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CORRECTION TO MY PAPER "UNIFORM AND STRONG ERGODIC THEOREMS IN BANACH SPACES"

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We use some notations and terminology from [1] and [4]. Let $C_0[0, 1]$ be the space of functions $f(x)$ continuous for $0 \le x \le 1$ which vanish at 0, with $\|f\| = \max |f(x)|$. For any real number $\beta > 0$ we define

$$Q_\beta f = (I - J_\beta)f, \quad (J_\beta f)(x) = [\Gamma(\beta)]^{-1} \int_0^x (x-u)^{\beta-1} f(u)\, du$$

for $f \varepsilon C_0[0, 1]$ and $0 \le x \le 1$. Then for each integer $n \ge 1$ the n^{th} iterate Q_β^n has the form

$$(Q_\beta^n f)(x) = f(x) - \int_0^x P_n(x-u, \beta) f(u)\, du$$

where

$$P_n(w, \beta) = \sum_{k=1}^{n} (-1)^{k-1} \binom{n}{k} [\Gamma(k\beta)]^{-1} w^{k\beta - 1}.$$

The kernel $P_n(w, \beta)$ used in the inequalities cited as Hille's estimates on pages 534 and 542 of [4] should be replaced by the Laguerre polynomial $L_n(w, \beta)$ (cf. [2], Chapter V):

$$L_n(w,\ \beta) = \sum_{k=0}^{n} (-1)^k \binom{n+\beta}{n-k} \frac{w^k}{k!}.$$

In this case, the estimate for $\|T_\beta^n\|$ on page 535 of [4], however, need not be satisfied. Instead, we define $T_\beta = \Gamma(\beta + 1) Q_1 J_\beta$ for $\beta \ge 3/2$. Since $\|J_\beta\| \le 1/\Gamma(\beta+1)$, it follows that $\|T_\beta^n\| \le \|Q_1^n\| = O(n^{1/4})$. T_β is compact, because J_β is compact as a Volterra integral operator. Therefore T_β turns out to be uniformly (C, α) ergodic for $\alpha > 1/4$ by Theorem 3.1 of [3]. Finally, the case $-1 < \beta < 3/2$ in Remark 2 of [4] should be replaced by the case $0 \le \beta \le 1$. This finishes the correction.

Incidentally, we remark that the operator T_β satisfies the following conditions:

(i) The point $\lambda = 1$ is either in the resolvent set $\rho(T_\beta)$ or else a simple pole of the resolvent $R(\lambda; T_\beta)$.

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(ii) The ascent and descent of $I - T_\beta$ are both equal to 1.
(iii) $I - T_\beta$ is a quasi-Fredholm operator.
(iv) $(I - T_\beta)^k C_0[0, 1]$ (and $(I - T_\beta)^k L_1(0, 1)$) is closed for any integer $k \geq 1$.
(v) $\mathrm{Ker}(I - T_\beta)^k + (I - T_\beta)^k C_0[0, 1]$ (and $\mathrm{Ker}(I - T_\beta)^k + (I - T_\beta)^k L_1(0, 1)$) is closed for any integer $k \geq 1$.

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