Note that the factor in brackets is the same as the last term on the right side of (1) with n replaced by $n - 1$ and $x + 1$ replaced by x. Hence for large x and N,

$$w(n, 1, N, x) \approx \frac{n}{N+n}\left(1 - \frac{x}{N}\right)^{n-1}$$

$$\cdot \left\{ \frac{\left(1 + \dfrac{n}{N-x}\right)^{N-x+n}}{\left(1 + \dfrac{n-1}{N}\right)^{N+n-1}} \right\} \sqrt{1 - \frac{n-1}{N+n-1}} \sqrt{1 + \frac{n-1}{N-x}}.$$

By the same limiting procedure as before,

$$\lim_{x=kN\to\infty} w(n, 1, N, kN) = \frac{n}{N}(1-k)^{n-1}. \tag{5}$$

In any small interval dk, there are Ndk possible values that x can assume; hence the probability that k lies in the interval dk is

$$p(k)\, dk = \frac{n}{N}(1-k)^{n-1}(N\, dk). \tag{6}$$

Therefore, $p(k) = n(1-k)^{n-1}$. This is exactly the result given by equation (4), but obtained in a somewhat different way.

From the symmetry of the problem, $\lim_{N\to\infty} W(n, 1, N, kN)$ is also the probability that in a large future sample at most a fraction k of the observations will be *less* than the *smallest* observation in the original trial sample of n units. Hence, a life-test of n units may be discontinued as soon as any unit fails and equation (3) will give the probability that in the future at most $100k\%$ of the units will fail in a time shorter than the length of the test. The graphs show W as a function of k for various values of n.

REFERENCE

[1] E. J. Gumbel and H. Von Schelling, "The distribution of the number of exceedances," *Annals of Math. Stat.*, Vol. 21 (1950), pp. 247–262.

CORRECTION TO "THE SAMPLING DISTRIBUTION OF THE RATIO OF TWO RANGES FROM INDEPENDENT SAMPLES"

By Richard F. Link

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In the note mentioned in the title (*Annals of Math. Stat.*, Vol. 21 (1950), pp. 112–116) the distribution given for the above mentioned ratio when the sample values are drawn from a rectangular distribution is correct only when $R \leqq 1$. This is pointed out in an article by P. R. Rider ("The distribution of the quotient of ranges in samples from a rectangular population," *Jour. Am. Stat. Assn.*, Vol. 46 (1951), pp. 502–507), who also gives the correct density of the ratio for $R \geqq 1$. The correct cumulative distribution for $R \geqq 1$ is

$$1 - R^{-n_2}\left\{\frac{Rn_2 n_1(n_1 - 1)}{(n_1 + n_2 - 1)(n_1 + n_2 - 2)} - \frac{n_1(n_1 - 1)(n_2 - 1)}{(n_1 + n_2)(n_1 + n_2 - 1)}\right\}.$$

ABSTRACTS OF PAPERS

(Abstracts of papers presented at the Blacksburg meeting of the Institute, March 19–21, 1952)

1. On the Approximation of Sampling Distributions by Punch Card Methods.
CARL F. KOSSACK AND LESTER L. HELMS, Purdue University.

This paper presents a procedure for obtaining empirical distributions, by punch card methods, of statistics for which the exact distribution or a usable approximation has not been found. The mechanization of random sampling of a univariate population has been described and extended to random sampling of a correlated multivariate population whose covariance matrix is given. This procedure has been applied to Wald's classification statistic in the univariate case, and the results noted.

2. Resolvable Incomplete Block Designs with Two Replications.
R. C. BOSE AND K. R. NAIR, University of North Carolina.

Incomplete block designs in which the blocks can be grouped in such a way that each group contains a complete replication may be called resolvable designs. They are useful from the point of view of recovery of inter-block information. It is therefore important to investigate resolvable designs involving a few replications. In this paper we consider a class of resolvable designs with two replications, which contains as a special case the well known square and rectangular lattices with two replications. Given a symmetrical balanced incomplete block design with u treatments, and r replications in which each pair occurs λ times, we can use the incidence matrix (n_{ij}) of this design to form a design of one class in the following way. Take a $u \times u$ square scheme, and in the cell (i, j) put x new treatments when $n_{ij} = 1$, and y new treatments when $n_{ij} = 0$. The total number of treatments obtained in this way is $v = u[rx + (u - r)y]$. The design is now constructed by taking the rows of the scheme for the blocks of the first replication, and the columns of the scheme for the blocks of the second replication. It has been shown that both the intra- and inter-block analysis can be carried out in a simple manner. The necessary formulae have been given, and the computational procedure illustrated by working out a numerical example.

3. Rank Analysis of Incomplete Block Designs. I. The Method of Paired Comparisons.
R. A. BRADLEY AND M. E. TERRY, Virginia Polytechnic Institute.

True preferences or ratings $\pi_{1u}, \cdots, \pi_{tu}$, $\Sigma_{i=1}^{t} \pi_{iu} = 1$, are assumed to exist for t treatments in the uth of g groups of experimental data in an experiment involving paired comparisons. For the uth group, the probability that treatment i is "better" than treatment j when they appear in a pair is postulated to be $\pi_{iu}/(\pi_{iu} + \pi_{ju})$.

Three tests of hypotheses are available and estimates of the treatment ratings may be