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NONPARAMETRIC SELECTION PROCEDURES FOR SYMMETRIC LOCATION PARAMETER POPULATIONS

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Various selection problems based on sample means have been considered by Bechhofer (*Ann. Math. Statist.* (1954), 16-39). Parallel selection procedures based on one-sample Hodges-Lehmann estimators (*Ann. Math. Statist.* (1963), 598-611) are proposed in this paper. These procedures possess more desirable finite sample properties and equivalent asymptotic properties as compared to rival procedures considered by Lehmann (*Math. Ann.* (1963), 268-275) and Puri and Puri (*Ann. Math. Statist.* (1969), 619-632).

1. Introduction and summary. Let $X_{[i]j}$ $(j = 1, \cdots, n; i = 1, \cdots, k)$ be independent observations from k populations with distribution functions (df's) $F(x - \theta_{[i]})$, $i = 1, 2, \cdots, k$. Let $\theta_1 \leqq \theta_2 \leqq \cdots \leqq \theta_k$ denote the ordered $\theta_{[i]}$'s. The following problems are considered:

(I) Select a "good" population, the ith population being regarded as good if $\theta_{[i]} > \theta_k - \Delta$, for some preassigned $\Delta > 0$ $(i = 1, 2, \cdots, k)$;

(II) select the best t populations, i.e., the populations with location parameters $\theta_{k-t+1}, \theta_{k-t+2}, \cdots, \theta_k$ *without* regard to order;

(III) select the best t populations *with* regard to order.

The above problems were considered by Bechhofer (1954) under the normality assumption on the $X_{[i]j}$'s. His approach, now known as the "*indifference zone*" approach selects the "*best*" populations with a guaranteed minimum probability P^* (preassigned) of correct selection when $(\theta_1, \cdots, \theta_k)$ lies in a subset, say S of the parameter space (here R^k, the k-dimensional Euclidean space). S is called the *preference zone* and $R^k - S$ the *indifference zone*. Robust procedures analogous to Bechhofer's but based on the joint ranks of the $nk X_{[i]j}$'s, were considered by Lehmann (1963), Bartlett and Govindarajulu (1968) and Puri and Puri (1969). All these procedures used the c-sample rank-order statistics for selection purposes. However, the slippage configuration of parameters used in these papers was not "least favourable" (to be explained later) for selecting the desired populations (see Rizvi and Woodworth (1970) for counterexamples). The slippage configuration as pointed out by Puri and Puri (1969) was least favourable only when the parameters satisfied the relation $\theta_{[i]} - \theta_{[j]} = O(n^{-\frac{1}{2}})$ for all $1 \leqq i \neq j \leqq k$.

In this note, we propose alternate procedures, based on one-sample Hodges-Lehmann (H-L) estimators (see [10]) of $\theta_{[i]}$'s under the additional assumption

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that F is symmetric about the origin. Our procedures give in all these cases least favourable configurations for finite samples without needing any restriction on the parameters. The procedures are given in Section 2, and it is proved that they are least favourable under the slippage configurations used by Bechhofer (1954) and others. The asymptotic relative efficiencies (ARE's) of our procedures with respect to the Bechhofer procedures are studied in Section 3. In all the three cases, these turn out to be the same as the ARE of a Chernoff–Savage test with respect to Student's t-test; the ARE results agree with those of Lehmann (1963), Puri and Puri (1969) and Randles (1970). The last author considers for problem (I) a procedure similar to ours based on $\binom{k}{2}$ two-sample H–L estimators (taking all possible pairs from the k-populations). A comparison between ours and Randles' procedure is made in Section 3.

One may remark in passing that there is another approach formulated by Gupta (1956, 1963) which does not require $(\theta_1, \cdots, \theta_k)$ to lie in some subset S of R^k. However, a subset of the given k populations is selected which is guaranteed to contain the best populations with probability not less than some preassigned P^*. Robust procedures based on this approach, known as "*subset selection*" procedures, are due to Gupta and McDonald (1970) (see also McDonald (1972)). These procedures are also based on the vector rankings of the $nkX_{[i]j}$'s. For a different kind of approach, one may also refer to Dudewicz (1971).

2. The selection procedures and least favourable configurations. Let $R_{ij} = \frac{1}{2} + \sum_{j'=1}^{n} u(|X_{[i]j}| - |X_{[i]j'}|)$, $j = 1, 2, \cdots, n$, $i = 1, 2, \cdots, k$, where $u(t) = 1, \frac{1}{2}$, or 0 as $t >, =,$ or < 0. Thus R_{ij} is the rank of $|X_{[i]j}|$ among $|X_{[i]1}|, \cdots, |X_{[i]n}|$ $(1 \leqq i \leqq k; 1 \leqq j \leqq n)$. Let $\mathbf{X}_{[i]} = (X_{[i]1}, \cdots, X_{[i]n})$. Consider the one-sample signed rank-statistics

$$h(\mathbf{X}_{[i]}) = \sum_{j=1}^{n} \operatorname{sgn}(X_{[i]j}) EJ(U_{nR_{ij}}), \qquad i = 1, 2, \cdots, k, \tag{2.1}$$

where sgn $t = 1, 0$, or -1 according as $t >, =,$ or < 0; $U_{n1} \leqq U_{n2} \leqq \cdots \leqq U_{nn}$ are the n ordered random variables forming a rectangular (0, 1) distribution, and, $J(u) = \Psi^{-1}((1+u)/2)$, $\Psi(x)$ being the df of a random variable satisfying $\Psi(x) + \Psi(-x) = 1$ for all real x.

The one-sample H–L estimators are given by

$$\hat{\theta}_{[i]}(\mathbf{X}_{[i]}) = \frac{\hat{\theta}_{[i]1}(\mathbf{X}_{[i]}) + \hat{\theta}_{[i]2}(\mathbf{X}_{[i]})}{2}, \tag{2.2}$$

$i = 1, 2, \cdots, k$, where $\hat{\theta}_{[i]1}(X_{[i]}) = \sup\{a: h(\mathbf{X}_{[i]} - a\mathbf{1}_n) > 0\}$, $\hat{\theta}_{[i]2}(\mathbf{X}_{[i]}) = \inf\{a: h(\mathbf{X}_{[i]} - a\mathbf{1}_n) < 0\}$, $\mathbf{1}_n = (1, \cdots, 1)$ is an n-tuple with all elements 1. We may note that all these statistics and estimators depend on n. The following property of location invariance (for proof see [10]) is satisfied by these estimators.

$$\hat{\theta}_{[i]}(\mathbf{X}_{[i]} + c\mathbf{1}_n) = \hat{\theta}_{[i]}(\mathbf{X}_{[i]}) + c, \qquad i = 1, 2, \cdots, k, \tag{2.3}$$

c some constant. In the particular case, when $J(u) = u$ or $\chi_1^{-1}(u)$ (the inverse of a chi-distribution with one degree of freedom) the statistics become the Wilcoxon signed-rank or normal-score statistics. In the former case,

$$\hat{\theta}_{[i]}(\mathbf{X}_{[i]}) = \text{med}_{1 \leqq j \leqq j' \leqq n} \frac{X_{[i]j} + X_{[i]j'}}{2}, \qquad i = 1, 2, \cdots, k .$$

Let $\hat{\theta}_{(1)} \leqq \hat{\theta}_{(2)} \leqq \cdots \leqq \hat{\theta}_{(k)}$ denote the ordered estimators. The selection procedures for problems (I)—(III) are now proposed as follows:

(I) Select the population corresponding to $\hat{\theta}_{(k)}$.

(II) Select the t populations associated with $(\hat{\theta}_{(k-t+1)}, \cdots, \hat{\theta}_{(k)})$.

(III) Select the t populations in problem (II) by classifying the population associated with $\hat{\theta}_{(k)}$ the "best", the one associated with $\hat{\theta}_{(k-1)}$ the "next best" and so on.

To study now the least favourable configuration of parameters, let $\boldsymbol{\theta} = (\theta_1, \cdots, \theta_k)$ and CS a subset of R^k where correct selection is made. A particular configuration of θ_i's is said to be least favourable if infimum (wrt $\boldsymbol{\theta} \in S$) of $P(\hat{\boldsymbol{\theta}} \in CS)$ is attained under that configuration ($\hat{\boldsymbol{\theta}} = (\hat{\theta}_1(\mathbf{X}_1), \cdots, \hat{\theta}_k(\mathbf{X}_k))$). From [10] the df of each θ_i is absolutely continuous wrt Lebesgue measure. Note also that the df of each $\hat{\theta}_i(\mathbf{X}_i)$ is stochastically non-decreasing in θ_i, i.e., for $\theta_i \leqq \theta_i'$ $(1 \leqq i \leqq k)$,

$$P\{\hat{\theta}_i(\mathbf{X}_i) \leqq x \mid \theta_i\} \geqq P\{\hat{\theta}_i(\mathbf{X}_i) \leqq x \mid \theta_i'\}, \qquad 1 \leqq i \leqq k . \tag{2.4}$$

This is because the lhs of (2.4) $\geqq P\{\hat{\theta}_i(\mathbf{X}_i) + (\theta_i' - \theta_i) \leqq x \mid \theta_i\}$

$$= P\{\hat{\theta}_i(\mathbf{X}_i + (\theta_i' - \theta_i)\mathbf{1}_n) \leqq x \mid \theta_i\} \quad \text{(using (2.3))}$$
$$= \text{rhs of (2.4)} .$$

Using a theorem of Barr and Rizvi (1966) it follows now that for problem (II)

$$P\{\hat{\boldsymbol{\theta}} \in CS \mid \boldsymbol{\theta}\} = P\{\max_{1 \leqq j \leqq k-t} \hat{\theta}_j(\mathbf{X}_j) < \min_{k-t+1 \leqq j \leqq k} \hat{\theta}_j(\mathbf{X}_j) \mid \boldsymbol{\theta}\} \tag{2.5}$$

is a non-increasing function of $\theta_1, \cdots, \theta_{k-t}$ and a non-decreasing function of $\theta_{k-t+1}, \cdots, \theta_k$. Thus if $\boldsymbol{\theta} \in S \subset R^k$, $P\{\hat{\boldsymbol{\theta}} \in CS \mid \boldsymbol{\theta}\}$ attains its infimum (wrt $\boldsymbol{\theta}$) when $\theta_1, \cdots, \theta_{k-t}$ attain their maximum possible values, while $\theta_{k-t+1}, \cdots, \theta_k$ attain their minimum possible values subject to $\boldsymbol{\theta} \in S$. Taking $S = \{\boldsymbol{\theta} \in R^k: \theta_{k-t+1} - \theta_{k-t} \geqq \Delta\}$ as in Bechhofer (1954), we are led to the following result for problem (II):

$$\begin{aligned} &\inf_{\boldsymbol{\theta} \in S} P(\hat{\boldsymbol{\theta}} \in CS \mid \boldsymbol{\theta}) \\ &\quad = P\{\hat{\boldsymbol{\theta}} \in CS \mid \theta_1 = \cdots = \theta_{k-t} = \theta_{k-t+1} - \Delta = \cdots = \theta_k - \Delta\} . \end{aligned} \tag{2.6}$$

For problem (I), take $S = R^k$; again a use of the Barr–Rizvi theorem gives

$$\begin{aligned} &\inf_{\boldsymbol{\theta} \in R^k} P(\hat{\boldsymbol{\theta}} \in CS \mid \boldsymbol{\theta}) \\ &\quad = \inf_{\boldsymbol{\theta} \in R^k} P(\text{select the population with parameter } \theta_k \mid \boldsymbol{\theta}) \\ &\quad = \inf_{\boldsymbol{\theta} \in R^k} P\{\max_{1 \leqq j \leqq k-1} \hat{\theta}_j(\mathbf{X}_j) < \hat{\theta}_k(\mathbf{X}_k) \mid \boldsymbol{\theta}\} \\ &\quad = P\{\max_{1 \leqq j \leqq k-1} \hat{\theta}_j(\mathbf{X}_j) < \hat{\theta}_k(\mathbf{X}_k) \mid \theta_1 = \cdots = \theta_{k-1} = \theta_k - \Delta\} . \end{aligned} \tag{2.7}$$

Finally for problem (III)

$$P(\hat{\boldsymbol{\theta}} \in CS \mid \boldsymbol{\theta}) = P\{\max_{1\leq j\leq k-t} \hat{\theta}_j(\mathbf{X}_j) < \hat{\theta}_{k-t+1}(\mathbf{X}_{k-t+1}) < \cdots < \hat{\theta}_k(\mathbf{X}_k) \mid \boldsymbol{\theta}\} . \tag{2.8}$$

Here S is taken to be (see Bechhofer (1954))

$$S = \{\boldsymbol{\theta} \in R^k : \theta_{i+1} - \theta_i \geqq \Delta, i = k-t, \cdots, k-1\} . \tag{2.9}$$

Let

$$a_j = \theta_j - \theta_{k-t+1} + \Delta , \qquad (j = 1, 2, \cdots, k-t)$$
$$a_j = \theta_j - \theta_{k-t+1} - (j - (k-t+1))\Delta , \qquad (j = k-t+1, \cdots, k) .$$

Define $\mathbf{Y}_j = \mathbf{X}_j - a_j \mathbf{1}_n$ $(j = 1, \cdots, k)$. Then, using (2.3), $\hat{\theta}_j(\mathbf{Y}_j) = \hat{\theta}_j(\mathbf{X}_j) - a_j$ $(j = 1, \cdots, k)$. Further, for $\boldsymbol{\theta} \in S$, $a_1 \leqq \cdots \leqq a_{k-t} \leqq 0 = a_{k-t+1} \leqq a_{k-t+2} \leqq \cdots \leqq a_k$. Using these, one gets

$$\begin{aligned} P\{\hat{\boldsymbol{\theta}} \in CS \mid \boldsymbol{\theta}\} &= P\{\max_{1\leq j\leq k-t} \hat{\theta}_j(\mathbf{X}_j) < \hat{\theta}_{k-t+1}(\mathbf{X}_{k-t+1}) < \cdots < \hat{\theta}_k(\mathbf{X}_k) \mid \boldsymbol{\theta}\} \\ &= P\{\max_{1\leq j\leq k-t} (\hat{\theta}_j(\mathbf{Y}_j) + a_j) < \hat{\theta}_{k-t+1}(\mathbf{Y}_{k-t+1}) \\ &\qquad + a_{k-t+1} < \cdots < \hat{\theta}_k(\mathbf{Y}_k) + a_k \mid \boldsymbol{\theta}\} \\ &\geqq P\{\max_{1\leq j\leq k-t} \hat{\theta}_j(\mathbf{Y}_j) < \hat{\theta}_{k-t+1}(\mathbf{Y}_{k-t+1}) < \cdots < \hat{\theta}_k(\mathbf{Y}_k) \mid \boldsymbol{\theta}\} . \end{aligned} \tag{2.10}$$

This leads to the result

$$\begin{aligned} \inf_{\boldsymbol{\theta} \in S} P(\hat{\boldsymbol{\theta}} \in CS \mid \boldsymbol{\theta}) = P\{&\max_{1\leq j\leq k-t} \hat{\theta}_j(\mathbf{X}_j) < \hat{\theta}_{k-t+1}(\mathbf{X}_{k-t+1}) < \cdots \\ &< \hat{\theta}_k(\mathbf{X}_k) \mid \theta_1 = \cdots = \theta_{k-t} = \theta_{k-t+1} - \Delta \\ &= \cdots \theta_k - t\Delta\} , \end{aligned} \tag{2.11}$$

as the rhs of (2.11) = the rhs of (2.10).

As anticipated, the least favourable configurations are the same as those of Bechhofer (1954) who uses the estimators $\bar{X}_i = (1/n) \sum_{i=1}^{n} X_{ij}$ $(i = 1, \cdots, k)$ instead of $\hat{\theta}_i(\mathbf{X}_i)$ $(1 \leqq i \leqq k)$. In the following section, we compare the asymptotic performances of the two procedures.

3. ARE. Following Lehmann (1963), we define the ARE of our selection procedures wrt other procedures as the limiting ratio of the reciprocals of the corresponding sample sizes required to achieve the same minimum probability of correct selection. Consider the sequence of parameter points for which

$$\Delta = \Delta^{(n)} = n^{-\frac{1}{2}}C + o(n^{-\frac{1}{2}}) , \qquad \text{where } C \text{ is some constant.} \tag{3.1}$$

If we now set for all these problems

$$\lim_{n\to\infty} \inf_{\boldsymbol{\theta} \in S} P\{\hat{\boldsymbol{\theta}} \in CS \mid \boldsymbol{\theta}\} = \gamma , \tag{3.2}$$

we find for the two sets of procedures the limiting sample sizes subject to (3.1) and (3.2). A general way of achieving this would be as follows.

It is known (see, e.g., [10] or [14]) that if θ_i's are true values of the parameters, under some regularity assumptions, $n^{\frac{1}{2}}(\hat{\theta}_i(\mathbf{X}_i) - \hat{\theta}_i)B(F)/A$ is asymptotically (as

$n \to \infty$) $N(0, 1)$, $i = 1, 2, \cdots, k$, where

$$A^2 = \tfrac{1}{4} \int_0^1 J^2(u)\, du\,, \qquad B(F) = \int_0^\infty \frac{d}{dx} J(2F(x) - 1)\, dF(x)\,.$$

These statistics are also mutually independent. Now, for all the problems (see (2.5), (2.7) and (2.8)) CS satisfies

$$(3.3) \qquad (y_1, \cdots, y_k) \in CS \Rightarrow (b(y_1 + a), \cdots, b(y_k + a)) \in CS$$

where a and b (> 0) are constants (CS being different under different situations). Using (3.3), one obtains

$$(3.4) \qquad P(\hat{\boldsymbol{\theta}} \in CS \mid \boldsymbol{\theta}) = P(n^{\frac{1}{2}}\hat{\boldsymbol{\theta}}B(F)/A \in CS \mid \boldsymbol{\theta}) = P(\mathbf{U}_n \in CS \mid \boldsymbol{\theta})\,,$$

where $\mathbf{U}_n = (U_{n1}, \cdots. U_{nk})$, $U_{ni} = A^{-1}n^{\frac{1}{2}}(\hat{\theta}_i - \theta_i)B(F) - A^{-1}n^{\frac{1}{2}}(\theta_k - \theta_i)B(F)$, ($i = 1, 2, \cdots, k$). Under the least favourable configurations of the parameters θ_i, for all the problems, by virtue of (3.1), $A^{-1}n^{\frac{1}{2}}(\theta_k - \theta_i)B(F) \to (\delta_k - \delta_i)B(F)/A$ ($i = 1, 2, \cdots, k$) as $n \to \infty$, where δ_i's are some constants. Invoking now the asymptotic normality of $\hat{\theta}_i$'s, one gets

$$(3.5) \qquad \lim_{n\to\infty} \inf_{\boldsymbol{\theta} \in S} P(\hat{\boldsymbol{\theta}} \in CS \mid \boldsymbol{\theta}) = P(\mathbf{Z} \in CS)$$

where $\mathbf{Z} = (Z_1, \cdots, Z_k)$, Z_i's being independent $N((\delta_k - \delta_i)B(F)/A, 1)$ variables, $1 \leqq i \leqq k$.

We know also that if the X_{ij}'s are homoscedastic with nonzero and finite variance σ^2, and $m \equiv m(n)$ is the sample size used ($m \to \infty$ as $n \to \infty$ and $\lim_{n\to\infty} m/n = e$ exists), then $m^{\frac{1}{2}}(\bar{X}_i - \theta_i)$ are independent asymptotically $N(0, \sigma^2)$ variables. Again, if (4.1) holds, $m^{\frac{1}{2}}(\theta_k - \theta_i)/\sigma \to (\delta_k - \delta_i)e^{\frac{1}{2}}/\sigma$ as $n \to \infty$, ($1 \leqq i \leqq k$). Also, if $\bar{\mathbf{X}} = (\bar{X}_1, \cdots, \bar{X}_k)$,

$$(3.6) \qquad \lim_{n\to\infty} \inf_{\boldsymbol{\theta} \in S} P(\bar{\mathbf{X}} \in CS \mid \boldsymbol{\theta}) = P(\mathbf{Y} \in CS)\,,$$

where $\mathbf{Y} = (Y_1, \cdots, Y_k)$, Y_i's independent $N((\delta_k - \delta_i)e^{\frac{1}{2}}/\sigma, 1)$ variables. Then if (4.2) holds, the lhs of (3.5) $=$ the lhs of (3.6) $= \gamma$, i.e., $P(\mathbf{Z} \in CS) = P(\mathbf{Y} \in CS) = \gamma$. Then we must have $B(F)/A = e^{\frac{1}{2}}/\sigma$, i.e., the efficiency of our procedures wrt Bechhofer procedures in all the three cases $= e = \sigma^2B^2(F)/A^2$, the Pitman efficiency of Chernoff–Savage test wrt Student's t-test.

Suppose F is absolutely continuous wrt Lebesgue measure with a density f. If now $J(u) = u$ (Wilcoxon case), then $e = 12\sigma^2[\int f^2(x)\, dx]^2$. In this case, it is proved by Hodges and Lehmann [8] that $e \geqq .864$ for all F, the lower bound being attained for some distribution with parabolic density. Also, $e = 3/\pi \approx .955$, when $F(x) = \Phi(x/\sigma)$, $\Phi(t)$ standard normal; the efficiency exceeds 1 for double exponential, Cauchy and logistic df. If $J(u) = \chi_1^{-1}(u)$ (normal score case), then

$$e = \sigma^2 \left[\int \frac{f^2(x)\, dx}{\phi(\Phi^{-1}(F(x)))} \right]^2,$$

where $\Phi(t) = \Phi'(t)$. It is known (see, e.g., Puri and Sen (1971, pages 117–118)) that in this case $e \geqq 1$ for all F, equality holding if and only if $F(x) = \Phi(x/\sigma)$. For details, one may refer to [8] and [9].

Finally, one may remark that the ARE expressions obtained by Randles (1970) are the same as ours. Besides, he does not need the symmetry of F. However Randles' procedure seems to be computationally more difficult than ours even for moderately large k and n. This is because Randles needs to compute $\binom{k}{2}$ two-sample H–L estimators each based on the ranks of $2n$ observations, whereas we need to compute k one-sample H–L estimators, each based on the ranks of n observations. One may also add that the symmetry assumption is valid for many well-known distributions like Normal, logistic, Cauchy or double-exponential. In the case $e \geqq 1$ (normal scores case), our procedure is usually asymptotically more efficient than the Bechhofer–Sobel procedure for which the ARE's (wrt means procedure) as computed by Dudewicz (1971) are < 1.

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