a random state; rather, the initial distribution of Fourier modes recurred, or very nearly recurred, at special points in time (known as "recurrence times"). It is now understood that such behavior indicates that the equations of the lattice are almost completely integrable (in the sense of [1]). The hope of elucidating these numerical results was one motivation for Kruskal and Zabusky to begin their study of the Korteweg-de Vries equation.

This section begins with a survey article by E. Atlee Jackson; his article is the only place in the literature where a physicist explains for mathematicians the reasons for his interests in lattice solitons. M. Toda's survey, in contrast to Jackson's general article, emphasizes isospectral flows and their relevance to lattices. The remaining four articles in this section present examples of current research in the mathematics of completely integrable discrete systems, and the physics of (not necessarily integrable) systems which support soliton-phenomena.

Reference: 1. V. I. Arnol'd and A. A. Avez, *Ergodic Problems of Classical Mechanics* (W. A. Benjamin, Inc., New York, 1968).