p. 310, line 1: change "is independent" to "is the distribution of two independent random variables".

CORRECTION TO "THE WAGR SEQUENTIAL T-TEST REACHES A DECISION WITH PROBABILITY ONE"

By Herbert T. David and William H. Kruskal

Two corrections to the paper of the above title (*Ann. Math. Stat.* Vol. 27 (1956), pp. 797–805) should be made.

(1) Page 803, line after (4.2): $K\sqrt{1 + K^2}$ should be replaced by $K/\sqrt{1 + K^2}$.
(2) Page 804, line 4: $v_n(A_n - R_n)$ should be replaced by $\sqrt{n}\ (A_n - R_n)$.

ABSTRACTS OF PAPERS

(Abstracts of papers presented for the Ames, Iowa Meeting of the Institute, April 3–5, 1958.)

41. Similar Tests of Hypotheses Concerning the Ratio of Mean to Standard Deviation in a Normal Population. Robert A. Wijsman, University of Illinois.

Let $X_1, \cdots, X_N$ be independent $N(\mu, \sigma^2)$ variables, and consider the hypothesis that μ/σ equals a given value against various alternatives. Let

$$T_1 = \sum X_i^2,\ T_2 = \sqrt{N}\,\bar{X}, \qquad T = (T_1, T_2),\ r = \sqrt{N}\,\mu/\sigma.$$

Then the density of T is $c(\sigma, r)h(t) \exp[-(t_1/2\sigma^2) + (r/\sigma)t_2]$ with $h(t) = (t_1 - t_2^2)^{n/2-1}$ if $t_1 \geqq t_2^2$ and $h = 0$ otherwise (we have put $n = N - 1$). Let the hypothesis be $r = r_0$. Associated with the exponential is a differential operator $D = \partial^2/\partial t_2^2 - 2r_0^2\,\partial/\partial t_1$. For a certain class C of functions G of t the test function $\alpha + \phi(t)$ with $\phi = h^{-1}DG$ will be similar and of size α. Conversely, to any similar test function $\alpha + \phi(t)$ there corresponds a $G \varepsilon C$, obtained by considering the differential equation $DG = h\phi$ as a heat (or diffusion) problem in one dimension, with a heat source density $h\phi$ which is a function of both position (t_2) and time (t_1), and solving the equation with help of the usual Green's function for the heat equation. Some of the unsolved problems concerning the search for an optimum similar test are indicated. (Rec. April 3, 1958)

(Abstracts of papers presented for the Los Angeles Meeting of the Institute, December 27–28, 1957.)

42. Demand for and Allocation of Engineering Personnel. I. Estimation of the Demand for Engineering Personnel, and General Formulation of the Allocation Problem. Rajendra Kashyap

Historical data for manpower and costs are analyzed for several types of contracts (prototype, initial, and follow-on contracts) with special regard to routines for (1) dissection of multiphase distributions with overlapping significant phases; (2) determination of standard patterns for incremental and cumulative manpower and costs; (3) estimation of total manpower and costs. As to (1), graphical procedures may be useful (*Gibrat, Daeves,* etc.). For (2), the *Pearson* curve types may be applied, or the *Edgeworth-Kapteyn* system, which is closely related to the application of Hermitian polynomials, a method that for several reasons may deserve preference above all competing devices. (3) is a typical regression problem, the affinity and the effectivity of the chosen approach to be checked by *Fisher's* and *Student's* tests respectively. The problem of allocation of engineering personnel involves the determination of an optimal scheme for the allocation of available personnel to meet the demand for these personnel by the engineering units. This allocation has to be satisfactory under surplus as well as under shortage conditions. The simple consideration of manpower transfer to alternative fields of engineering activities shows clearly that optimization is necessarily an overall group problem. It can be described by an objective function considering competitive ability ratings in various fields, under the aspect of some suitable optimality criterion concerning costs, output or parametric quality-level. Thus the complex problem is formally reduced to one in linear programming. (Received March 14, 1958.)

43. Demand for and Allocation of Engineering Personnel. II. Integral-Valued Solutions of Allocation Problems. HERMAN W. VON GUERARD

Analysis of proportional representation, allocation or elimination of units is bound to integral-valued solutions. In consequence, proportionality, in general, can be approached only, and that leads to a problem of optimization. Unfortunately, that does not provide by itself the criterion for the least deviation from proportionality. Rounding procedures, in general, are not satisfactory. The main issue is, in terms of political elections, that no party is presumed to score less by the only reason that the total number of seats has been increased (postulate of monotony). Other criteria, based on least squares or on minimizing *Gram's* determinant (i.e. maximizing linear dependence), are subject to the same considerations. The best expedient may be seen in requiring maximum likelihood to straight proportionality, and that is equivalent to sampling with replacement (the homogeneous case). The still more important procedure of sampling without replacement leads to *d'Hondt's* scheme (the inhomogeneous case), which is equivalent to maximum likelihood after adding one unit to each of the initial frequencies, i.e. to the popular votes per party. Most of the related theorems can be easily visualized by multidimensional geometry of numbers (*Minkowski*), where *d'Hondt's* method of successive divisions is represented by successive penetrations of a vector through hyperplanes. (Received March 14, 1958.)

(Abstracts of papers presented for the Cambridge, Massachusetts Meeting of the Institute, August 25–30, 1958.)

44. On the Asymptotic Minimax Character of the Sample d.f. of Vector Chance Variables. J. KIEFER AND J. WOLFOWITZ, Cornell University. (By title)

Let $\mathfrak{F}$ (resp., $\mathfrak{F}^c$) denote the class of all d.f.'s (resp., continuous d.f.'s) on Euclidean m-space R^m. Let $X_1, \cdots, X_n$ be independent chance m-vectors with common unknown d.f. F. The space D of decisions (values of the estimate of F) is any space of real functions d on R^m which includes all possible realizations of the sample d.f. S_n of $X_1, \cdots, X_n$. Let ϕ_n^* be the decision function which always makes decision S_n. Dvoretzky, Kiefer and Wolfowitz showed in *Ann. Math. Stat.*, 1956, that, when $m = 1, \phi_n^*$ is asymptotically minimax (as $n \to \infty$) for estimating F in $\mathfrak{F}$ or $\mathfrak{F}^c$, for any of a wide class of loss functions. In the present paper analogous results are proved when $m > 1$, despite the fact that S_n no longer has the distribution-free property it has when $m = 1$. The resulting nonconstancy of the risk function $r(F, \phi_n^*)$ for F in $\mathfrak{F}^c$ and even the simplest loss functions, presents new difficulties in the minimax proof when $m > 1$: for example, the method of proof necessitates showing that $r(F, \phi_n^*)$ approaches a limit as $n \to \infty$, *uniformly* for F in an appropriately dense subset of $\mathfrak{F}^c$; the authors' results in *Trans. Amer. Math. Soc.*, 1958, are used in proving this. (Received March 21, 1958.)

45. **Optimum Designs in Regression Problems.** J. KIEFER AND J. WOLFOWITZ, Cornell University. (By title)

Suppose Y_{x_i}, $i = 1, \cdots, n$, are independent random variables with $EY_x = \sum_1^k a_j f_j(x)$ for $x \in \mathfrak{X}$, where the f_j are known and the a_j are the unknown regression coefficients; Var $(Y_x) = v(x)\sigma^2$, where v is known. We consider the optimum allocation of the x_i for problems of statistical inference (1) about a_k, (2) about the s parameters $a_{k-s+1}, \cdots, a_k$, (3) about the whole function $\sum a_j f_j$. Algorithms are obtained which facilitate the computation of optimum designs (for several different optimality criteria, in the case of (2)). Examples are given which show the great simplification to be achieved by the use of these algorithms, over a more direct approach. For example, in case (1) the problem is solved by finding the best Chebyshev approximation to f_k of the form $\sum_1^{k-1} c_j f_j$ and locating the x_j, with appropriate frequencies, at points of maximum absolute deviation of the best approximation from f_k; in the example $\mathfrak{X} = [-1, 1]$, $f_j(x) = x^{j-1}$, $k = h + 1$, the optimum design locates a fraction $1/h$ of the observations at each of -1 and 1 and a fraction $1/2h$ of the observations at $\cos(j\pi/h)$, $1 \leq j \leq h - 1$, and, as h increases, the relative efficiency of the often used "equal spacing" designs tends rapidly to zero. (Received April 17, 1958.)

46. **Uniqueness of the L_2 Association Scheme.** S. S. SHRIKHANDE, University of North Carolina.

A partially balanced incomplete block design with $v = s^2$ treatments is said to have L_2 association scheme (R. C. Bose and T. Shimamoto, *Journal of the American Statistical Association*, 47: 151–184, 1952), if the treatments can be arranged in an $s \times s$ square such that any two treatments in the same row or the same column are 1-associates, whereas all the other pairs are 2-associates. In this case it is easily seen that $n_1 = 2s - 2$, $n_2 = (s - 1)^2$, $p_{11}^1 = s - 1$, $p_{11}^2 = 2$, where the symbols have the usual meanings. It is now proved that for a P.B.I.B. with s^2 treatments with the above values for n_1, n_2, p_{11}^1 and p_{11}^2, the association scheme is of L_2 type for all $s \geq 3$ excepting $s = 4$. It can be shown that a necessary condition for existence of a symmetrical P.B.I.B. with above parameters, when s is even, is that $r - 2\lambda_1 + \lambda_2$ must be a perfect square and further $(r - \lambda_1 + (s - 1)(\lambda_1 - \lambda_2), -1) = 1$ for every odd prime p, where the last symbol stands for the Hilbert norm-residue symbol. The result contained in the last sentence, can also be obtained from a paper submitted by M. N. Vartak to the *Annals of Mathematical Statistics*. Here r, λ_1, λ_2 have the usual meaning. (Received May 26, 1958.)

47. **On the existence of Wald's sequential test.** ROBERT A. WIJSMAN, University of Illinois.

In the literature on Wald's sequential probability ratio test the question of existence of stopping bounds, given the two error probabilities, has never been answered. Granted existence, the uniqueness has been shown by L. Weiss (*Ann. Math. Stat.* Vol. 27 (1956) pp. 1178–1181) in the case that the probability ratio is continuous. Let α_1, α_2, be the two error probabilities, and let $\alpha = (\alpha_1, \alpha_2)$. In the case of continuous probability ratio, and in the discrete case with suitable randomization, α_1 and α_2 are continuous functions of the stopping bounds. Let C be the non-increasing (and convex) curve of points α produced by coincident stopping bounds, and let A be the set in the α-plane bounded by C and the coordinate axes. Consider a point (α_1^*, α_2^*) on C, and separate the stopping bounds in a way which keeps α_1 constant. Since α_2 is a continuous function of the separation d between the bounds, with $\alpha_2(0) = \alpha_2^*$, $\alpha_2(\infty) = 0$, every value α_2 between 0 and α_2^* is assumed for some d. It follows that for every α in A there exist stopping bounds. In the continuous case it is known from Weiss' work that α_2 decreases monotonically from α_2^* to 0, as d increases from 0 to ∞. In that case, for the existence of stopping bounds it is also necessary that $\alpha \in A$. (Received August 16, 1957; revised June 16, 1958.)

NEWS AND NOTICES

Readers are invited to submit to the Secretary of The Institute news items of interest

Personal Items

Gertrude Mary Cox, director of the Institute of Statistics, Consolidated University of North Carolina, was awarded an honorary Doctor of Science degree by Iowa State College during its Founder's Day centennial observance; she was cited as "teacher, researcher, leader and administrator in the field of statistics."

George Waddel Snedecor, who was primarily responsible for the development of the Iowa State College Statistical Laboratory, was awarded an honorary Doctor of Science degree by the college during its Founder's Day centennial observance and cited as "teacher, author, pioneer in experimental statistics." He has been a visiting professor at North Carolina State College, in the Institute of Statistics, since 1957.

Allan G. Anderson has resigned his position as Chief Statistician at the General Tire & Rubber Company, Akron, Ohio, to accept a position as Professor and Head of the Department of Mathematics at Western Kentucky State College, Bowling Green, Kentucky.

Dr. Ernst P. Billeter has been appointed Professor of Statistics and Automation at the University of Fribourg (Switzerland). He has also been elected Director of the Institute for Research in Automation, which has recently been founded at this University. The aim of this Institute is to do basic research work in application of automation in business and to introduce businessmen and their staff members, as well as students in economics, into the general methods of programming electronic data processing machines. Furthermore, this Institute will help businessmen in solving their problems in operations research, market research, and statistical quality control.

Dr. Uttam Chand has a new position as Officer on Special Duty (Training) in the Central Statistical Organisation (Cabinet Sectt.), New Delhi, India.

Dr. Frank A. Haight, formerly of Auckland University College, New Zealand, has returned to the United States to become Associate Mathematician at the Institute of Transportation and Traffic Engineering, U. C. L. A.