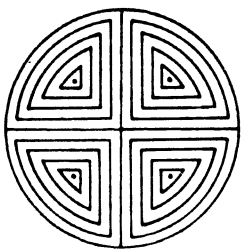
The Rocky Mountain Journal of Mathematics

Volume 8, 1978



Published by
The Rocky Mountain Mathematics Consortium
Department of Mathematics
Arizona State University
Tempe, Arizona

The Rocky Mountain Mathematics Consortium is a nonprofit corporation with headquarters at Arizona State University. Its institutional members are:

University of Alberta
Arizona State University
University of Arizona
Brigham Young University
Colorado State University
University of Colorado
University of Denver
Idaho State University
University of Kansas
Montana State University
University of Montana
University of Nebraska
New Mexico State University
University of New Mexico
Northern Arizona University
Texas Tech University
University of Texas at El Paso
University of Texas at San Antonio
University of Utah
Utah State University
University of Wyoming

Printed by The University Press
Brigham Young University
Provo, Utah, U.S.A.

ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 8, Numbers 1 and 2,
Winter and Spring 1978

PREFACE

This issue contains papers presented at, or arising from, a Conference on the Theory and Application of Solitons, held in Tucson, January 5–10, 1976. The term "soliton" was coined in the early 1960's by Martin Kruskal and Norman Zabusky; it designates a special type of wave which retains its identity under nonlinear interactions with other waves.

Solitons are, first of all, localized solutions of nonlinear evolution equations ("solitary waves"). Pre-1960 experience with nonlinear phenomena suggested that nonlinear interaction of such waves should lead to their disintegration; however, computer studies revealed the surprising fact that distortion is not inevitable: solitons, like particles (whence the suffix "-on") emerge unchanged after interaction. Since the early discoveries of Kruskal and Zabusky, intensive investigations of soliton phenomena have considerably deepened our understanding of nonlinear waves. During this same period, the physical sciences have made increasing use of the soliton's properties as clarifying concepts in the nonlinear (as opposed to near-linear) analysis of diverse processes. Moreover, these studies have led to novel problems and results in fields of mathematics once thought far removed from the theory of nonlinear waves; indeed, substantial advances in "pure" mathematics were made by scientists concerned with applied problems.

The primary goal of the Conference was to promote contact between mathematicians and physicists working on various aspects of the soliton phenomenon. It was felt that mathematicians, in their further researches, should make use of the physical intuition and insight which had led to the present successes. On the other hand, this field presents a unique opportunity for mathematics to contribute to a developing area of physics, since in this field the mathematical techniques and their applications are evolving simultaneously. Accordingly, we asked the speakers and authors to stress the utility of the mathematics involved, and to identify mathematical problems arising in current physics research. The response has been gratifyng. The reader of this volume will find beautiful applications in fairly final form, as well as numerous open problems ranging from detailed questions about a specific equation to entire theories yet to be constructed.

The volume is organized, as were the sessions at the Conference, into four major topics: mathematical techniques associated with continuous

problems (p.d.e.'s); discrete problems (physical lattices and systems of o.d.e.'s); waves in plasmas; quantization and quantum field theory. Each of these sections is preceded by a short introduction aimed at orienting the nonspecialist reader. This volume is not a comprehensive survey of the current work on solitons, and we decided against overburdening our introductions with lengthy reference lists. Apologies are due the authors of important works which we did not cite in our summaries, but we feel that the interested reader, following the leads provided in this volume, will soon discover and study all the fundamental papers of the subject.

We are indebted to many people for their help with the planning and running of this conference: Drs. Agins, Pasta, and Pell at N.S.F. for their support (financial and otherwise); the main lecturers, Drs. A. Jackson, D. Kaup, A. Neveu, A. Newell and F. Tappert; the session chairmen, Drs. J. Ford, J. Greene, M. Kac, and A. Scott; all contributors to this volume for the care with which they explained ideas often not available in expository form. We apologize for the delay in publication; this was occasioned by the size of the volume and by numerous revisions requested in an attempt to make these proceedings as useful as possible to the non-expert.

For day-to-day help during the conference, we are particularly indebted to Dr. Warren Ferguson. For editorial assistance, we thank Dr. Wm. Scott, Managing Editor of this Journal; and for secretarial help, Debbie Pearlman and Laurinda Queen.

H. Flaschka and D. W. McLaughlin
Tucson, April 15, 1977