ABSTRACTS OF PAPERS

(Additional abstracts of papers presented at the Detroit meeting of the Institute, September 7–10, 1956)

1. **Further Applications of Information Theory to Multivariate Analysis and Statistical Inference, (Preliminary Report),** Morton Kupperman, The George Washington University, (By Title).

A generalized statistic based on the Kullback-Leibler measure of information is defined as

$$2nI^* = 2n \int f(x, \theta^*) \log \frac{f(x, \theta^*)}{f(x, \theta_0)} \, d\lambda(x),$$

where the vector θ^* of h components is any consistent, asymptotically normal, efficient estimator and θ_0 is specified. $2nI^*$ is used to test the hypothesis H_0: The sample is from a specified multivariate multiparameter population (not necessarily normal). The asymptotic distribution of $2nI^*$ under H_0 is chi-square with h d.f. I^* is modified to test the hypothesis H_0': r ($\geqq 2$) samples are from the same general multivariate population, parameters not specified; its asymptotic distribution under H_0' is chi-square with $(r - 1)h$ d.f. Corresponding results are obtained for divergence-statistics based on the divergence $J(1, 2)$. Large-sample distributions of I^* and J^* under alternative hypotheses are approximated by noncentral chi-square distributions.

For any multivariate multiparameter distribution admitting sufficient statistics, $-\log \lambda \equiv \hat{I}$, where λ is the likelihood-ratio criterion and $\hat{I}$ uses maximum-likelihood estimators.

Information theory is applied to hypothesis testing, Pearson's chi-square test of goodness of fit, and the derivation of exact sampling distributions of sufficient statistics. It is shown that the set of sufficient estimators of population parameters appearing explicitly in any Koopman-Pitman distribution (admitting sufficient statistics) are distributed jointly in a Koopman-Pitman distribution. (Received July 19, 1956.)

2. **Generalization of Thompson's Distribution,** André G. Laurent, Michigan State University.

Generalization of Thompson's Distribution. 1.1.) Let $X = (X_1, \cdots, X_N)$ be $N(m, \sigma)$ distributed, $\bar{X} = \Sigma X_i/N$, $s^2 = \Sigma(X_i - \bar{X})^2/N$ and $t = (X_i - \bar{X})/s$. It is well known (W. R. Thompson. 1935) that $t^2/(N - 1)$ is Incomplete Beta distributed and that this distribution is also the conditional distribution of any X_i, given $\bar{X}$ and s. Three generalizations of that result are presented. 1.2.) If $\xi = (\xi_1, \cdots, \xi_k)'$ is a subsample from X, the p.d.f. of $t = (\xi - \bar{X})/s$ is $[1 - t(\vartheta_{ij}/N + 1/N(N - k))t']^{(N-k-3)/2} \Gamma[(N - 1)/2]/\pi^{k/2}\Gamma[(N - k - 1)/2] N^{(k-1)/2}(N - k)^{1/2}$. This provides also the conditional distribution of ξ, given $\bar{X}$ and s. 2.1.) If a vector $X = (X', \cdots, X^p)$ is $N(m, \Sigma)$ distributed and if $\xi = (\xi', \cdots, \xi^p)$ is any observation from a sample $(X) = (X_1, \cdots, X_N)$ 'with mean m'' and covariance matrix S, the cond. p.d.f. of ξ, given m' and S, is $[1 - (\xi - m')S^{-1}(\xi - m')']^{(N-p-3)/2} \mid S \mid^{-1/2}[(N - 1)\pi]^{-1/2} \Gamma[(N - 1)/2]/\Gamma[(N - 1 - p)/2]$ 2.2.) The latter result is generalized to the case where a subsample $(\xi) = (\xi_1, \cdots, \xi_k)'$ is drawn from (X). The conditional distribution of (ξ), given m' and S, is a generalized multivariate Incomplete Beta distribution. These results make it possible to obtain and study the *U.M.V.* unbiased estimates of functions of the populations parameters, with obvious applications in the fields of S.Q.C., bombing problems, etc., and tolerance regions investigations. (Received July 23, 1956.)

3. A New Class of PBIB Designs, Dale M. Mesner, Purdue University and Michigan State University.

In partially balanced incomplete block (PBIB) designs of Latin square type with g constraints and n^2 treatments, algebraic expressions in the integers n and g give values of the parameters n_i and p^i_{jk} if n and g are positive. Some negative values of n and g lead to negative parameter values which cannot occur in a design, but others give non-negative values which differ from those for any known designs and suggest the existence of a new type of design with n^2 treatments. Five families of the new designs, referred to here as the "negative Latin square" type, are constructed, based on association schemes with 16, 64, 81 (two schemes) and 100 treatments. Necessary and sufficient conditions are first given that associate classes in an association scheme may be combined to give a scheme with fewer classes. For n^2 equal to 16, 64, or 81, the new schemes are constructed by combining classes in association schemes having $n - 1$ classes of $n + 1$ treatments each, constructed from the field $GF(n^2)$ by a method similar to that of Sprott [*Can. J. Math.*, Vol. 7 (1955), 369–381]. The scheme with 100 treatments is found by an enumeration method. (Received July 23, 1956.)

4. Some Results for Inverting Patterned Matrices, A. E. Sarhan, Egyptian Medical Research Laboratories, Cairo, Egypt, and B. G. Greenberg, University of North Carolina, (By Title).

Employing the results given in a paper by Ukita "On the Characterization of Diagonal Matrix of 2-Type and its Application to Order Statistics," generalizations can be made for various cases given in the paper by Roy and Sarhan "On Inverting a Class of Patterned Matrices," *Biometrika*, June 1956. In addition, generalizations for getting the inverse of complex matrices which can be partitioned into submatrices of the form $[D\alpha + \lambda J]$ are obtained. Matrices of this nature are frequently encountered in varied statistical applications, involving least squares, experimental design and order statistics. (Received July 23, 1956.)

5. On the Solution of the Functional Equation of Farrell's Market, A. Charnes and O. P. Aggarwal, Purdue University.

A. Charnes and M. Farrell in June 1953 derived a la R. Bellman a functional equation describing the optimal pricing strategy of a seller in a recurring market of Markovian reaction characteristics of "kinky oligopoly" type suggested by Farrell (*Econometrica* Vol. 22, No. 3, July 1954) in his approximate im kleinen analysis of an econometric controversy on optimality of holding price constant. They established (unpublished) existence and uniqueness of continuous solutions of at most exponential growth for this functional equation, a type still not subsumed in any yet treated (cf. S. Karlin *Naval Research Logistics Quarterly* Vol. 2, No. 4, Dec. 1955).

This paper develops explicitly the exact solution for all relevant parameter ranges. Hence (1) a new method of approximately solving "extremal" functional equations by employing piece-wise quadratic "forcing" terms is suggested and partially implemented, (2) an im grossen resolution of the econometric controversy is at hand. (Received August 29, 1956.)

6. Randomization and Experimentation, W. J. Youden, National Bureau of Standards.

Randomization, often specified as an indispensable requirement in experimental design, is required only when the order or position of the experimental unit influences the per-

formance of the unit. Randomization, when required, may give an arrangement that is obviously undesirable and one that may doom the particular experimental program. A system of constrained randomization is proposed that eliminates the undesirable arrangements without sacrificing the customary gains achieved by randomization. **(Received September 14, 1956.)**

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