ESTIMATION OF LOCATION AND SCALE PARAMETERS BY ORDER STATISTICS FROM SINGLY AND DOUBLY CENSORED SAMPLES[1] PART II. TABLES FOR THE NORMAL DISTRIBUTION FOR SAMPLES OF SIZE $11 \leqq n \leqq 15$

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1. Introduction. In a previous paper [2], estimation of the mean and standard deviation from singly and doubly censored samples drawn from the normal distribution were considered for samples $n \leqq 10$. The generalization of an alternative estimate for these parameters was also obtained.

In the present work, all calculations and tables obtained for the corresponding items in Part I are extended up to $n \leqq 15$.

The method is to obtain the best linear unbiased estimates of the mean and standard deviation by taking the best linear combination of the ordered observations. The variances and covariances of the order statistics for samples $11 \leqq n \leqq 15$ which are required in carrying out these calculations are obtained from Table I in [2].

Further investigation of the efficiency of the alternative estimate under varied degrees of censoring shows that the alternative estimate proposed by Gupta [1] is better than previously supposed when judged by doubly censored samples rather than singly censored samples alone.

2. Tables. Table I gives the coefficients for the best linear estimates of the mean and standard deviation for the normal population from samples of size $11 \leqq n \leqq 15$ undergoing all possible conditions of Type II censoring. Estimation from complete or singly censored samples are simply special cases and are given in the table

$$(r_1 = r_2 = 0, \text{ and } r_1 \text{ or } r_2 = 0).$$

The best linear estimates of the mean and standard deviations are obtained by using

$$\mu^* = \sum_{i=r_1+1}^{n-r_2} a_{1i} y_{(i)},$$

$$\sigma^* = \sum_{i=r_1+1}^{n-r_2} a_{2i} y_{(i)},$$

where

$$y_{(1)} < y_{(2)} < \cdots, < y_{(n)}.$$

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n = 11

Table I. The coefficients of the most efficient linear systematic statistics of the mean and standard deviation in censored samples of sizes 11 ≤ n ≤ 15 from normal populations

$r_1 = 0$, r_2		$y_{(1)}$	$y_{(2)}$	$y_{(3)}$	$y_{(4)}$	$y_{(5)}$	$y_{(6)}$	$y_{(7)}$	$y_{(8)}$	$y_{(9)}$	$y_{(10)}$	$y_{(11)}$		$r_1 = 1$, r_2
0	μ^*	.0909	.0909	.0909	.0909	.0909	.0909	.0909	.0909	.0909	.0909	.0909		
	σ^*	-.1883	-.1115	-.0760	-.0481	-.0234	.0000	.0234	.0481	.0760	.1115	.1883		
1	μ^*	.0781	.0841	.0869	.0891	.0910	.0928	.0945	.0963	.0982	.1891			
	σ^*	-.2149	-.1256	-.0843	-.0519	-.0233	.0038	.0309	.0593	.0911	.3149			
2	μ^*	.0592	.0744	.0814	.0869	.0917	.0962	.1005	.1049	.3047	3.1882	-2.1882	μ^*	8
	σ^*	-.2463	-.1417	-.0934	-.0555	-.0220	.0095	.0409	.0736	.4349	3.0023	-3.0023	σ^*	
3	μ^*	.0320	.0609	.0741	.0845	.0935	.1020	.1101	.4430	1.8131	-.0776	-.7355	μ^*	7
	σ^*	-.2852	-.1610	-.1038	-.0589	-.0194	.0178	.0545	.5562	1.8473	-.3251	-1.5222	σ^*	
4	μ^*	-.0082	.0415	.0642	.0820	.0974	.1116	.6116	1.2227	.0631	.0019	-.2877	μ^*	6
	σ^*	-.3357	-.1854	-.1163	-.0621	-.0146	.0299	.6842	1.3689	-.1119	-.2362	-1.0208	σ^*	
5	μ^*	-.0698	.0128	.0504	.0797	.1049	.8220	.8646	.1026	.0738	.0407	-.0818	μ^*	5
	σ^*	-.4045	-.2175	-.1317	-.0647	-.0061	.8246	1.0717	-.0195	-.0985	-.1881	-.7656	σ^*	
6	μ^*	-.1702	-.0323	.0303	.0786	1.0937	.6157	.1116	.0971	.0813	.0632	.0311	μ^*	4
	σ^*	-.5053	-.2627	-.1519	-.0657	.9857	.8517	.0301	-.0272	-.0882	-.1569	-.6095	σ^*	
7	μ^*	-.3516	-.1104	-.0016	1.4636	.4301	.1093	.1022	.0948	.0868	.0776	.0992	μ^*	3
	σ^*	-.6687	-.3331	-.1807	1.1825	.6715	.0611	.0154	-.0307	-.0796	-.1345	-.5032	σ^*	
8	μ^*	-.7445	-.2712	2.0157	.2859	.1025	.0998	.0970	.0941	.0909	.0872	.1426	μ^*	2
	σ^*	-.9862	-.4630	1.4492	.5130	.0836	.0440	.0061	-.0320	-.0722	-.1173	-.4252	σ^*	
9	μ^*	-2.0245	3.0245	.1710	.0938	.0940	.0941	.0942	.0941	.0940	.0938	.1710	μ^*	1
	σ^*	-1.9065	1.9065	.3647	.1033	.0656	.0320	.0000	-.0320	-.0656	-.1033	-.3647	σ^*	
				$y_{(10)}$	$y_{(9)}$	$y_{(8)}$	$y_{(7)}$	$y_{(6)}$	$y_{(5)}$	$y_{(4)}$	$y_{(3)}$	$y_{(2)}$		

$r_1 = 2$, r_2		$y_{(3)}$	$y_{(4)}$	$y_{(5)}$	$y_{(6)}$	$y_{(7)}$	$y_{(8)}$	$y_{(9)}$		$r_1 = 3$, r_2
2	μ^*	.2529	.0986	.0990	.0990	.0990	.0986	.2529		
	σ^*	-.6113	-.0952	-.0465	.0000	.0465	.0952	.6113		
3	μ^*	.1978	.0927	.0983	.1032	.1079	.4001			
	σ^*	-.7445	-.1095	-.0482	.0102	.0682	.8239			
4	μ^*	.1031	.0834	.0981	.1117	.6036	1.9486	-.9486	μ^*	6
	σ^*	-.9395	-.1287	-.0484	.0276	1.0890	4.2179	-4.2179	σ^*	
5	μ^*	-.0736	.0675	.0999	.9062	.9504	.0966	-.0470	μ^*	5
	σ^*	-1.2584	-.1573	-.0453	1.4610	2.2169	-.1019	-2.1150	σ^*	
6	μ^*	-.4631	.0359	1.4273	.5616	.1125	.1024	.2235	μ^*	4
	σ^*	-1.8863	-.2083	2.0946	1.4723	.0202	-.0867	-1.4057	σ^*	
7	μ^*	-1.7312	2.7312	.3413	.1057	.1059	.1057	.3413	μ^*	3
	σ^*	-3.7473	3.7473	1.0451	.0764	.0000	-.0764	-1.0451	σ^*	
				$y_{(8)}$	$y_{(7)}$	$y_{(6)}$	$y_{(5)}$	$y_{(4)}$		

$r_1 = 4$, r_2		$y_{(5)}$	$y_{(6)}$	$y_{(7)}$
4	μ^*	.4421	.1157	.4421
	σ^*	-2.2233	.0000	2.2233
5	μ^*	.0000	1.0000	
	σ^*	-4.4466	4.4466	

n = 12

Table I. (Continued)

$r_1 = 0$, r_2		$y_{(1)}$	$y_{(2)}$	$y_{(3)}$	$y_{(4)}$	$y_{(5)}$	$y_{(6)}$	$y_{(7)}$	$y_{(8)}$	$y_{(9)}$	$y_{(10)}$	$y_{(11)}$	$y_{(12)}$		$r_1 = 1$, r_2
0	μ^*	.0833	.0833	.0833	.0833	.0833	.0833	.0833	.0833	.0833	.0833	.0833	.0833		
	σ^*	-.1748	-.1061	-.0749	-.0506	-.0294	-.0097	.0097	.0294	.0506	.0749	.1061	.1748		
1	μ^*	.0726	.0775	.0796	.0813	.0828	.0842	.0855	.0868	.0882	.0896	.1719			
	σ^*	-.1972	-.1185	-.0827	-.0548	-.0305	-.0079	.0142	.0367	.0608	.0881	.2919			
2	μ^*	.0574	.0693	.0747	.0789	.0825	.0859	.0891	.0923	.0956	.2745	3.4554	-2.4554	μ^*	9
	σ^*	-.2232	-.1324	-.0911	-.0590	-.0310	-.0050	.0203	.0461	.0733	.4020	3.0970	-3.0970	σ^*	
3	μ^*	.0360	.0581	.0682	.0759	.0827	.0888	.0948	.1006	.3950	1.9966	-.1242	-.8725	μ^*	8
	σ^*	-.2545	-.1487	-.1007	-.0633	-.0308	-.0007	.0286	.0582	.5119	1.9252	-.3546	-1.5706	σ^*	
4	μ^*	.0057	.0428	.0595	.0724	.0836	.0938	.1036	.5386	1.3737	-.0373	-.0298	-.3812	μ^*	7
	σ^*	-.2937	-.1686	-.1119	-.0678	-.0296	.0058	.0400	.6259	1.4442	-.1345	-.2554	-1.0542	σ^*	
5	μ^*	-.0382	.0210	.0477	.0684	.0861	.1022	.7128	.9961	.0870	.0542	.0162	-.1536	μ^*	6
	σ^*	-.3448	-.1939	-.1255	-.0726	-.0267	.0155	.7479	1.1482	-.0390	-.1151	-.2024	-.7918	σ^*	
6	μ^*	-.1048	-.0109	.0313	.0637	.0915	.9292	.7331	.1018	.0844	.0652	.0430	-.0275	μ^*	5
	σ^*	-.4146	-.2274	-.1428	-.0774	-.0210	.8833	.9313	.0121	-.0423	-.1011	-.1684	-.6316	σ^*	
7	μ^*	-.2125	-.0609	.0070	.0589	1.2075	.5361	.1035	.0939	.0839	.0729	.0601	.0496	μ^*	4
	σ^*	-.5171	-.2749	-.1659	-.0820	1.0399	.7559	.0435	.0009	-.0430	-.0903	-.1442	-.5228	σ^*	
8	μ^*	-.4059	-.1472	-.0321	1.5852	.3819	.0996	.0946	.0896	.0843	.0785	.0717	.0998	μ^*	3
	σ^*	-.6836	-.3493	-.1996	1.2324	.6042	.0654	.0295	-.0059	-.0423	-.0814	-.1259	-.4435	σ^*	
9	μ^*	-.8225	-.3249	2.1474	.2580	.0932	.0912	.0892	.0872	.0851	.0827	.0798	.1336	μ^*	2
	σ^*	-1.0075	-.4874	1.4948	.4660	.0827	.0503	.0198	-.0102	-.0410	-.0738	-.1112	-.3825	σ^*	
10	μ^*	-2.1728	3.1728	.1566	.0856	.0858	.0859	.0860	.0860	.0859	.0858	.0856	.1566	μ^*	1
	σ^*	-1.9474	1.9474	.3334	.0989	.0671	.0391	.0128	-.0128	-.0391	-.0671	-.0989	-.3334	σ^*	
				$y_{(11)}$	$y_{(10)}$	$y_{(9)}$	$y_{(8)}$	$y_{(7)}$	$y_{(6)}$	$y_{(5)}$	$y_{(4)}$	$y_{(3)}$	$y_{(2)}$		

$r_1 = 2$, r_2		$y_{(3)}$	$y_{(4)}$	$y_{(5)}$	$y_{(6)}$	$y_{(7)}$	$y_{(8)}$	$y_{(9)}$	$y_{(10)}$		$r_1 = 3$, r_2
2	μ^*	.2309	.0895	.0898	.0899	.0899	.0898	.0895	.2309		
	σ^*	-.5437	-.0933	-.0545	-.0179	.0179	.0545	.0933	.5437		
3	μ^*	.1883	.0840	.0880	.0914	.0947	.0978	.3558			
	σ^*	-.6440	-.1061	-.0587	-.0142	.0292	.0733	.7204			
4	μ^*	.1206	.0759	.0858	.0947	.1032	.5197	2.3903	-1.3903	μ^*	7
	σ^*	-.7811	-.1225	-.0631	-.0076	.0465	.9278	4.4525	-4.4525	σ^*	
5	μ^*	.0076	.0633	.0832	.1015	.7444	1.2001	.0699	-.2700	μ^*	6
	σ^*	-.9829	-.1451	-.0676	.0045	1.1911	2.3763	-.1421	-2.2342	σ^*	
6	μ^*	-.1993	.0417	.0806	1.0771	.7412	.1016	.0836	.0736	μ^*	5
	σ^*	-1.3139	-.1796	-.0719	1.5654	1.6109	-.0111	-.1124	-1.4874	σ^*	
7	μ^*	-.6489	-.0018	1.6507	.4812	.1029	.0970	.0907	.2282	μ^*	4
	σ^*	-1.9673	-.2429	2.2102	1.1775	.0488	-.0223	-.0950	-1.1090	σ^*	
8	μ^*	-2.0971	3.0971	.3097	.0950	.0953	.0953	.0950	.3097	μ^*	3
	σ^*	-3.9063	3.9063	.8780	.0828	.0273	-.0273	-.0828	-.8780	σ^*	
				$y_{(9)}$	$y_{(8)}$	$y_{(7)}$	$y_{(6)}$	$y_{(5)}$	$y_{(4)}$		

$r_1 = 4$, r_2		$y_{(5)}$	$y_{(6)}$	$y_{(7)}$	$y_{(8)}$		$r_1 = 5$, r_2
4	μ^*	.3973	.1027	.1027	.3973		
	σ^*	-1.5849	-.0497	.0497	1.5849		
5	μ^*	.1962	.1034	.7004			
	σ^*	-2.3874	-.0468	2.4342			
6	μ^*	-.4893	1.4893	.5000	.5000	μ^*	5
	σ^*	-4.7696	4.7696	4.8738	-4.8738	σ^*	
				$y_{(7)}$	$y_{(6)}$		

n = 13

Table I. (Continued)

$r_1 = 0$, r_2		$y_{(1)}$	$y_{(2)}$	$y_{(3)}$	$y_{(4)}$	$y_{(5)}$	$y_{(6)}$	$y_{(7)}$	$y_{(8)}$	$y_{(9)}$	$y_{(10)}$	$y_{(11)}$	$y_{(12)}$	$y_{(13)}$		$r_1 = 1$, r_2
0	μ^*	.0769	.0769	.0769	.0769	.0769	.0769	.0769	.0769	.0769	.0769	.0769	.0769	.0769		
	σ^*	-.1632	-.1013	-.0735	-.0520	-.0335	-.0164	.0000	.0164	.0335	.0520	.0735	.1013	.1632		
1	μ^*	.0679	.0718	.0735	.0749	.0761	.0771	.0781	.0792	.0802	.0813	.0824	.1576			
	σ^*	-.1824	-.1122	-.0806	-.0563	-.0353	-.0160	.0026	.0212	.0404	.0612	.0850	.2724			
2	μ^*	.0552	.0648	.0691	.0724	.0752	.0778	.0803	.0827	.0852	.0877	.2497	3.7044	-2.7044	μ^*	10
	σ^*	-.2043	-.1243	-.0884	-.0607	-.0368	-.0148	.0063	.0273	.0490	.0723	.3743	3.1823	-3.1823	σ^*	
3	μ^*	.0380	.0555	.0633	.0693	.0745	.0792	.0836	.0880	.0924	.3564	2.1686	-.1687	-.9999	μ^*	9
	σ^*	-.2301	-.1382	-.0970	-.0653	-.0379	-.0128	.0113	.0352	.0598	.4750	1.9951	-.3810	-1.6141	σ^*	
4	μ^*	.0144	.0430	.0557	.0655	.0739	.0816	.0888	.0958	.4813	1.5160	.0123	-.0600	-.4683	μ^*	8
	σ^*	-.2616	-.1549	-.1071	-.0703	-.0386	-.0095	.0182	.0456	.5781	1.5114	-.1546	-.2727	-1.0841	σ^*	
5	μ^*	-.0185	.0259	.0457	.0610	.0740	.0857	.0968	.6294	1.1209	.0712	.0352	-.0070	-.2203	μ^*	7
	σ^*	-.3011	-.1754	-.1191	-.0758	-.0386	-.0046	.0278	.6867	1.2161	-.0561	-.1298	-.2151	-.8151	σ^*	
6	μ^*	-.0659	.0020	.0322	.0553	.0750	.0928	.8085	.8453	.0916	.0716	.0496	.0239	-.0820	μ^*	6
	σ^*	-.3528	-.2015	-.1339	-.0819	-.0374	.0032	.8042	1.0015	-.0034	-.0556	-.1127	-.1786	-.6512	σ^*	
7	μ^*	-.1371	-.0330	.0132	.0484	.0784	1.0301	.6382	.0969	.0853	.0730	.0595	.0437	.0035	μ^*	5
	σ^*	-.4236	-.2363	-.1528	-.0888	-.0341	.9355	.8296	.0287	-.0116	-.0538	-.0998	-.1528	-.5401	σ^*	
8	μ^*	-.2516	-.0876	-.0151	.0400	1.3143	.4755	.0955	.0889	.0820	.0746	.0666	.0572	.0597	μ^*	4
	σ^*	-.5276	-.2859	-.1785	-.0964	1.0884	.6826	.0505	.0172	-.0164	-.0515	-.0896	-.1335	-.4594	σ^*	
9	μ^*	-.4561	-.1817	-.0610	1.6988	.3439	.0912	.0876	.0840	.0802	.0763	.0719	.0668	.0983	μ^*	3
	σ^*	-.6969	-.3638	-.2165	1.2773	.5509	.0669	.0377	.0093	-.0192	-.0489	-.0811	-.1181	-.3976	σ^*	
10	μ^*	-.8946	-.3753	2.2699	.2353	.0853	.0839	.0824	.0809	.0794	.0778	.0759	.0737	.1254	μ^*	2
	σ^*	-1.0266	-.5094	1.5360	.4278	.0808	.0536	.0283	.0039	-.0207	-.0462	-.0738	-.1055	-.3483	σ^*	
11	μ^*	-2.3101	3.3101	.1445	.0788	.0790	.0791	.0791	.0791	.0791	.0791	.0790	.0788	.1445	μ^*	1
	σ^*	-1.9845	1.9845	.3076	.0947	.0673	.0434	.0213	.0000	-.0213	-.0434	-.0673	-.0947	-.3076	σ^*	
				$y_{(12)}$	$y_{(11)}$	$y_{(10)}$	$y_{(9)}$	$y_{(8)}$	$y_{(7)}$	$y_{(6)}$	$y_{(5)}$	$y_{(4)}$	$y_{(3)}$	$y_{(2)}$		

n = 13 Table I. (Continued)

$r_1 = 2$, r_2		$y_{(3)}$	$y_{(4)}$	$y_{(5)}$	$y_{(6)}$	$y_{(7)}$	$y_{(8)}$	$y_{(9)}$	$y_{(10)}$	$y_{(11)}$		$r_1 = 3$, r_2
2	μ^*	.2125	.0819	.0822	.0823	.0823	.0823	.0822	.0819	.2125		
	σ^*	-.4911	-.0904	-.0584	-.0287	.0000	.0287	.0584	.0904	.4911		
3	μ^*	.1784	.0770	.0799	.0825	.0848	.0871	.0892	.3211			
	σ^*	-.5698	-.1017	-.0636	-.0283	.0058	.0398	.0748	.6431			
4	μ^*	.1272	.0700	.0770	.0833	.0892	.0949	.4584	2.8102	-1.8102	μ^*	8
	σ^*	-.6724	-.1157	-.0695	-.0266	.0146	.0555	.8142	4.6615	-4.6615	σ^*	
5	μ^*	.0479	.0597	.0732	.0854	.0969	.6370	1.4405	.0415	-.4820	μ^*	7
	σ^*	-.8133	-.1341	-.0763	-.0229	.0282	1.0184	2.5175	-.1774	-2.3402	σ^*	
6	μ^*	-.0822	.0437	.0680	.0901	.8805	.9169	.0883	.0639	-.0691	μ^*	6
	σ^*	-1.0214	-.1597	-.0846	-.0154	1.2810	1.7328	-.0380	-.1350	-1.5598	σ^*	
7	μ^*	-.3174	.0164	.0607	1.2404	.6209	.0970	.0865	.0752	.1204	μ^*	5
	σ^*	-1.3635	-.1994	-.0952	1.6581	1.2929	.0255	-.0417	-.1115	-1.1652	σ^*	
8	μ^*	-.8233	-.0391	1.8624	.4246	.0939	.0902	.0862	.0820	.2231	μ^*	4
	σ^*	-2.0397	-.2735	2.3132	.9927	.0610	.0095	-.0423	-.0958	-.9251	σ^*	
9	μ^*	-2.4408	3.4408	.2838	.0863	.0866	.0866	.0866	.0863	.2838	μ^*	3
	σ^*	-4.0488	4.0488	.7622	.0841	.0414	.0000	-.0414	-.0841	-.7622	σ^*	
				$y_{(10)}$	$y_{(9)}$	$y_{(8)}$	$y_{(7)}$	$y_{(6)}$	$y_{(5)}$	$y_{(4)}$		

$r_1 = 4$, r_2		$y_{(5)}$	$y_{(6)}$	$y_{(7)}$	$y_{(8)}$	$y_{(9)}$		$r_1 = 5$, r_2
4	μ^*	.3615	.0923	.0924	.0923	.3615		
	σ^*	-1.2545	-.0674	.0000	.0674	1.2545		
5	μ^*	.2377	.0900	.0977	.5745			
	σ^*	-1.6839	-.0755	.0184	1.7410			
6	μ^*	-.0422	.0861	.9562	1.0000	.0000	μ^*	6
	σ^*	-2.5323	-.0873	2.6196	5.2487	-5.2487	σ^*	
7	μ^*	-.9632	1.9632	.4498	.1003	.4498	μ^*	5
	σ^*	-5.0555	5.0555	2.6243	.0000	-2.6243	σ^*	
				$y_{(8)}$	$y_{(7)}$	$y_{(6)}$		

n = 14

Table I. (Continued)

$r_1 = 0$, r_2		$y_{(1)}$	$y_{(2)}$	$y_{(3)}$	$y_{(4)}$	$y_{(5)}$	$y_{(6)}$	$y_{(7)}$	$y_{(8)}$	$y_{(9)}$	$y_{(10)}$	$y_{(11)}$	$y_{(12)}$	$y_{(13)}$	$y_{(14)}$		$r_1 = 1$, r_2
0	μ^*	.0714	.0714	.0714	.0714	.0714	.0714	.0714	.0714	.0714	.0714	.0714	.0714	.0714	.0714		
	σ^*	-.1532	-.0968	-.0717	-.0526	-.0362	-.0212	-.0070	.0070	.0212	.0362	.0526	.0717	.0968	.1532		
1	μ^*	.0637	.0669	.0683	.0694	.0704	.0712	.0721	.0728	.0736	.0745	.0753	.0762	.1455			
	σ^*	-.1698	-.1065	-.0784	-.0568	-.0384	-.0216	-.0056	.0100	.0259	.0426	.0609	.0820	.2556			
2	μ^*	.0530	.0609	.0643	.0670	.0692	.0713	.0732	.0751	.0770	.0789	.0809	.2291	3.9374	-2.9374	μ^*	11
	σ^*	-.1885	-.1171	-.0854	-.0612	-.0404	-.0215	-.0036	.0140	.0319	.0505	.0707	.3506	3.2597	-3.2597	σ^*	
3	μ^*	.0388	.0529	.0592	.0639	.0680	.0717	.0752	.0785	.0819	.0852	.3247	2.3304	-.2112	-1.1191	μ^*	10
	σ^*	-.2102	-.1292	-.0933	-.0658	-.0423	-.0209	-.0006	.0192	.0393	.0601	.4438	2.0584	-.4048	-1.6536	σ^*	
4	μ^*	.0199	.0426	.0526	.0602	.0667	.0726	.0782	.0835	.0887	.4350	1.6505	-.0121	-.0888	-.5496	μ^*	9
	σ^*	-.2361	-.1434	-.1023	-.0709	-.0440	-.0196	.0035	.0260	.0487	.5382	1.5721	-.1726	-.2882	-1.1112	σ^*	
5	μ^*	-.0057	.0288	.0440	.0557	.0655	.0744	.0828	.0908	.5637	1.2394	.0556	.0167	-.0291	-.2827	μ^*	8
	σ^*	-.2678	-.1604	-.1129	-.0765	-.0455	-.0174	.0092	.0350	.6363	1.2771	-.0713	-.1430	-.2267	-.8362	σ^*	
6	μ^*	-.0411	.0102	.0328	.0500	.0646	.0777	.0899	.7159	.9525	.0811	.0591	.0345	.0057	-.1329	μ^*	7
	σ^*	-.3077	-.1815	-.1256	-.0829	-.0466	-.0137	.0172	.7407	1.0642	-.0171	-.0675	-.1231	-.1878	-.6688	σ^*	
7	μ^*	-.0915	-.0158	.0175	.0429	.0643	.0835	.8992	.7365	.0897	.0765	.0623	.0466	.0281	-.0397	μ^*	6
	σ^*	-.3599	-.2084	-.1414	-.0903	-.0469	-.0077	.8546	.8949	.0158	-.0227	-.0635	-.1084	-.1606	-.5555	σ^*	
8	μ^*	-.1670	-.0537	-.0040	.0338	.0655	1.1255	.5661	.0908	.0827	.0743	.0652	.0552	.0435	.0222	μ^*	5
	σ^*	-.4317	-.2444	-.1618	-.0990	-.0457	.9825	.7515	.0379	.0065	-.0256	-.0596	-.0970	-.1402	-.4734	σ^*	
9	μ^*	-.2879	-.1127	-.0360	.0218	1.4148	.4277	.0883	.0834	.0784	.0733	.0677	.0616	.0544	.0651	μ^*	4
	σ^*	-.5372	-.2959	-.1898	-.1094	1.1322	.6244	.0540	.0270	.0003	-.0271	-.0560	-.0876	-.1242	-.4107	σ^*	
10	μ^*	-.5027	-.2142	-.0886	1.8054	.3129	.0840	.0812	.0785	.0758	.0729	.0699	.0665	.0625	.0957	μ^*	3
	σ^*	-.7091	-.3771	-.2318	1.3180	.5075	.0669	.0426	.0191	-.0040	-.0276	-.0525	-.0797	-.1112	-.3610	σ^*	
11	μ^*	-.9616	-.4228	2.3843	.2163	.0787	.0776	.0765	.0753	.0742	.0730	.0717	.0703	.0685	.1180	μ^*	2
	σ^*	-1.0441	-.5293	1.5734	.3961	.0785	.0552	.0338	.0133	-.0070	-.0275	-.0492	-.0728	-.1002	-.3203	σ^*	
12	μ^*	-2.4378	3.4378	.1341	.0730	.0731	.0732	.0733	.0733	.0733	.0733	.0732	.0731	.0730	.1341	μ^*	1
	σ^*	-2.0182	2.0182	.2858	.0907	.0667	.0459	.0270	.0089	-.0089	-.0270	-.0459	-.0667	-.0907	-.2858	σ^*	
				$y_{(13)}$	$y_{(12)}$	$y_{(11)}$	$y_{(10)}$	$y_{(9)}$	$y_{(8)}$	$y_{(7)}$	$y_{(6)}$	$y_{(5)}$	$y_{(4)}$	$y_{(3)}$	$y_{(2)}$		

n = 14

Table I. (Continued)

$r_1 = 2$, r_2		$y_{(3)}$	$y_{(4)}$	$y_{(5)}$	$y_{(6)}$	$y_{(7)}$	$y_{(8)}$	$y_{(9)}$	$y_{(10)}$	$y_{(11)}$	$y_{(12)}$		$r_1 = 3$, r_2
2	μ^*	.1969	.0755	.0758	.0759	.0759	.0759	.0759	.0758	.0755	.1969		
	σ^*	-.4488	-.0872	-.0602	-.0354	-.0117	.0117	.0354	.0602	.0872	.4488		
3	μ^*	.1689	.0711	.0734	.0753	.0771	.0788	.0804	.0820	.2930			
	σ^*	-.5125	-.0972	-.0656	-.0367	-.0090	.0182	.0457	.0744	.5827			
4	μ^*	.1287	.0651	.0703	.0749	.0792	.0834	.0874	.4111	3.2094	-2.2094	μ^*	9
	σ^*	-.5926	-.1092	-.0718	-.0375	-.0048	.0273	.0596	.7291	4.8497	-4.8497	σ^*	
5	μ^*	.0695	.0565	.0661	.0748	.0829	.0907	.5593	1.6712	.0123	-.6835	μ^*	8
	σ^*	-.6975	-.1244	-.0791	-.0376	.0018	.0403	.8965	2.6442	-.2087	-2.4355	σ^*	
6	μ^*	-.0206	.0440	.0606	.0755	.0895	.7510	1.0874	.0734	.0439	-.2047	μ^*	7
	σ^*	-.8420	-.1444	-.0881	-.0365	.0123	1.0987	1.8416	-.0617	-.1552	-1.6247	σ^*	
7	μ^*	-.1668	.0247	.0527	.0779	1.0115	.7583	.0893	.0750	.0596	.0178	μ^*	6
	σ^*	-1.0559	-.1728	-.0996	-.0330	1.3612	1.3951	.0053	-.0588	-.1262	-1.2153	σ^*	
8	μ^*	-.4285	-.0084	.0405	1.3964	.5397	.0905	.0837	.0766	.0690	.1105	μ^*	5
	σ^*	-1.4081	-.2172	-.1160	1.7413	1.0932	.0424	-.0061	-.0555	-.1074	-.9667	σ^*	
9	μ^*	-.9875	-.0756	2.0631	.3818	.0860	.0835	.0809	.0781	.0750	.2147	μ^*	4
	σ^*	-2.1051	-.3010	2.4061	.8643	.0660	.0266	-.0125	-.0522	-.0937	-.7987	σ^*	
10	μ^*	-2.7647	3.7647	.2621	.0791	.0794	.0795	.0795	.0794	.0791	.2621	μ^*	3
	σ^*	-4.1778	4.1778	.6765	.0831	.0489	.0162	-.0162	-.0489	-.0831	-.6765	σ^*	
				$y_{(11)}$	$y_{(10)}$	$y_{(9)}$	$y_{(8)}$	$y_{(7)}$	$y_{(6)}$	$y_{(5)}$	$y_{(4)}$		

$r_1 = 4$, r_2		$y_{(5)}$	$y_{(6)}$	$y_{(7)}$	$y_{(8)}$	$y_{(9)}$	$y_{(10)}$		$r_1 = 5$, r_2
4	μ^*	.3320	.0839	.0841	.0841	.0839	.3320		
	σ^*	-1.0495	-.0738	-.0244	.0244	.0738	1.0495		
5	μ^*	.2456	.0807	.0857	.0903	.4978			
	σ^*	-1.3227	-.0839	-.0195	.0439	1.3821			
6	μ^*	.0837	.0755	.0895	.7512	1.4921	-.4921	μ^*	7
	σ^*	-1.7721	-.0983	-.0088	1.8791	5.5823	-5.5823	σ^*	
7	μ^*	-.2720	.0657	1.2063	.7061	.0910	.2029	μ^*	6
	σ^*	-2.6618	-.1229	2.7848	2.8338	-.0406	-2.7931	σ^*	
8	μ^*	-1.4198	2.4198	.4098	.0902	.0902	.4098	μ^*	5
	σ^*	-5.3115	5.3115	1.8560	.0441	-.0441	-1.8560	σ^*	
				$y_{(9)}$	$y_{(8)}$	$y_{(7)}$	$y_{(6)}$		

$r_1 = 6$, r_2		$y_{(7)}$	$y_{(8)}$
6	μ^*	.5000	.5000
	σ^*	-5.6716	5.6716

n = 15

Table I. (Continued)

$r_1 = 0$, r_2		$y_{(1)}$	$y_{(2)}$	$y_{(3)}$	$y_{(4)}$	$y_{(5)}$	$y_{(6)}$	$y_{(7)}$	$y_{(8)}$	$y_{(9)}$	$y_{(10)}$	$y_{(11)}$	$y_{(12)}$	$y_{(13)}$	$y_{(14)}$	$y_{(15)}$		$r_1 = 1$, r
0	μ^*	.0667	.0667	.0667	.0667	.0667	.0667	.0667	.0667	.0667	.0667	.0667	.0667	.0667	.0667	.0667		
	σ^*	-.1444	-.0927	-.0699	-.0526	-.0379	-.0247	-.0122	.0000	.0122	.0247	.0379	.0526	.0699	.0927	.1444		
1	μ^*	.0599	.0627	.0639	.0648	.0655	.0662	.0669	.0675	.0682	.0688	.0695	.0702	.0709	.1351			
	σ^*	-.1590	-.1013	-.0760	-.0568	-.0404	-.0256	-.0116	.0019	.0154	.0293	.0440	.0602	.0791	.2409			
2	μ^*	.0508	.0574	.0602	.0624	.0642	.0659	.0675	.0690	.0704	.0719	.0735	.0751	.2116	4.1564	-3.1564	μ^*	12
	σ^*	-.1752	-.1108	-.0825	-.0610	-.0427	-.0262	-.0106	.0044	.0195	.0349	.0512	.0690	.3300	3.3306	-3.3306	σ^*	
3	μ^*	.0390	.0506	.0556	.0595	.0628	.0657	.0685	.0711	.0737	.0763	.0790	.2982	2.4829	-.2519	-1.2311	μ^*	11
	σ^*	-.1937	-.1214	-.0897	-.0655	-.0450	-.0265	-.0091	.0078	.0246	.0417	.0598	.4169	2.1162	-.4265	-1.6897	σ^*	
4	μ^*	.0234	.0418	.0498	.0560	.0611	.0658	.0701	.0743	.0784	.0824	.3969	1.7779	-.0356	-.1163	-.6260	μ^*	10
	σ^*	-.2154	-.1336	-.0977	-.0705	-.0473	-.0264	-.0068	.0122	.0310	.0502	.5042	1.6273	-.1890	-.3024	-1.1359	σ^*	
5	μ^*	.0030	.0305	.0425	.0516	.0593	.0663	.0727	.0789	.0849	.5104	1.3522	.0403	-.0011	-.0501	-.3413	μ^*	9
	σ^*	-.2414	-.1481	-.1071	-.0760	-.0496	-.0258	-.0035	.0180	.0393	.5940	1.3324	-.0850	-.1549	-.2373	-.8553	σ^*	
6	μ^*	-.0244	.0155	.0330	.0462	.0574	.0674	.0767	.0856	.6425	1.0549	.0706	.0467	.0200	-.0115	-.1807	μ^*	8
	σ^*	-.2733	-.1654	-.1181	-.0822	-.0518	-.0244	.0012	.0258	.6882	1.1208	-.0293	-.0781	-.1325	-.1963	-.6847	σ^*	
7	μ^*	-.0621	-.0046	.0205	.0395	.0555	.0698	.0830	.7983	.8308	.0823	.0677	.0519	.0342	.0134	-.0803	μ^*	7
	σ^*	-.3136	-.1870	-.1315	-.0894	-.0538	-.0219	.0079	.7892	.9536	.0045	-.0326	-.0722	-.1162	-.1677	-.5694	σ^*	
8	μ^*	-.1155	-.0326	.0036	.0309	.0539	.0743	.9854	.6537	.0857	.0764	.0666	.0561	.0443	.0305	-.0131	μ^*	6
	σ^*	-.3664	-.2146	-.1482	-.0979	-.0555	-.0174	.9001	.8129	.0269	-.0029	-.0339	-.0670	-.1036	-.1464	-.4859	σ^*	
9	μ^*	-.1950	-.0732	-.0203	.0196	.0531	1.2157	.5093	.0848	.0789	.0728	.0664	.0595	.0518	.0428	.0338	μ^*	5
	σ^*	-.4390	-.2518	-.1700	-.1082	-.0562	1.0252	.6892	.0430	.0177	-.0078	-.0342	-.0624	-.0935	-.1298	-.4223	σ^*	
10	μ^*	-.3217	-.1364	-.0560	.0043	1.5097	.3890	.0818	.0781	.0744	.0706	.0666	.0623	.0575	.0519	.0677	μ^*	4
	σ^*	-.5459	-.3050	-.2002	-.1211	1.1722	.5767	.0555	.0331	.0111	-.0110	-.0339	-.0582	-.0850	-.1163	-.3721	σ^*	
11	μ^*	-.5462	-.2448	-.1148	1.9058	.2873	.0777	.0756	.0735	.0714	.0693	.0671	.0647	.0620	.0587	.0927	μ^*	3
	σ^*	-.7201	-.3892	-.2458	1.3552	.4712	.0661	.0453	.0256	.0062	-.0131	-.0331	-.0543	-.0778	-.1050	-.3311	σ^*	
12	μ^*	-1.0242	-.4676	2.4918	.2002	.0730	.0721	.0712	.0704	.0695	.0686	.0676	.0666	.0654	.0640	.1113	μ^*	2
	σ^*	-1.0601	-.5477	1.6077	.3693	.0761	.0558	.0373	.0197	.0026	-.0145	-.0321	-.0508	-.0714	-.0954	-.2968	σ^*	
13	μ^*	-2.5574	3.5574	.1252	.0679	.0681	.0682	.0682	.0682	.0682	.0682	.0682	.0682	.0681	.0679	.1252	μ^*	1
	σ^*	-2.0493	2.0493	.2672	.0869	.0656	.0474	.0308	.0152	.0000	-.0152	-.0308	-.0474	-.0656	-.0869	-.2672	σ^*	
				$y_{(14)}$	$y_{(13)}$	$y_{(12)}$	$y_{(11)}$	$y_{(10)}$	$y_{(9)}$	$y_{(8)}$	$y_{(7)}$	$y_{(6)}$	$y_{(5)}$	$y_{(4)}$	$y_{(3)}$	$y_{(2)}$		

Table I. (Continued)

n = 15

$r_1 = 2$ r_2		$y_{(3)}$	$y_{(4)}$	$y_{(5)}$	$y_{(6)}$	$y_{(7)}$	$y_{(8)}$	$y_{(9)}$	$y_{(10)}$	$y_{(11)}$	$y_{(12)}$	$y_{(13)}$		$r_1 = 3$ r_2
2	μ^*	.1835	.0701	.0703	.0704	.0705	.0705	.0705	.0704	.0703	.0701	.1835		
	σ^*	-.4139	-.0838	-.0607	-.0395	-.0195	.0000	.0195	.0395	.0607	.0838	.4139		
3	μ^*	.1601	.0662	.0679	.0695	.0708	.0721	.0734	.0746	.0758	.2697			
	σ^*	-.4668	-.0927	-.0660	-.0417	-.0187	.0037	.0261	.0490	.0730	.5341			
4	μ^*	.1274	.0609	.0649	.0684	.0717	.0748	.0778	.0808	.3734	3.5892	-2.5892	μ^*	10
	σ^*	-.5314	-.1032	-.0721	-.0438	-.0170	.0090	.0349	.0612	.6624	5.0207	-5.0207	σ^*	
5	μ^*	.0815	.0536	.0608	.0673	.0733	.0790	.0845	.5000	1.8923	-.0171	-.8752	μ^*	9
	σ^*	-.6129	-.1160	-.0792	-.0457	-.0142	.0164	.0468	.8049	2.7587	-.2368	-2.5219	σ^*	
6	μ^*	.0150	.0436	.0555	.0662	.0762	.0857	.6579	1.2523	.0577	.0238	-.3338	μ^*	8
	σ^*	-.7200	-.1323	-.0878	-.0475	-.0095	.0272	.9699	1.9396	-.0828	-.1734	-1.6834	σ^*	
7	μ^*	-.0854	.0289	.0481	.0654	.0815	.8614	.8925	.0804	.0629	.0441	-.0799	μ^*	7
	σ^*	-.8679	-.1539	-.0987	-.0486	-.0017	1.1708	1.4867	-.0125	-.0741	-.1395	-1.2605	σ^*	
8	μ^*	-.2467	.0063	.0375	.0655	1.1375	.6535	.0857	.0764	.0666	.0561	.0617	μ^*	6
	σ^*	-1.0872	-.1847	-.1132	-.0486	1.4336	1.1828	.0264	-.0198	-.0674	-.1178	-1.0042	σ^*	
9	μ^*	-.5335	-.0324	.0205	1.5454	.4801	.0840	.0793	.0745	.0694	.0639	.1487	μ^*	5
	σ^*	-1.4486	-.2334	-.1346	1.8166	.9544	.0508	.0137	-.0235	-.0619	-.1023	-.8312	σ^*	
10	μ^*	-1.1426	-.1111	2.2537	.3478	.0792	.0775	.0756	.0737	.0716	.0693	.2054	μ^*	4
	σ^*	-2.1646	-.3259	2.4905	.7692	.0678	.0363	.0055	-.0254	-.0571	-.0905	-.7058	σ^*	
11	μ^*	-3.0706	4.0706	.2436	.0731	.0733	.0734	.0734	.0734	.0733	.0731	.2436	μ^*	3
	σ^*	-4.2953	4.2953	.6102	.0810	.0528	.0261	.0000	-.0261	-.0528	-.0810	-.6102	σ^*	
				$y_{(12)}$	$y_{(11)}$	$y_{(10)}$	$y_{(9)}$	$y_{(8)}$	$y_{(7)}$	$y_{(6)}$	$y_{(5)}$	$y_{(4)}$		

$r_1 = 4$ r_2		$y_{(5)}$	$y_{(6)}$	$y_{(7)}$	$y_{(8)}$	$y_{(9)}$	$y_{(10)}$	$y_{(11)}$		$r_1 = 5$ r_2
4	μ^*	.3073	.0770	.0772	.0772	.0772	.0770	.3073		
	σ^*	-.9085	-.0756	-.0374	.0000	.0374	.0756	.9085		
5	μ^*	.2424	.0736	.0770	.0802	.0832	.4435			
	σ^*	-1.1001	-.0854	-.0377	.0088	.0552	1.1592			
6	μ^*	.1339	.0686	.0774	.0859	.6342	1.9724	-.9724	μ^*	8
	σ^*	-1.3838	-.0987	-.0367	.0236	1.4956	5.8824	-5.8824	σ^*	
7	μ^*	-.0644	.0601	.0791	.9252	.9607	.0776	-.0383	μ^*	7
	σ^*	-1.8515	-.1186	-.0327	2.0028	3.0212	-.0764	-2.9448	σ^*	
8	μ^*	-.4928	.0435	1.4494	.5846	.0865	.0804	.2486	μ^*	6
	σ^*	-2.7788	-.1547	2.9335	2.0090	.0169	-.0670	-1.9590	σ^*	
9	μ^*	-1.8586	2.8586	.3769	.0820	.0821	.0820	.3769	μ^*	5
	σ^*	-5.5431	5.5431	1.4614	.0604	.0000	-.0604	-1.4614	σ^*	
				$y_{(10)}$	$y_{(9)}$	$y_{(8)}$	$y_{(7)}$	$y_{(6)}$		

$r_1 = 6$ r_2		$y_{(7)}$	$y_{(8)}$	$y_{(9)}$
6	μ^*	.4557	.0887	.4557
	σ^*	-3.0248	.0000	3.0248
7	μ^*	.0000	1.0000	
	σ^*	-6.0497	6.0497	

Table II. Variances and Covariances of the Estimates of the

of Sizes $11 \leq n \leq 15$

n	r_1	r_2	0	1	2	3	4	5	6	7	8	9	10
11	0	V(μ*)	.0909	.0929	.0966	.1031	.1143	.1342	.1718	.2504	.4493	1.2179	
		V(σ*)	.0517	.0601	.0704	.0836	.1013	.1262	.1640	.2279	.3577	.7550	
		Cov(μ*, σ*)	.0000	.0041	.0102	.0195	.0336	.0559	.0936	.1644	.3251	.8777	
	1	V(μ*)		.0943	.0975	.1033	.1143	.1356	.1805	.2929	.7093		
		V(σ*)		.0712	.0854	.1043	.1311	.1717	.2408	.3817	.8140		
		Cov(μ*, σ*)		.0000	.0066	.0172	.0343	.0637	.1194	.2453	.6696		
	2	V(μ*)			.0997	.1044	.1144	.1364	.1926	.3997			
		V(σ*)			.1052	.1330	.1753	.2469	.3928	.8389			
		Cov(μ*, σ*)			.0000	.0115	.0321	.0718	.1623	.4662			
	3	V(μ*)				.1075	.1153	.1369	.2202				
		V(σ*)				.1764	.2496	.3982	.8512				
		Cov(μ*, σ*)				.0000	.0239	.0806	.2748				
	4	V(μ*)					.1191	.1372					
		V(σ*)					.3999	.8564					
		Cov(μ*, σ*)					.0000	.0908					
12	0	V(μ*)	.0833	.0849	.0878	.0926	.1004	.1136	.1363	.1784	.2650	.4809	1.3044
		V(σ*)	.0469	.0538	.0620	.0723	.0855	.1033	.1283	.1663	.2305	.3610	.7601
		Cov(μ*, σ*)	.0000	.0033	.0081	.0152	.0254	.0406	.0645	.1045	.1791	.3469	.9202
	1	V(μ*)		.0861	.0885	.0929	.1004	.1139	.1393	.1915	.3195	.7864	
		V(σ*)		.0627	.0737	.0878	.1067	.1336	.1745	.2438	.3854	.8195	
		Cov(μ*, σ*)		.0000	.0052	.0130	.0250	.0440	.0762	.1364	.2710	.7212	
	2	V(μ*)			.0903	.0939	.1007	.1140	.1421	.2115	.4614		
		V(σ*)			.0884	.1082	.1361	.1786	.2505	.3971	.8450		
		Cov(μ*, σ*)			.0000	.0084	.0222	.0460	.0909	.1918	.5263		
	3	V(μ*)				.0963	.1018	.1140	.1457	.2617			
		V(σ*)				.1369	.1804	.2539	.4032	.8581			
		Cov(μ*, σ*)				.0000	.0153	.0454	.1141	.3438			
	4	V(μ*)					.1049	.1144	.1524				
		V(σ*)					.2549	.4059	.8647				
		Cov(μ*, σ*)					.0000	.0379	.1699				
	5	V(μ*)						.1175					
		V(σ*)						.8667					
		Cov(μ*, σ*)						.0000					

Mean (μ^*) and Standard Deviation (σ^*) for Censored Samples
from Normal Populations

n	r_1	r_2	0	1	2	3	4	5	6	7	8	9	10	11	
13	0	$V(\mu^*)$	.0769	.0782	.0805	.0841	.0899	.0991	.1141	.1395	.1858	.2799	.5118	1.3868	
		$V(\sigma^*)$	.0429	.0486	.0554	.0636	.0739	.0872	.1050	.1302	.1688	.2329	.3640	.7646	
		$Cov(\mu^*, \sigma^*)$	.0000	.0027	.0066	.0121	.0198	.0309	.0472	.0725	.1146	.1925	.3668	.9589	
	1	$V(\mu^*)$		.0792	.0811	.0845	.0900	.0992	.1152	.1444	.2036	.3466	.8613		
		$V(\sigma^*)$		.0559	.0647	.0756	.0898	.1088	.1358	.1768	.2465	.3886	.8243		
		$Cov(\mu^*, \sigma^*)$		.0000	.0041	.0101	.0190	.0322	.0530	.0876	.1518	.2944	.7679		
	2	$V(\mu^*)$			.0826	.0854	.0903	.0992	.1159	.1500	.2324	.5237			
		$V(\sigma^*)$			.0761	.0909	.1106	.1387	.1813	.2536	.4007	.8502			
		$Cov(\mu^*, \sigma^*)$			.0000	.0064	.0162	.0320	.0587	.1084	.2184	.5803			
	3	$V(\mu^*)$				.0874	.0914	.0993	.1164	.1584	.3078				
		$V(\sigma^*)$				.1112	.1399	.1836	.2574	.4073	.8639				
		$Cov(\mu^*, \sigma^*)$				.0000	.0107	.0294	.0648	.1442	.4054				
	4	$V(\mu^*)$					.0939	.1002	.1167	.1780					
		$V(\sigma^*)$					.1843	.2591	.4108	.8714					
		$Cov(\mu^*, \sigma^*)$					.0000	.0216	.0716	.2397					
	5	$V(\mu^*)$						.1032	.1168						
		$V(\sigma^*)$						.4118	.8747						
		$Cov(\mu^*, \sigma^*)$						.0000	.0793						
14	0	$V(\mu^*)$	.0714	.0725	.0743	.0772	.0816	.0883	.0988	.1155	.1435	.1938	.2950	.5419	1.
		$V(\sigma^*)$	.0395	.0444	.0500	.0568	.0650	.0753	.0886	.1065	.1318	.1702	.2350	.3666	.
		$Cov(\mu^*, \sigma^*)$	.0000	.0023	.0055	.0099	.0159	.0242	.0360	.0533	.0799	.1239	.2048	.3851	.
	1	$V(\mu^*)$		.0733	.0749	.0775	.0817	.0883	.0992	.1175	.1505	.2164	.3736	.9337	
		$V(\sigma^*)$		.0505	.0576	.0664	.0773	.0915	.1106	.1376	.1789	.2488	.3914	.8284	
		$Cov(\mu^*, \sigma^*)$		.0000	.0034	.0081	.0149	.0246	.0390	.0612	.0981	.1660	.3158	.8105	
	2	$V(\mu^*)$			.0762	.0784	.0821	.0884	.0993	.1193	.1594	.2543	.5858		
		$V(\sigma^*)$			.0668	.0781	.0929	.1127	.1408	.1836	.2561	.4038	.8547		
		$Cov(\mu^*, \sigma^*)$			.0000	.0050	.0124	.0236	.0411	.0704	.1243	.2427	.6293		
	3	$V(\mu^*)$				.0800	.0831	.0887	.0993	.1212	.1737	.3567			
		$V(\sigma^*)$				.0933	.1137	.1425	.1863	.2604	.4108	.8689			
		$Cov(\mu^*, \sigma^*)$				.0000	.0079	.0206	.0422	.0825	.1714	.4609			
	4	$V(\mu^*)$					.0852	.0896	.0994	.1237	.2104				
		$V(\sigma^*)$					.1430	.1875	.2626	.4148	.8770				
		$Cov(\mu^*, \sigma^*)$					.0000	.0141	.0411	.1020	.3022				
	5	$V(\mu^*)$						.0922	.0997	.1286					
		$V(\sigma^*)$						.2633	.4166	.8812					
		$Cov(\mu^*, \sigma^*)$						.0000	.0338	.1496					
	6	$V(\mu^*)$							.1022						
		$V(\sigma^*)$							.8825						
		$Cov(\mu^*, \sigma^*)$							.0000						

Table II. (Continued)

n	r_1 \ r_2		0	1	2	3	4	5	6	7	8	9	10	11	12	13
15	0	$V(\mu^*)$	.0667	.0676	.0691	.0714	.0748	.0799	.0875	.0992	.1176	.1480	.2022	.3101	.5713	1.5404
		$V(\sigma^*)$	.0366	.0408	.0456	.0512	.0580	.0662	.0765	.0899	.1078	.1332	.1718	.2368	.3689	.7723
		$Cov(\mu^*, \sigma^*)$	.0000	.0019	.0046	.0082	.0130	.0195	.0284	.0409	.0590	.0868	.1325	.2163	.4020	1.0273
	1	$V(\mu^*)$		.0683	.0696	.0717	.0749	.0799	.0877	.1001	.1207	.1574	.2296	.4004	1.0037	
		$V(\sigma^*)$		.0460	.0519	.0590	.0678	.0787	.0929	.1121	.1393	.1807	.2508	.3939	.8322	
		$Cov(\mu^*, \sigma^*)$		.0000	.0028	.0067	.0120	.0194	.0299	.0453	.0689	.1079	.1791	.3355	.8496	
	2	$V(\mu^*)$			.0706	.0724	.0753	.0800	.0877	.1007	.1239	.1697	.2769	.6471		
		$V(\sigma^*)$			.0594	.0684	.0798	.0946	.1145	.1427	.1856	.2584	.4065	.8586		
		$Cov(\mu^*, \sigma^*)$			.0000	.0040	.0098	.0181	.0304	.0496	.0812	.1389	.2649	.6740		
	3	$V(\mu^*)$				.0738	.0762	.0804	.0877	.1011	.1279	.1908	.4069			
		$V(\sigma^*)$				.0802	.0953	.1157	.1446	.1886	.2629	.4138	.8731			
		$Cov(\mu^*, \sigma^*)$				.0000	.0061	.0153	.0298	.0541	.0987	.1962	.5112			
	4	$V(\mu^*)$					.0780	.0813	.0879	.1014	.1340	.2472				
		$V(\sigma^*)$					.1161	.1455	.1901	.2655	.4182	.8817				
		$Cov(\mu^*, \sigma^*)$					.0000	.0099	.0270	.0590	.1295	.3585				
	5	$V(\mu^*)$						.0835	.0886	.1016	.1487					
		$V(\sigma^*)$						.1906	.2667	.4205	.8866					
		$Cov(\mu^*, \sigma^*)$						.0000	.0196	.0644	.2126					
	6	$V(\mu^*)$							.0911	.1017						
		$V(\sigma^*)$							.4212	.8888						
		$Cov(\mu^*, \sigma^*)$							.0000	.0704						

This table is a continuation of Table II in [2]. The entries in Table I, as well as in Table II of this present paper, have been rounded to four decimal places for convenience. Readers who desire more precision may obtain copies of the original tables containing eight decimal places from the authors. The results in the eight-decimal-place table are exact to seven places but rounding may cause some of the figures in the eighth place to be a few units in error.

If the coefficients of an estimate are sought for a value of r_1 not given in the table, the same procedure can be followed as that mentioned in Part I of this series.

The variances of the estimates and their covariances are given in Table II in terms of σ^2. This table is a continuation of Table III in [2] and the results are given to only four decimal places for convenience.

Table III shows the efficiency of the estimates for every case of censoring relative to the corresponding estimate obtained by complete samples.

3. Alternative estimate. The alternative estimate was proposed by Gupta [1] to replace the best linear estimate when sample sizes are greater than 10 and censoring was from one side only. This estimate was generalized to the case of double censoring in Part I of this present series. The variance of the alternative estimates and their efficiencies relative to the best linear estimate for samples of sizes 12 and 15 under every case of censoring are given in Table IV.

The authors know of no instance where the alternative estimates have been compared previously for sample sizes this large.

4. Comments. The conclusions mentioned in [2], Section 5, hold true here and, in fact, appear much stronger for increasing sample size. Several points are worth emphasis:

(1) In estimation of the mean, the relative efficiency holds up—about 65 per cent or better—as long as the median value remains known. (For an even n, it is about 70 per cent or better as long as the two middle values are uncensored.) This result was anticipated because the asymptotic efficiency of the median is $2/\pi = 63.7\%$.

Another way of presenting this same finding can be seen clearly from Fig. 1 which shows the relative efficiency of the best linear estimate of μ under all conditions of censoring a sample of size 15 from the normal distribution. Each one of the curves shows the efficiency of the estimate of the mean for a certain number of known elements $[k = n - (r_1 + r_2)]$ for all possible values of r_1 and r_2. The efficiency attains its maximum whenever the middle element is known.

(2) From Table III, one can see that, for fixed values of censoring from one side, the efficiency of the estimate of the standard deviation decreases approximately in equal amounts with each increment in the number of censored elements on the opposite side.

This is illustrated by Fig. 2 which shows the relative efficiency of the best linear estimate of σ under all conditions of censoring a sample of size 15 from the

Table III

Percentage efficiencies of the estimate of the mean (μ^*) and standard deviation (σ^*) for censored samples **relative** to uncensored samples in a normal population for $11 \leq n \leq 15$
(Continuation of Table IV in reference [2].)

n	r_1 \ r_2		0	1	2	3	4	5	6	7	8	9	10
11	0	μ^*	100.0	97.89	94.13	88.20	79.55	67.74	52.92	36.31	20.23	7.46	
		σ^*	100.0	86.02	73.43	61.82	51.04	40.95	31.51	22.68	14.45	6.85	
	1	μ^*		96.36	93.29	87.98	79.54	67.06	50.37	31.04	12.82		
		σ^*		72.55	60.53	49.55	39.44	30.10	21.47	13.54	6.35		
	2	μ^*			91.20	87.04	79.46	66.64	47.21	22.75			
		σ^*			49.14	38.86	29.48	20.93	13.16	6.16			
	3	μ^*				84.56	78.84	66.39	41.29				
		σ^*				29.31	20.71	12.98	6.07				
	4	μ^*					76.32	66.28					
		σ^*					12.93	6.04					

12	0	μ^*	100:0	98.16	94.95	90.04	82.98	73.38	61.13	46.72	31.45	17.33	6.39
		σ^*	100.0	87.18	75.57	64.84	54.80	45.39	36.53	28.18	20.33	12.98	6.17
	1	μ^*		96.78	94.13	89.74	82.97	73.14	59.83	43.52	26.08	10.60	
		σ^*		74.78	63.63	53.40	43.92	35.09	26.87	19.22	12.16	5.72	
	2	μ^*			92.25	88.75	82.75	73.08	58.63	39.40	18.06		
		σ^*			53.01	43.33	34.44	26.25	18.71	11.81	5.55		
	3	μ^*				86.49	81.89	73.08	57.21	31.85			
		σ^*				34.25	25.99	18.47	11.63	5.46			
	4	μ^*					79.43	72.84	54.68				
		σ^*					18.39	11.55	5.42				
	5	μ^*						70.91					
		σ^*						5.41					

Table III (continued)

n	r_1	r_2	0	1	2	3	4	5	6	7	8	9	10	11	12	13
13	0	μ^*	100.0	98.36	95.60	91.44	85.55	77.61	67.43	55.13	41.39	27.48	15.03	5.55		
		σ^*	100.0	88.16	77.39	67.39	58.02	49.19	40.84	32.94	25.47	18.41	11.78	5.61		
	1	μ^*		97.11	94.80	91.08	85.51	77.55	66.80	53.27	37.78	22.19	8.93			
		σ^*		76.66	66.28	56.70	47.77	39.42	31.59	24.25	17.40	11.03	5.20			
	2	μ^*			93.10	90.09	85.17	77.55	66.38	51.28	33.10	14.69				
		σ^*			56.32	47.20	38.76	30.93	23.65	16.91	10.70	5.04				
	3	μ^*				88.03	84.20	77.43	66.10	48.58	24.99					
		σ^*				38.56	30.65	23.36	16.66	10.53	4.96					
	4	μ^*					81.88	76.80	65.93	43.21						
		σ^*					23.27	16.55	10.44	4.92						
	5	μ^*						74.54	65.86							
		σ^*						10.41	4.90							

14	0	μ^*	100.0	98.53	96.11	92.53	87.54	80.87	72.31	61.85	49.78	36.85	24.21	13.18	4.87
		σ^*	100.0	89.00	78.95	69.59	60.79	52.48	44.59	37.10	29.98	23.22	16.82	10.78	5.14
	1	μ^*		97.39	95.34	92.14	87.45	80.86	72.03	60.79	47.45	33.01	19.12	7.65	
		σ^*		78.28	68.56	59.55	51.12	43.20	35.74	28.71	22.09	15.88	10.09	4.77	
	2	μ^*			93.79	91.16	87.03	80.82	71.92	59.86	44.82	28.08	12.19		
		σ^*			59.19	50.57	42.55	35.06	28.06	21.52	15.43	9.79	4.62		
	3	μ^*				89.29	86.00	80.55	71.90	58.92	41.12	20.03			
		σ^*				42.35	34.76	27.73	21.21	15.18	9.62	4.55			
	4	μ^*					83.86	79.70	71.88	57.75	33.95				
		σ^*					27.63	21.08	15.05	9.53	4.51				
	5	μ^*						77.44	71.64	55.56					
		σ^*						15.01	9.49	4.48					
	6	μ^*							69.91						
		σ^*							4.48						

Table III (continued)

n	r_1 \ r_2		0	1	2	3	4	5	6	7	8	9	10	11	12	13
15	0	μ^*	100.0	98.68	96.52	93.40	89.11	83.42	76.15	67.21	56.70	45.04	32.97	21.50	11.67	4.33
		σ^*	100.0	89.72	80.30	71.51	63.21	55.35	47.88	40.76	33.97	27.49	21.33	15.47	9.93	4.74
	1	μ^*		97.62	95.79	92.99	88.97	83.42	76.04	66.62	55.23	42.36	29.03	16.65	6.64	
		σ^*		79.69	70.54	62.04	54.06	46.54	39.42	32.68	26.30	20.28	14.60	9.30	4.40	
	2	μ^*			94.37	92.04	88.50	83.31	76.02	66.22	53.79	39.28	24.07	10.30		
		σ^*			61.70	53.53	45.89	38.73	32.00	25.68	19.73	14.18	9.01	4.27		
	3	μ^*				90.33	87.46	82.93	76.01	65.93	52.12	34.94	16.38			
		σ^*				45.70	38.43	31.66	25.33	19.43	13.93	8.85	4.20			
	4	μ^*					85.49	81.95	75.86	65.73	49.75	26.97				
		σ^*					31.55	25.18	19.27	13.80	8.76	4.15				
	5	μ^*						79.80	75.25	65.61	44.82					
		σ^*						19.22	13.74	8.71	4.13					
	6	μ^*							73.19	65.56						
		σ^*							8.70	4.12						

normal distribution. In this figure, the graphs for $r_1 = 0, 1, \cdots, 12$ show a parallelism as r_1 changes. Thus, for any corresponding value of r_2 the efficiency decreases by about the same amount for each change in the value of r_1.

(3) Using Table III again and reading the entries for σ^* in diagonal fashion, one can see that, for a given n and fixed uncensored sample size ($r_1 + r_2 =$ constant), the efficiency of the best estimate of σ is remarkably constant independently of how r_1 and r_2 are chosen. In other words, there is practically no difference in efficiency irrespective of the proportion of the relative censoring from either side.

This can be observed very clearly in Fig. 2. The approximate horizontal lines show constancy of the relative efficiencies of σ^* for the known elements (k) of the sample whatever may be the individual values of r_1 and r_2.

(4) From Table III (and graphs similar to Fig. 2), one can construct the following table showing how the efficiency in estimating σ^* varies with the number of uncensored values for each sample size to serve as a rough guide in censoring.

Rough guide for assessing approximate efficiency (per cent) of estimate of σ*

Sample Size n	*Number of uncensored observations in sample, or $k = n - (r_1 + r_2)$													
	2	3	4	5	6	7	8	9	10	11	12	13	14	15
11	6	13	21	30	39	50	61	73	86	100				
12	6	12	19	27	35	44	54	64	75	87	100			
13	5	11	17	24	31	39	48	57	67	77	88	100		
14	5	10	15	22	28	36	43	51	59	69	78	89	100	
15	4	9	14	20	26	32	39	46	54	62	71	80	90	100

* These values are rounded averages of different combinations of censoring and are within 2 or 3 per cent in almost all cases.

The information in this rough guide for censoring is also illustrated in Fig. 3. The efficiency in estimating σ^* for varying proportions of censoring is shown in the graph for samples of size 10, 15, 20 and large n. The latter value was obtained from Gupta [1] and represented single censoring. However, as stated previously, the efficiency in estimating σ^* depends primarily upon the proportion of uncensored elements irrespective of the side and can be used in this way.

(5) Figs. 4 and 5 show the efficiencies of the alternative estimate from a sample of size 15 relative to the correspondingly best linear estimate for the mean and the standard deviation respectively.

Judging from these figures, the worst efficiencies of the alternative estimate for estimating both the mean and standard deviations are attained for singly censored samples (i.e., only r_1 or $r_2 = 0$). Thus, the alternative estimate is relatively more precise when applied to doubly censored samples. The alternative estimate was proposed by judging the results of a comparison of efficiencies using a singly

Table IV

Variances and Relative Efficiencies of Alternative Estimates of the Mean ($\mu^{*\prime}$) and Standard Deviation ($\sigma^{*\prime}$) for Censored Samples of Size 12 and 15 from a Normal Population

n	r_1	r_2		0	1	2	3	4	5	6	7	8	9	10	11	12	13
12	0	($\mu^{*\prime}$)	V.	.08333333	.08532138	.08921286	.09569251	.10617803	.12326618	.15197055	.20294226	.30233969	.53167098	1.30441131			
			R.E.	100.00	99.51	98.38	96.72	94.59	92.13	89.71	87.90	87.64	90.44	100.00			
		($\sigma^{*\prime}$)	V.	.04693011	.05541733	.06584693	.07855768	.09459627	.11566732	.14474943	.18762545	.25730631	.39089620	.76011855			
			R.E.	99.89	97.03	94.20	92.04	90.42	89.29	88.66	88.66	89.60	92.36	100.00			
	1	($\mu^{*\prime}$)	V.		.08688517	.08998634	.09535109	.10440203	.11990788	.14777036	.20261677	.33120159	.78642201				
			R.E.		99.11	98.38	97.39	96.20	95.03	94.25	94.50	96.48	100.00				
		($\sigma^{*\prime}$)	V.		.06478228	.07715733	.09281482	.11337921	.14183827	.18414527	.25419403	.39403316	.81947829				
			R.E.		96.77	95.48	94.58	94.15	94.19	94.75	95.93	97.81	100.00				
	2	($\mu^{*\prime}$)	V.			.09205743	.09613108	.10369458	.11794105	.14676609	.21537209	.46137626					
			R.E.			98.13	97.67	97.11	96.68	96.85	98.21	100.00					
		($\sigma^{*\prime}$)	V.			.09299998	.11391167	.14281363	.18574083	.25691562	.39999265	.84501770					
			R.E.			95.09	94.97	95.30	96.14	97.51	99.28	100.00					
	3	($\mu^{*\prime}$)	V.				.09854217	.10405259	.11637233	.14734277	.26167710						
			R.E.				97.77	97.79	97.99	98.85	100.00						
		($\sigma^{*\prime}$)	V.				.14326687	.18682141	.25894808	.40403858	.85814724						
			R.E.				95.53	96.54	98.04	99.80	100.00						
	4	($\mu^{*\prime}$)	V.					.10664702	.11546756	.15239128							
			R.E.					98.38	99.08	100.00							
		($\sigma^{*\prime}$)	V.					.25964368	.40599317	.86468798							
			R.E.					98.17	99.98	100.00							
	5	($\mu^{*\prime}$)	V.						.11751619								
			R.E.						100.00								
		($\sigma^{*\prime}$)	V.						.86669069								
			R.E.						100.00								

15	0	(μ*1) V.	.06676667	.06783620	.06996743	.07323737	.07806417	.08512564	.09551426	.11105404	.13495703	.17330497	.23882285	.36232523	.63848625	1.54037885
		R.E.	100.00	99.60	98.71	97.46	95.84	93.88	91.66	89.32	87.12	85.40	84.66	85.58	89.47	100.00
		(σ*1) V.	.03667132	.04199425	.04825549	.05545771	.06393382	.07415317	.08679542	.10290237	.12417377	.15360043	.19699885	.26744195	.40210996	.77230522
		R.E.	99.90	97.23	94.53	92.38	90.64	89.25	88.15	87.35	86.85	86.75	87.19	88.54	91.74	100.00
	1	(μ*1) V.		.06883025	.07060707	.07337808	.07754855	.08378381	.09319554	.10773719	.13110231	.17103630	.24649116	.41734709	1.00373313	
		R.E.		99.22	98.57	97.70	96.63	95.38	94.08	92.88	92.06	92.01	93.17	95.95	100.00	
		(σ*1) V.		.04757932	.05459531	.06286426	.07280110	.08506272	.10067027	.12130831	.14998813	.19270808	.26346042	.40451706	.83217195	
		R.E.		96.62	95.12	93.93	93.08	92.54	92.31	92.40	92.86	93.75	95.21	97.38	100.00	
	2	(μ*1) V.			.07192272	.07413167	.07764224	.08314965	.09187723	.10612952	.13071264	.17732697	.28290628	.64711601		
		R.E.			98.23	97.70	97.02	96.23	95.45	94.86	94.81	95.72	97.89	100.00		
		(σ*1) V.			.06297264	.07309736	.08556985	.10141243	.12232669	.15136196	.19461599	.26642265	.41072321	.85860349		
		R.E.			94.28	93.63	93.28	93.26	93.57	94.26	95.38	96.99	98.98	100.00		
	3	(μ*1) V.				.07566586	.07837611	.08298324	.09085279	.10482738	.13179671	.19342211	.40692760			
		R.E.				97.54	97.25	96.88	96.54	96.46	97.05	98.65	100.00			
		(σ*1) V.				.08580912	.10194081	.12319669	.15265533	.19648232	.26920271	.41556039	.87312546			
		R.E.				93.42	93.50	93.93	94.73	95.97	97.66	99.58	100.00			
	4	(μ*1) V.					.08009668	.08351673	.09014086	.10361683	.13537627	.24716376				
		R.E.					97.36	97.40	97.49	97.89	98.99	100.00				
		(σ*1) V.					.12350319	.15333514	.19764253	.27106519	.41878810	.88168661				
		R.E.					94.02	94.90	96.20	97.94	99.85	100.00				
	5	(μ*1) V.						.08536772	.09008179	.10250554	.14874275					
		R.E.						97.86	98.35	99.13	100.00					
		(σ*1) V.						.19802892	.27197904	.42061691	.88658942					
		R.E.						96.26	98.05	99.97	100.00					
	6	(μ*1) V.							.09184730	.10169465						
		R.E.							99.17	100.00						
		(σ*1) V.							.42120833	.88884100						
		R.E.							100.00	100.00						

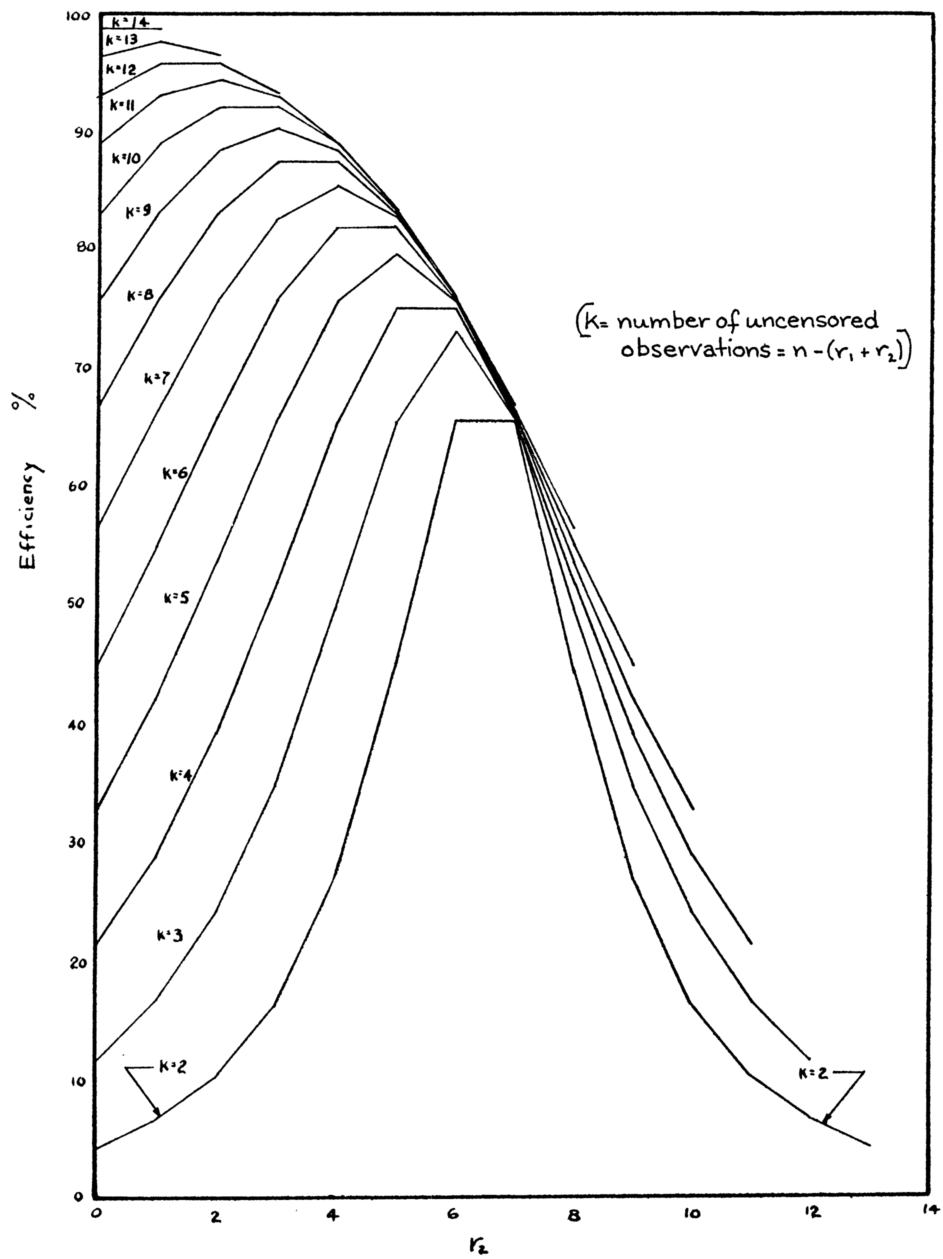

FIG. 1. Relative efficiency of the best linear estimate of μ under all conditions of censoring a sample of size 15 from the normal distribution.

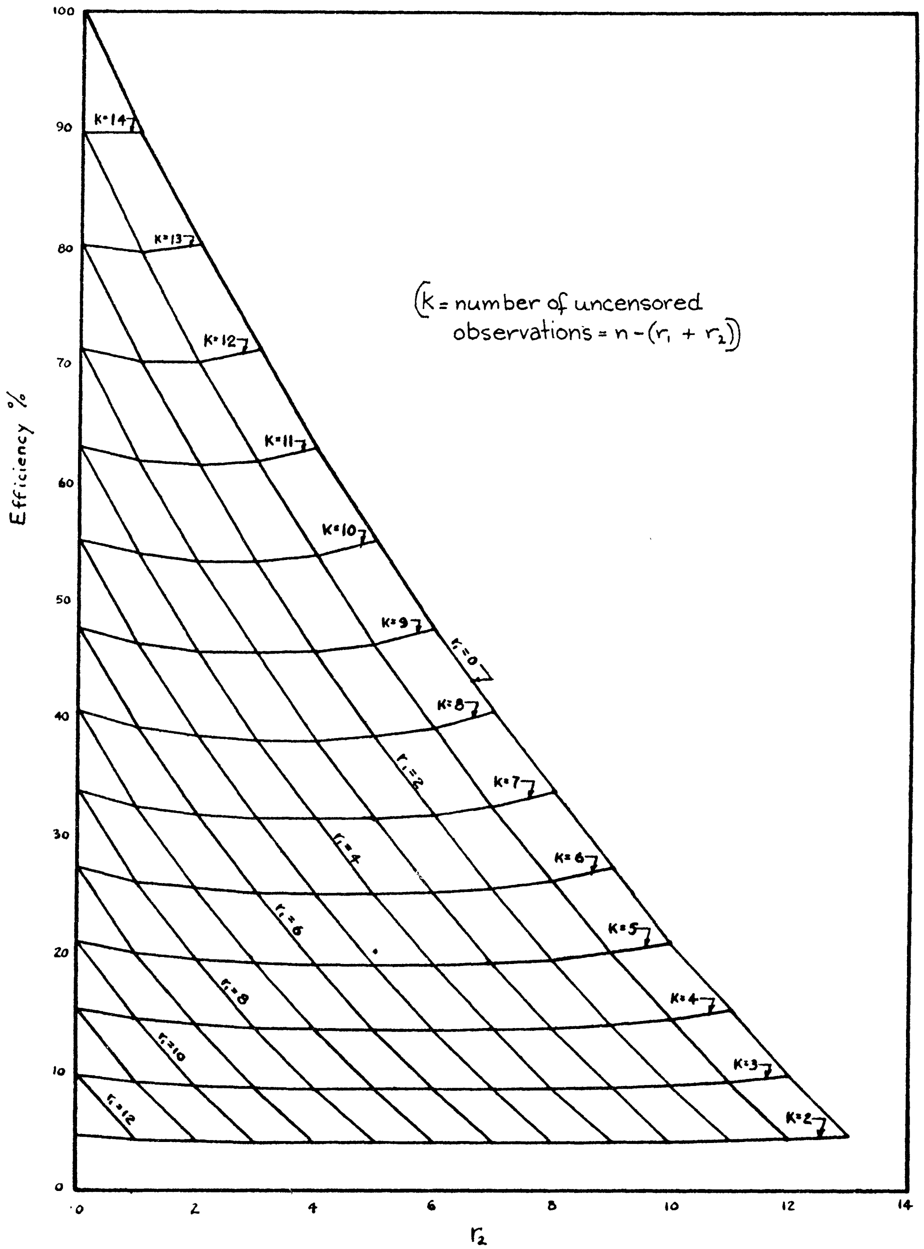

FIG. 2. Relative efficiency of the best linear estimate of σ under all conditions of censoring a sample of size 15 from the normal distribution.

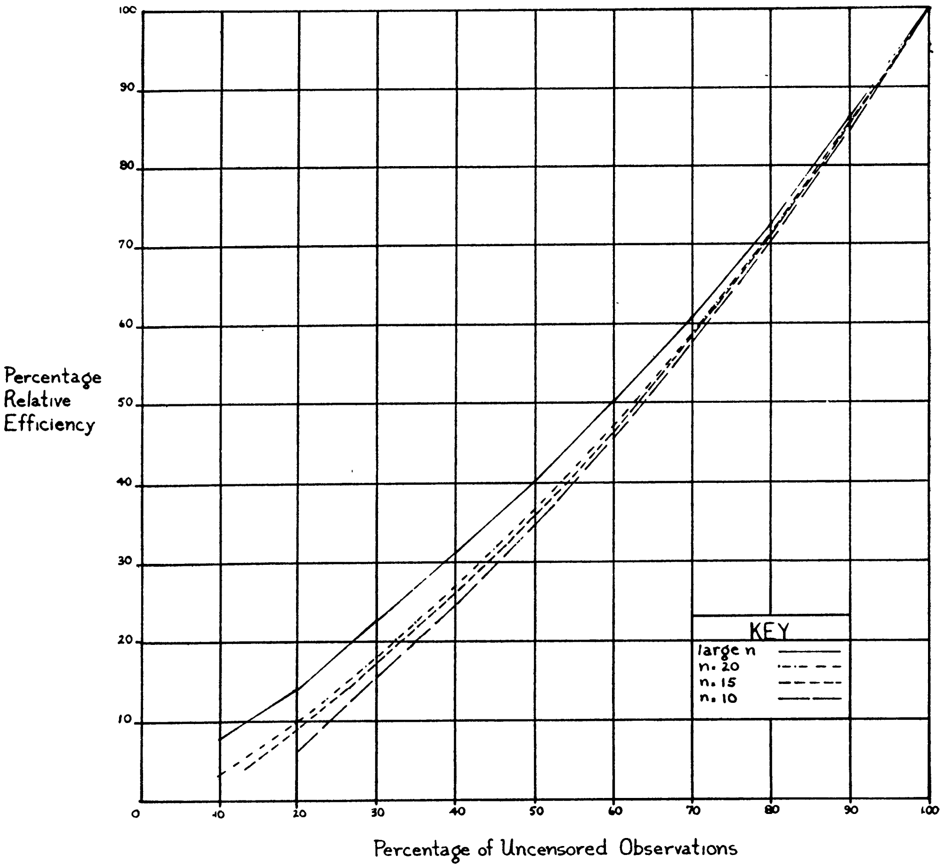

FIG. 3. Approximate efficiencies in estimating σ^* for censored samples of size 10, 15, 20 and large n.

censored sample. The present graphs show that the alternative estimate is even better than previously supposed.

Also, one can observe that for $r_1 = 0$, the alternative estimate of σ is more efficient than the corresponding estimate of the mean. In fact for other values of r_1, the efficiencies are much more concentrated for the former than for the latter. Again, the drop in efficiency for the estimate of the mean is much sharper than that for the standard deviation. This shows that the alternative estimate also appears better if one judges its value by considering its efficiency in estimation of the standard deviation rather than of the mean alone.

Addendum. The extension of Tables I, II, and III of this paper are now available for $16 \leqq n \leqq 20$ in eight decimal places. Copies may be obtained from the authors.

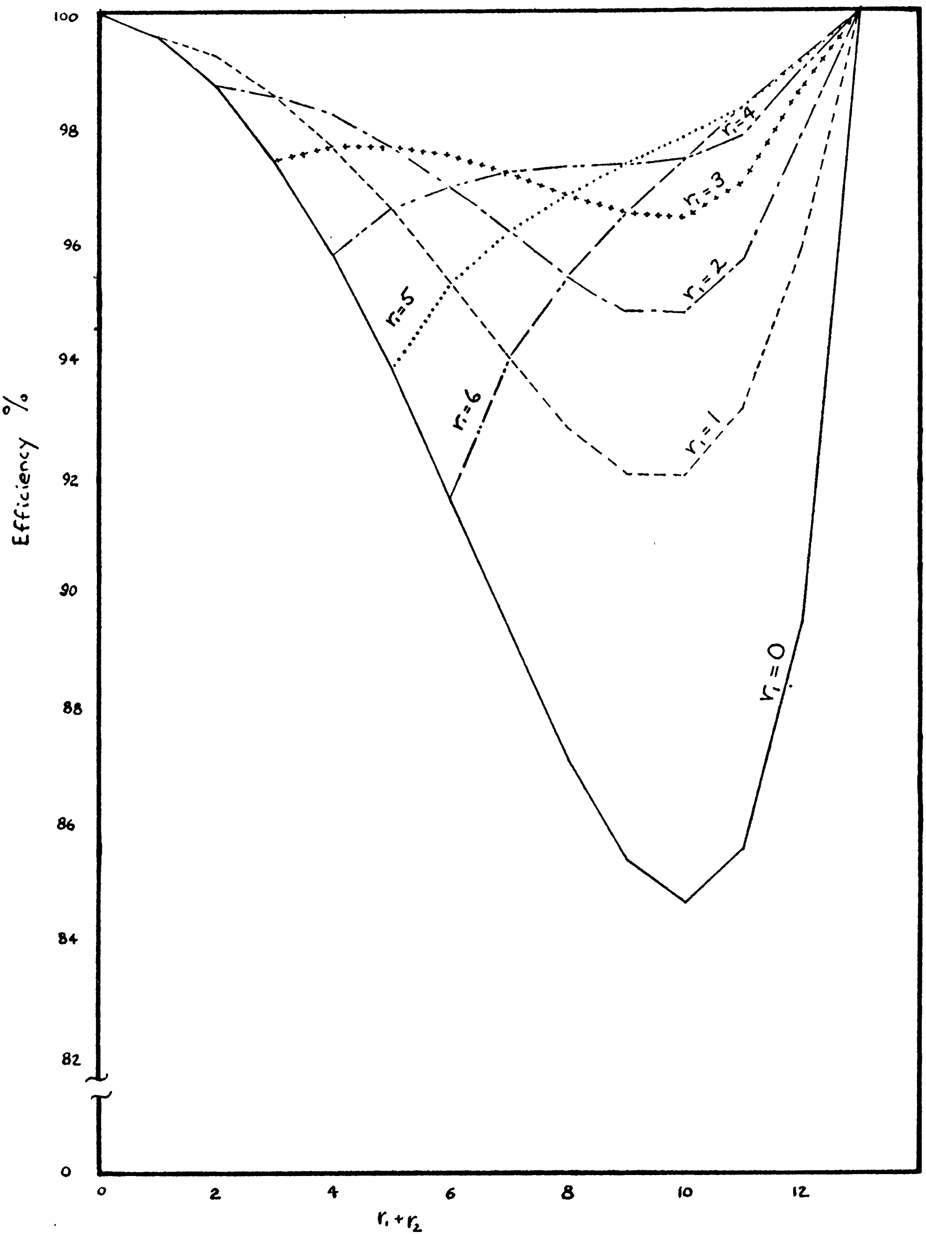

FIG. 4. Relative efficiency of the alternative estimate of μ under conditions of censoring a sample of size 15 from the normal population.

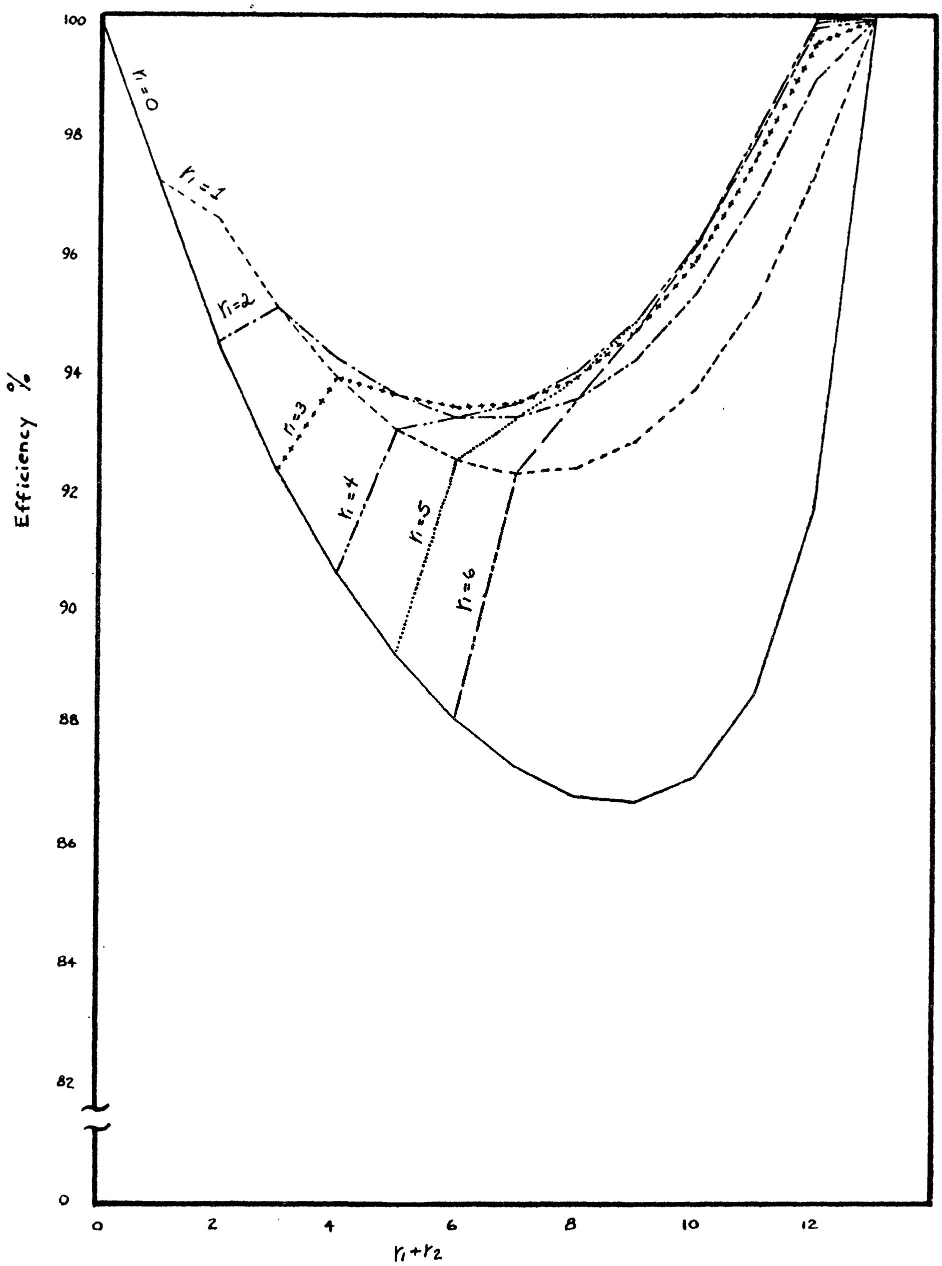

FIG. 5. Relative efficiency of the alternative estimate of σ under conditions of censoring a sample of size 15 from the normal population.

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