JULIUS R. BLUM 1922-19821

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On the thirteenth of April, 1982, as he worked in his office at the University of California, Davis, Julius R. Blum was felled by a massive heart attack. His untimely death was much more than a local tragedy. Julius Blum was truly an international figure with friends and co-workers all over the world. While the impact of his prolific research record was substantial, the impact he had on his profession went well beyond his research contributions. He had a rare ability to bring the creative talents that served him so well in his mathematical and statistical investigations to bear upon thorny practical problems of the administrative or organizational type. He played a key role in the development of programs in statistics at several major universities. We expect that his overall leadership in the profession over a distinguished thirty-year career will have an influence that will persist long after the year of his death.

Julius Blum was born in Nuremberg, Germany, in 1922, the son of Abraham M. and Antonia B. Blum. Even at the age of six, he stubbornly insisted that he would be a professional mathematician. While his early years were relatively normal, his life changed abruptly in 1937, the year his parents arranged for him to leave Germany under the sponsorship of his uncle in the United States. Julius served in the United States Army during World War II and began his mathematical studies in earnest at Berkeley after the war. He was a Phi Beta Kappa at Berkeley, receiving the A.B. Degree in mathematics with highest honors in 1949. He received his Ph.D. in statistics from Berkeley in 1953, writing a dissertation under the direction of M. Loeve.

He spent his first six postdoctoral years at Indiana University and spent the three years that followed as a member of the technical staff at the Sandia Corporation. These were very fertile research years for Blum. In 1963 alone, he published eleven research papers, all of which appeared in leading journals in mathematics and statistics. Also in 1963, he assumed the dual role of Professor of Mathematics and Chairman of the Department of Mathematics at the University of New Mexico. During his six years as department chair, he helped to develop a small and relatively unknown department into a vital and extremely active group of research scientists. During this period, his distinctive administrative style became apparent. The hallmarks of the Blum style were openness, forthrightness, and an absolute insistence on quality. His years in New Mexico were happy, exciting years for him, and he often spoke fondly of Albuquerque and the many friends he made there. He left New Mexico in 1974, accepting an appointment in the Mathematics Department at the University of Wisconsin,

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By action of the Council of the Institute of Mathematical Statistics, this issue of $\it The\ Annals\ of\ Statistics$ is dedicated to the memory of

JULIUS R. BLUM



Milwaukee. He enjoyed working with several new collaborators at Milwaukee and launched a series of deep and highly technical investigations in several new areas.

In 1976, Blum accepted a one-year appointment as Program Director for Statistics at the National Science Foundation. The broad range of his research interests made him an ideal choice for this task. During that year, he expanded the reputation he enjoyed for his breadth, judgement, and taste. He was also a very fair-minded man, and he made a strong effort to support promising young researchers as well as more established workers. He spent the next two years at the University of Arizona, where he chaired the Committee on Statistics. He established a statistical consulting center at the University of Arizona, broadened the range of statistics offerings there, and oversaw a modest growth in the statistics faculty in spite of increasing financial constraints on the University.

The Intercollege Division of Statistics was established on the Davis campus in January 1979. The Division first offered a full range of coursework during fall quarter 1979, with Julius Blum as chair. Blum joined the University of California, Davis as Associate Dean for Statistics in July, 1979. Blum immediately dedicated himself to the recruitment of senior faculty, to the development of a full graduate program, and to nurturing and developing a fledgling statistical consulting unit. In three short years at Davis, he made substantial progress toward achieving these goals, and left a legacy which continues to inspire and motivate his colleagues there.

Julius Blum left behind an impressive record of more than eighty publications touching many areas of probability and statistics. One may classify his research interests into three categories: probability, ergodic theory and mathematical statistics. We discuss his contributions to each of these areas in turn.

His earliest work was on almost sure convergence [1] and used martingale techniques. These results were applied to obtain conditions on the almost sure convergence of stochastic approximations of Robbins-Monro (1951) procedures. The results obtained are still quoted in current literature. We state the basic result of this research. Let H(y|x) be a family of distribution functions depending on the real parameter x. The regression function $M(x) = \int y dH(y|x)$ is taken to be unknown to the experimenter. It is assumed that M(x) is Lebesgue measurable and satisfies

- A. $|M(x)| \le c + d|x|$ where $c, d \ge 0$
- B. $\int [y M(x)]^2 dH(y \mid x) \le \sigma^2 < \infty$
- C. $M(x) < \alpha$ for $x < \theta$, $M(x) > \alpha$ for $x > \theta$
- D. $\inf_{\delta_1 \le |x-\theta| \le \delta_2} |M(x) \alpha| > 0$ for every pair (δ_1, δ_2) with $0 < \delta_1 < \delta_2 < \infty$.

Let $\{a_n\}$ be a sequence of positive numbers such that (a) $\sum_{n=1}^{\infty} a_n = \infty$, (b) $\sum_{n=1}^{\infty} a_n^2 < \infty$. Suppose the x_1 is arbitrary and that the sequence of random variables are recursively given by

$$x_{n+1} = x_n + a_n(\alpha - y_n),$$

where y_n is a random variable having conditional distribution $H(y | x_n)$ given

 x_n, x_{n-1}, \dots, x_1 . Under these assumptions, Blum showed that x_n converges to θ with probability one as $n \to \infty$.

Another frequently cited work in Blum's bibliography was written jointly with H. Chernoff, M. Rosenblatt, and H. Teicher [12], and dealt with central limit theorems for interchangeable processes. This paper made use of the celebrated result of De Finetti (1937) on interchangeable processes as mixtures of independent, identically distributed sequences, and established necessary and sufficient conditions for asymptotic normality of the partial sums. One of the results in the paper states that if $\{X_n\}$ is an interchangeable sequence with mean zero and variance one, the central limit theorem holds for the process if and only if, for $i \neq j$, $EX_iX_j = 0$ and $E\{[X_i^2 - 1][X_j^2 - 1]\} = 0$.

Julius Blum published his first papers in ergodic theory ([14] and [15] with D. L. Hanson) in 1960. His paper on invariant probability measures [15] was a particularly important work showing that each ergodic probability measure is an extreme point of the set of invariant probability measures, and that (in reasonable cases) each invariant probability measure can be represented as an integral over the set of ergodic probability measures. Blum developed a lasting interest in ergodic theory and, from 1960 until his death, about half his research was in ergodic theory.

Blum's interests spanned ergodic theory; he worked on a wide variety of topics within it and obtained results on most of them. He obtained ([21] with D. L. Hanson) the first result showing what it means, in terms of a mixing condition, for a stochastic process to have no dependence on the infinite past (or infinite future); this work applies to automorphisms of Kolmogorov. He obtained an interesting collection of results involving roots of transformations: [43] with H. D. Brunk and D. L. Hanson, and [37] and [39] with N. Friedman.

Though Blum obtained other results on various topics in ergodic theory, perhaps the one cohesive theme that runs through most of this work is the investigation of the ergodic theorem and, more particularly, the ergodic theorem for subsequences and for weighted averages. His work here begins in [14] with Hanson in which it is observed that a measure preserving transformation is strongly mixing if and only if the mean ergodic theorem applies to each integrable function and each subsequence of its "translates." He worked on the ergodic theorem for weighted averages in [34] with Hanson and in [61] with B. Eisenberg. His work on the ergodic theorem for subsequences, some of it technically quite difficult, involved various co-authors and continued up until his death: [60] with Eisenberg and L. S. Hahn, [65] and [75] with Eisenberg, and a sequence of papers with J. Reich, including [71] and [72].

While Julius Blum's primary research interests were in the areas of ergodic theory and probability, he also explored a variety of questions in statistics, making a number of important contributions. A good deal of his early work in statistics was joint work with Judah Rosenblatt. In a series of papers [32, 35, 47, 48], Blum and Rosenblatt studied fixed-width confidence intervals (FWCI), identifying problems in which FWCI cannot be constructed and problems in which multistage sampling is required. They were able to provide simpler proofs of several results on the nonexistence of FWCI by showing that FWCI were impossible for smaller families of distributions than considered in the original

proofs. For example, they reproved the Bahadur-Savage result on nonexistence of FWCI when estimating the mean of a distribution belonging to the class of all distributions with finite mean by restricting attention to distributions concentrated on two points. They demonstrated that one sample procedure could not provide a FWCI for the mean of an IFR distribution by demonstrating this fact for the exponential distribution. In this latter problem, as in many similar problems, they explicitly displayed two-stage or multistage procedures leading to the desired FWCI.

In collaboration with V. Susarla and G. Walter, Blum investigated a number of nonparametric estimation problems [69, 70, 73, 76, 77, 79, 80, 86]. The basic idea involved in much of this work is that of inverting a known functional relationship. Blum used this approach successfully in some of his early work; for example, in [6], he considered the problem of approximating a distribution function F from the sequence $\{a_n\}$, where

(1)
$$a_n = \int f_n(y) \ dF(y),$$

with f_n known. Blum's solution to this problem involved approximating these integrals by finite sums employing extreme values of the functions $\{f_n\}$ over subintervals. He then identified a class of step functions which converge to F at all its continuity points. The approach outlined above provided a natural entré into the problem of estimating a mixing distribution [70, 79]; indeed, the solution outlined by Blum and Susarla in [70] has precisely this flavor. Although clusters of Blum's papers have common themes or techniques, his scope was broad, and his forays into nonparametric estimation included studies involving Bayesian nonparametrics using mixtures of Dirichlet priors [73], nonparametric maximum likelihood estimation using Grenander's method of sieves [86], and nonparametric estimation using delta sequences [76] and differential operators [79].

Much of Blum's statistical work was dedicated to extending or refining the work of others. There are, however, occasions in which he introduced a new and fundamental notion into the statistical literature. An example of this is his paper with J. Rosenblatt [42] on the use of partial prior information in which the idea of T-minimax estimation was first defined in a general way. A decision rule δ_0 is said to be T-minimax if

$$\sup_{\tau \in T} r(\tau, \, \delta_0) = \inf_{\delta} \sup_{\tau \in T} r(\tau, \, \delta),$$

where T is a fixed class of prior distributions, and $r(\tau, \delta)$ is the Bayes risk of the rule δ relative to the prior τ . In [42], Blum and Rosenblatt identified T-minimax rules in several problems, and proved a certain continuity result concerning T-minimax rules in problems with contaminated priors. As shown in a number of subsequent studies (Randles and Hollander, 1971; Watson, 1974), the notion of T-minimaxity is appropriate and tractable in a variety of different settings.

A noteworthy feature of Julius Blum's research was his numerous collaborations. The joy of interacting with other creative minds was as important to him as the challenge of the problems themselves. Though six of his first seven papers were solo-authored, 74 of his last 79 papers were co-authored, and there were two 10-year periods in his life when all his published research was co-authored. He was highly imaginative, a veritable fount of ideas, and it is no wonder that his co-authors (all 34 of them) found joint research efforts with Julius Blum to be extremely rewarding.

Although Julius Blum's life was marred by a number of personal tragedies, he never lost his zest for living. His personal characteristics included uncommon generosity and a genuine concern about social injustice. He was a very social, outgoing person, and it has been conjectured that no one in Statistics had more personal friends and acquaintances in the profession than did Julius Blum. What he enjoyed most was people and mathematics. He loved to travel, he loved to laugh, he loved to eat—he was open to all life had to offer. Indeed, he was so full of life that, since the thirteenth of April, 1982, many of his friends have found the silence deafening. He is survived by his wife, Andrea Hicks Blum, by his two sons, Mark and Howard, and by his granddaughter, Robin. They all brought great joy into his life. May they find comfort and satisfaction in this tribute to the man and his work.

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