ABSTRACTS OF PAPERS

(Additional abstract of a paper presented at the Pittsburgh meeting of the Institute, March 19–21, 1959.)

33. The Comparison of the Sensitivities of Similar Experiments: Model II of the Analysis of Variance.

D. E. W. SCHUMANN AND R. A. BRADLEY, University of Stellenbosch, South Africa, and Virginia Polytechnic Institute.

When alternative scales of measurement or experimental techniques are available for experimentation, it is desirable to use that scale or technique more sensitive to the exhibition of treatment differences in the experiment. Sometimes it will be desirable to do preliminary experimentation in order to choose the more sensitive technique.

We consider parallel experiments with samples from the same experimental treatments and similar experimental designs. The experiments must be conducted so as to be independent in probability and to be appropriate for use of analysis of variance.

In earlier work we have considered Model I of the analysis of variance [*Biometrics* 13(4), 1957] and the comparison was based essentially on the distribution of the ratio of the F-statistics from the two experiments, each F assumed to have the non-central variance-ratio distribution. The distribution of the ratio was approximated by the distribution of the ratio of two central variance ratios with appropriately adjusted degrees of freedom.

We consider Model II of the analysis of variance here and the more sensitive procedure is the one with the relatively larger component of variance for treatments. Now the ratio of two central variance ratios is directly required. Additional tables have been provided for use with either Model I or Model II and an example is included. A confidence interval is also provided.

(Abstracts of papers not presented at any meeting of the Institute.)

1. Simultaneous Comparison of Tests.

R. R. BAHADUR, Indian Statistical Institute. (By title)

Let Ω be a parameter space of points θ and let z be a sample point with distribution P_θ. Let Ω_0 be a subset of Ω. A sequence $T^{(1)}(Z), T^{(2)}(Z), \cdots$, of real valued statistics is then said to be standard (for testing Ω_0) if (I) $\lim_{n\to\infty} P_\theta(T^{(n)} \geqq x) = L(x)$ (say) for every x and every $\theta \varepsilon \Omega_0$, (II) $\log L(x) = -ax^2[1 + O(1)]$ as $x \to \infty$, where $0 < a < \infty$, and (III) for each θ in $\Omega - \Omega_0$, $T^{(n)}/n^{\frac{1}{2}} \to b(\theta)$ with probability one as $n \to \infty$, where $0 < b < \infty$. It is pointed out that if $\{T_1^{(n)}\}$ and $\{T_2^{(n)}\}$ are standard sequences then, for any given $\theta \varepsilon \Omega - \Omega_0$, $\varphi = a_1b_1^2/a_2b_2^2$ serves as the asymptotic efficiency of $\{T_1^{(n)}\}$ relative to $\{T_2^{(n)}\}$, with attainment of an assigned significance level as the criterion. In particular,

$$L_1(T_1^{(n)})/L_2(T_2^{(n)}) \to \infty \; (0)$$

with probability one as $n \to \infty$ if $\varphi < 1 \; (>1)$. Again, with $N_i^- = \inf\{m: L_i(T_i^{(m)}) \leqq l\}$ and $N_i^+ = \inf\{m: L_i(T_i^{(n)}) \leqq l \text{ for all } n \geqq m\}$, $(i = 1, 2)$, both ratios N_2^-/N_1^+ and $N_2^+/N_1^- \to \varphi$ with probability one as $l \to 0$. The paper discusses, as examples, the simultaneous comparisons of the sign and t tests of a mean; of the Wald-Wolfowitz test, the Smirnov test, and the t test for two samples; and of the Kruskal-Wallis test and the F test for k samples. The first mentioned example is also studied, under normality assumptions and using the exact levels $L_i^{(n)}(T_i^{(n)})$ rather than the asymptotic levels $L_i(T_i^{(n)})$, in another paper (*Ann. Math. Stat.*, 1959, p. 623, (abstract)).

2. Mill's Ratio and Linear Truncation for Some Pearson Curves. WILLIAM FELLING AND WALDO A. VEZEAU, St. Louis University. (By title)

Any statistical distribution is completely determined when the parameters of the distribution are known. The determination of these parameters, when some variates are deliberately excluded from the sample population, presents an interesting problem in statistical point estimation. The author has developed estimate equations of the parent population parameters when samples are assumed taken from a Beta distribution, and the sample has been truncated on either or both tails of the distribution. The estimate equations are developed using methods of maximum likelihood. Since these equations involve Mill's Ratio of truncated area to bounding ordinates, the use of continued fractions is employed to obtain bounds on this ratio. These bounds are obtained from approximants to the continued fractions and the use of successively higher ordered approximants increases the accuracy of the bounds as estimates of Mill's Ratio.

3. On Cumulative Function Theory. JOSEPH M. MOSER AND WALDO A. VEZEAU, St. Louis University. (By title)

New cumulative functions are developed in this paper by using the Bernoulli differential equation, $dy/dx + Py = Qy^n$, where y is the cumulative frequency function $F(x)$. This is a generalization of Burr's differential equation.

The nth cumulative moments for some of these newly developed cumulative frequency functions and for cumulative frequency functions developed by other authors are discussed.

It is shown that when the method of curve fitting by using moments fails, other methods can be used, such as, interpolation, ratio M and others. Ratio M is developed because the function $[x^{-k} + 1]^{-r}$ could not be handled in curve fitting by moments. Ratio M is the ratio of the abscissa of the mode to the abscissa of the median. By means of a chart of values of M the values of the parameters are chosen.

For purposes of simplification of theory, the Stieltjes integral is introduced to define cumulative moments.

Finally, a discussion of reliability functions is presented. A reliability function is defined to be $R(x) = 1 - F(x)$. The moments defined for $R(x)$ are very similar to those for cumulative functions and can be expressed in terms of cumulative moments of $F(x)$.

4. On Sampling Distributions Derived by Cumulative Characteristic Function Methods (Preliminary Report). JOSE R. PADRO AND WALDO A. VEZEAU, St. Louis University. (By title)

New relations and properties of cumulative characteristic functions are presented. Derivations of sampling distributions for particular cumulative functions are given.

5. An Extension of the Theory of Cumulative Frequency Functions to N Variables. SR. MARY ALBERTA UZENDOSKI AND WALDO A. VEZEAU, St. Louis University. (By title)

It is the purpose of this work to generalize the existing theory of moments, characteristic functions, and moment-generating functions to multivariate cumulative distributions. Statistical independence and dependence served as a basis for further subdivision of the extension.

For the independent case the moment theory consisted of a definition of a cumulative moment about the origin, as well as about $x_i = a_i$, a more general point than heretofore given.

Unlike the earlier definition of a cumulative moment for the dependent case, the moment was defined in terms of marginal distributions. Thus the definitions for special functions to be used in the formulation of the moment definition were eliminated. Although the definitions of the moments for the two cases differ, they are composed of the same number of terms; and, if statistical independence is assumed in the dependent case, it reduces to the cumulative moment as defined for a function of n independent variables. Moreover, the relation between cumulative moments was found to be in accord with that for the independent case.

Similarly, an extension was made of the theory of cumulative characteristic and moment-generating functions found in previous theoretical work.