PROBABILITY INTEGRALS OF MULTIVARIATE NORMAL AND MULTIVARIATE t[1]

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1. Introduction. The evaluation of multivariate normal probability integrals is of special importance to the statistician dealing with multivariate problems. The joint distribution of several dependent continuous variates is often assumed to be a multivariate normal distribution, and the multivariate normal distribution also provides an approximation to the multinomial distribution for a large sample size. Applications of the multivariate normal distribution are numerous throughout statistical literature. An extensive list of applications of the bivariate normal probability distribution is given by D. B. Owen [8].[3] The present paper gives a survey of the work on multivariate probability integral and related functions starting with the bivariate case and includes the author's recent work on the probability integrals of the multivariate normal and a multivariate analogue of Student's t. Two new tables of the probability integrals of the equicorrelated multivariate normal are given at the end of the paper (Tables I and II).

2. Bivariate normal integral. W. F. Sheppard was perhaps the first statistician who concerned himself with the evaluation of the bivariate probability integral. In his paper [11] published in 1900, Sheppard obtained the exact probability that both the standardized correlated normal variates are positive and also discussed the calculation of certain V functions for evaluating the probability integral in general. In 1901, Pearson [31] published a method for evaluating the integral as a power series in ρ involving tetrachoric functions. The function computed by Pearson and his associates is:

$$\frac{d}{N} = \int_h^\infty \int_k^\infty f(x, y; \rho)\, dx\, dy \tag{1}$$

where

$$f(x, y; \rho) = [1/2\pi(1 - \rho^2)^{\frac{1}{2}}] \exp [-\tfrac{1}{2}\{x^2 - 2\rho xy + y^2\}/(1 - \rho^2)].$$

Pearson obtained the following tetrachoric series expansion:

$$d/N = \sum_{j=0}^{\infty} \rho^j \tau_j(h) \tau_j(k) \tag{2}$$

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[3] The numbers in brackets [ ] refer to the separate bibliography on the multivariate normal integrals and related topics which follow this paper.

where $\tau_j(x)$ is

$$\tau_j(x) = j\text{th tetrachoric function} = H_{j-1}(x)e^{-x^2/2}/[(j!)^{\frac{1}{2}}(2\pi)^{\frac{1}{2}}] \tag{3}$$

and $H_j(x)$ is the Hermite polynomial defined by

$$\begin{aligned} H_j(x) &= e^{x^2/2}[(-D)^j e^{-x^2/2}] = x^j - \binom{j}{2} x^{j-2} + \binom{j}{4} 3x^{j-4} \\ &+ \cdots + (-1)^s \binom{j}{2s} 3\cdot 5 \cdots (2s-1)x^{j-2s} + \cdots. \end{aligned} \tag{4}$$

The series (2) converges too slowly for high values of ρ and thus is not very satisfactory. For high values of $|\rho|$, the table of d/N was calculated by quadrature using the relation

$$\frac{d}{N} = \frac{1}{2\pi}\int_h^\infty e^{-x^2/2}\,dx \int_{(k-\rho x)/(1-\rho^2)^{\frac{1}{2}}}^\infty e^{-y^2/2}\,dy. \tag{5}$$

The function d/N is tabulated in Pearson's *Tables for Statisticians and Biometricians*, Part II, (1938), where references are given to several papers published in Biometrika, which originally contained tables of d/N for selected values of h, k, and ρ. The tetrachoric functions are also tabulated up to τ_{19}. Pearson's tables for d/N are given for $h, k = 0(.1)2.6$, $\rho = -1(.05)1$, to six decimal places for positive ρ and to seven decimal places for negative ρ. Pearson was interested in computing the tetrachoric correlation coefficient from a four-fold table under the assumption that the variates follow the normal distribution. If ρ is the correlation in the bivariate normal population and the observed cell frequencies in the first and second row are (a, b) and (c, d), respectively, such that $a + b + c + d = N$, then we can get an estimate ρ_t of ρ. (Pearson used the symbol r instead of ρ) from a solution of the Equation (2). Hence the notation d/N instead of our $L(h, k; \rho)$. In terms of the dichotomy represented by the four-fold table h and k are the solutions of

$$\int_h^\infty \frac{e^{-x^2/2}}{(2\pi)^{\frac{1}{2}}}\,dx = \frac{b+d}{N}, \qquad \int_k^\infty \frac{e^{-x^2/2}}{(2\pi)^{\frac{1}{2}}}\,dx = \frac{c+d}{N}.$$

It should be pointed out that tetrachoric series has been discovered by many writers. Apparently the priority goes to Mehler [26]. Also the exact relation for the probability that both the correlated random variables are positive is given by Stieltjes [203] as mentioned by Cramér [200], page 290. This result is

$$\int_0^\infty \int_0^\infty f(x, y; \rho)\,dx\,dy = \tfrac{1}{4} + (1/2\pi) \arcsin \rho. \tag{6}$$

C. Nicholson considered the evaluation of the probability integral for two variables in 1943 [7] (and a geometrical approach in an earlier paper [202] in 1941). He discussed the properties of the V function given by

$$V(h, q) = \int_0^h f(x)\,dx \int_0^{qx/h} f(y)\,dy, \tag{7}$$

where

$$f(x) = [1/(2\pi)^{\frac{1}{2}}]e^{-x^2/2}.$$

The function V depends on only two parameters h and q. Nicholson [7] tabulated the function $V(h, q)$ for $h, q = 0(.1)3$ and for $q = \infty$, to six decimal places.

Nicholson used the following expansion for computing $V(h, q)$. When $q < h$

$$V(h, q) = \frac{1}{2\pi}\left\{B_0 \frac{q}{h} - B_1 \frac{q^3}{3h^3} + B_2 \frac{q^5}{5h^5} - B_3 \frac{q^7}{7h^7} + \cdots\right\}, \tag{8}$$

where

$$B_n = \frac{1}{n!\,2^n}\int_0^h x^{2n+1}e^{-x^2/2}\,dx = 1 - e^{-h^2/2}\sum_{j=0}^{n}\frac{h^{2j}}{j!\,2^j} \tag{9}$$

and used the identity

$$V(h, q) + V(q, h) = \int_0^h f(x)\,dx \int_0^q f(y)\,dy \tag{10}$$

for obtaining $V(h, q)$ when $q > h$. Incidentally, Nicholson's table can also be used to find the distribution of the ratio of two normal chance variables. The following two approximations for $V(h, q)$ were obtained by Cadwell [5]

$$V(h, q) \simeq (\alpha/2\pi)\{1 - \exp(-hq/2\alpha)\} \tag{11}$$

$$V(h, q) \simeq \frac{\alpha}{2\pi}\left[1 - \exp\left\{-\frac{h^2 \tan\alpha}{2\alpha} + h^4\frac{\alpha\tan\alpha(3 + \tan^2\alpha) - 3\tan^2\alpha}{24\alpha^2}\right\}\right] \tag{12}$$

where $\alpha = \tan^{-1}(q/h)$. Some results on the maximum error of these approximations were also given by Cadwell [5].

In 1943, H. H. Germond and his associates C. Hastings and M. Piedam started the tabulation of $V(h, q)$. This table [6] was extended by Germond to cover $V(h, q)$ for $h = 0(.01)4$ and $\lambda = q/h = .1(.1)1$ to 5 decimal places. Germond's tables of $V(h, \lambda h)$ and their extension by the National Bureau of Standards are given in the Tables of the Bivariate Normal Distribution and Related Functions [2]. These tables [2] were issued in 1959 and tabulate the function $L(h, k; \rho)$ which represents the probability content of an infinite rectangle $[h \leqq x < \infty, k \leqq y < \infty]$ with sides parallel to the axes. The ranges of tabulation is $h, k = 0(.1)4$, $\rho = 0(.05).95(.01)1$, $6D$ and $h, k = 0(.1)h_n, k_n$, $-\rho = 0(.05).95(.01)1$; $7D$ where $L(h_n, k_n; -\rho) \leqq \frac{1}{2}\cdot 10^{-7}$ if h_n and k_n are both less than 4 (Tables I and II with r for ρ). The functions $V(h, \lambda h)$ and $V(\lambda h, h)$ are tabulated for $h = 0(.01)4(.02)4.6(.1)5.6$ and ∞; $\lambda = .1(.1)1$; $7D$ with last place uncertain by two units (Tables III and IV).

We will now summarize the formulas connecting the V function and the L

function defined by

$$L(h, k; \rho) = \int_h^\infty dx \int_k^\infty f(x, y; \rho)\, dy = L(k, h; \rho). \tag{13}$$

$$L(h, k; \rho) = V\{h, [(k - \rho h)/(1 - \rho^2)^{\frac{1}{2}}]\} + V\{k, [(h - \rho k)/(1 - \rho^2)^{\frac{1}{2}}]\} + H \tag{14}$$

where

$$H = -\tfrac{1}{2}[F(h) + F(k) - \tfrac{3}{2}] + \text{arc sin}\, \rho/2\pi \quad \text{and} \quad F(x) = [1/(2\pi)^{\frac{1}{2}}] \int_{-\infty}^{x} e^{-y^2/2}\, dy. \tag{15}$$

It should be pointed out here that in connection with some practical projects, tables of the bivariate normal probability distribution [4] were also computed in the Statistical Laboratory, University of California by L. A. Aroian, E. Fix and M. Johnsen. These tables [4] have been included in the National Bureau of Standards Tables [2]. They were reviewed in MTAC (See [4]).

D. B. Owen considered the evaluation of the integral of the bivariate normal in a research report [8] issued in August 1956. He denoted the probability content over the infinite rectangle $[-\infty, h; -\infty, k]$ by $B(h, k; \rho)$ and introduced the related function $T(h, a)$. These functions are given by

$$B(h, k; \rho) = \frac{1}{2\pi(1 - \rho^2)^{\frac{1}{2}}} \int_{-\infty}^{k} dy \int_{-\infty}^{h} f(x, y; \rho)\, dx \tag{16}$$

$$T(h, a) = \frac{1}{2\pi} \int_0^a \frac{\exp\{-\frac{1}{2}h^2(1 + x^2)\}}{1 + x^2}\, dx. \tag{17}$$

The relationship between the T and V function is

$$T(h, a) = \text{arc tan}\, a/2\pi - V(h, ah). \tag{18}$$

Owen derived various relations connecting the $B(h, k; \rho)$ and $T(h, a)$. His derivations are analytical in contrast to the heavily geometrical ones by Sheppard and Nicholson. The function $T(h, a)$ and its differences are tabulated by Owen in [8] and [9] (with different intervals of tabulation in [9] for $a = 0(.025)1, \infty$, and $h = 0(.01)3.50(.05)4.75$. Also $T(0, a)$ is tabulated separately for $a = 0(.005)1$. Using his tables and relations one can obtain the volumes under a bivariate surface over any polygon. It should be noted that Cadwell [5] had earlier described the use of the V-function to find volumes under a bivariate surface over any polygon.

Two approximations with some discussion of their maximum absolute errors have been given for the $T(h, a)$ function by Mallows [24] who also obtained upper bounds for this function. For $h \geqq 0$, $0 \leqq a < 1$, the two approximations given by Mallows straddle the true values and have maximum absolute errors .0078 and .0014 respectively at $h = 0$. Both his approximations and upper bounds are given as functions of the cumulative distribution function and density of the unit normal variate. We give below the two approximations $T^*(h, a)$ and $T^{**}(h, a)$.

For $-\infty < h, a < \infty$

$$T(h, a) \simeq T^*(h, a) = \{\tfrac{1}{2} - G(|h|)\}G(am(|h|)) \tag{18a}$$

$$\text{where} \quad m(h) = G^{-1}(\tfrac{1}{4}) + G^{-1}(G(h)/2),$$

$$\text{and where} \quad G(x) = \int_0^x g(t)\, dt \quad \text{and} \quad g(t) = [1/(2\pi)^{\frac{1}{2}}]e^{-t^2/2}.$$

For $|a| < 1$, an improved approximation

$$T(h, a) \simeq T^{**}(h, a) = \{\tfrac{1}{2} - G(h)\}G\{a\alpha(h) - a^3\beta(h)\} \tag{18b}$$

$$\text{where} \quad \alpha(h) = 2g(h)/(1 - 2G(h)) \quad \text{and} \quad \beta(h) = \alpha(h) - m(h).$$

Ruben [37] showed that the evaluation of the integral $L(h, k; \rho)$ over the infinite rectangle is essentially equivalent to the evaluation of the probability content of an off-center sector under a centered standard circular normal distribution. He introduced $W(c_0, \theta)$ to denote the latter when the vertex C of the sector is at a distance c_0 from the origin and θ is the angle subtended at C by the two bounding lines, one of which coincides with one of the new axes. The two parameter function $W(c_0, \theta)$ is related to the T and V functions as follows.

$$T(c_0 \sin\theta, \cot\theta) + W(c_0, \theta) = \tfrac{1}{2} - \tfrac{1}{2}F(c_0 \sin\theta), \qquad 0 \leqq \theta \leqq \pi/2 \tag{19}$$

$$V(c_0 \sin\theta, c_0 \cos\theta) - W(c_0, \theta) = -\tfrac{1}{4} - \theta/2\pi + \tfrac{1}{2}F(c_0 \sin\theta), \qquad 0 \leqq \theta \leqq \pi/2. \tag{20}$$

Ruben gave a series expansion for $W(c_0, \theta)$ similar to the series expansions given by Owen and Nicholson for $T(h, a)$ and $V(h, \lambda h)$, respectively. Ruben also obtained a continued fraction expansion for $W(c_0, \theta)$.

Pólya's inequalities and approximation for the bivariate integral. At the first Berkeley Symposium held in 1945 and 1946, Pólya [10] presented a paper in which he gave three results related to the bivariate normal integral. He defined a function $R(h, k)$ which is related to the V function as

$$R(h, k) = V(h, k) - \tfrac{1}{2}\int_0^h f(x)\, dx + (1/2\pi) \arctan (h/k) \tag{21}$$

and obtained an enveloping divergent series for $R(h, k)$ when $h, k > 0$. This essentially reproduces an expansion first given by Sheppard [11]. Note that the function $R(h, k)$ of formula (21) is the same as function $W(c_0, \theta)$ defined earlier.

Pólya also obtained inequalities (bounds) for $L(h, k; \rho)$ and gave an approximation for $L(a, a'; b, b'; \rho) = \int_a^b \int_{a'}^{b'} f(x, y; \rho)\, dx\, dy$.

3. Generalization of the tetrachoric series. Kendall [57] discussed the generalization of the tetrachoric series to more than two variables and gave explicit results for the case of three variables. (See (23) below.) A generalization similar to Kendall's [57] but for the density function was given by Kibble [21]. Using a term by term integration of Kibble's series, Somerville [42] evaluated the multivariate normal integral for the case of equal correlation. Kibble's generalization of Mehler's theorem [26] gives the standardized multivariate normal density function as

$$
\begin{aligned}
f(x_1, x_2, \cdots, x_n; \{\rho_{ij}\}) &= \frac{1}{(2\pi)^{n/2}} \exp\left(-\tfrac{1}{2}\sum_{i=1}^{n} x_i^2\right) \\
&\quad \cdot \sum_{r=0}^{\infty} \left\{\frac{1}{r!} \sum C \rho_{ij}\, \rho_{kl} \cdots\cdot H_{h_1}(x_1) \cdots H_{h_n}(x_n)\right\}.
\end{aligned} \tag{22}
$$

For the summation inside the curly brackets on the right hand side of (22) we have the Conditions (i) the number of the ρ's is r; (ii) the summation is over all different sets $i, j, k, l, \cdots$ from 1 to n subject only to the condition $i \neq j, k \neq l$: (iii) C is the number of possible permutations of $\rho_{il}, \rho_{kl}, \cdots$; that is $C = r!/(p!\, q! \cdots)$ if p of the ρ's have the same pair of subscripts i, j; q have another pair and so on; and (iv) h_t is the total number of times t occurs among the subscripts $i, j, k, \cdots$. We will now state Kendall's result for the integral for the case of three variables, viz.

$$
\begin{aligned}
&\int_{h_1}^{\infty}\int_{h_2}^{\infty}\int_{h_3}^{\infty} f(x, y, z; \{\rho_{ij}\})\, dx\, dy\, dz \\
&\quad = \sum \frac{\rho_{12}^{j}\, \rho_{23}^{k}\, \rho_{13}^{l}}{j!\, k!\, l!} [H_{j+k-1}(h_1) f(h_1) H_{j+l-1}(h_2) f(h_2) H_{k+l-1}(h_3) f(h_3)].
\end{aligned} \tag{23}
$$

Moran [61] gave the tetrachoric expansion for four variables similar to above for the case when all the lower limits are equal to zero. His result is

$$
\begin{aligned}
&\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\int_0^{\infty} f(x_1, x_2, x_3, x_4; \{\rho_{ij}\})\, dx_1\, dx_2\, dx_3\, dx_4 \\
&= \sum_{l,m,n,p,q,r=0}^{\infty} (-1)^{l+m+n+p+q+r} \frac{G_{l+m+n}\, G_{l+p+q}\, G_{m+p+r}\, G_{n+q+r}}{l!\, m!\, n!\, p!\, q!\, r!} \\
&\qquad\qquad \cdot \rho_{12}^{l}\, \rho_{13}^{m}\, \rho_{14}^{n} \rho_{23}^{p}\, \rho_{24}^{q}\, \rho_{34}^{r} \\
&= \tfrac{1}{16} + (1/8\pi)(\rho_{12} + \rho_{13} + \rho_{14} + \rho_{23} + \rho_{24} + \rho_{34}) \\
&\qquad + (1/4\pi^2)(\rho_{12}\,\rho_{34} + \rho_{13}\,\rho_{24} + \rho_{14}\,\rho_{23}) + \cdots.
\end{aligned} \tag{24}
$$

In (24), $G_0 = \frac{1}{2}$ and

$$
\begin{aligned}
G_s &= 0 \quad \text{if } s \text{ is even,} \\
&= (2m)!(-i)/((2\pi)^{\frac{1}{2}} 2^m (m!)) \quad \text{if } s \text{ is odd and equal to } 2m + 1.
\end{aligned}
$$

4. The trivariate normal integral. The evaluation of the trivariate integral in terms of tetrachoric series expansion as given by Kendall [57] is stated under the generalizations of tetrachoric series expansions above (see (23)) in this paper. It is well known that the tetrachoric series expansions for the multivariate normal integral converge only very slowly for high values of $|\rho_{ij}|$.

Owen [8] briefly discussed the evaluation of the trivariate integral for the general case (particular cases of the trivariate, are discussed above and below) and gave the formula for the standardized trivariate normal integral as follows:

$$\begin{aligned}(25)\quad &\int_{-\infty}^{j}\int_{-\infty}^{k}\int_{-\infty}^{h} f(x, y, z; \{\rho_{ij}\})\, dx\, dy\, dz \\ &= \int_{-\infty}^{j} f(z)B\left(\frac{k-\rho_{23}z}{(1-\rho_{23}^2)^{\frac{1}{2}}}, \frac{h-\rho_{13}z}{(1-\rho_{13}^2)^{\frac{1}{2}}}; \frac{\rho_{12}-\rho_{13}\rho_{23}}{(1-\rho_{13}^2)^{\frac{1}{2}}(1-\rho_{23}^2)^{\frac{1}{2}}}\right) dz\end{aligned}$$

where

$$\begin{aligned}(26)\quad &B(h, k; \rho) \\ &= \frac{1}{2\pi(1-\rho^2)^{\frac{1}{2}}}\int_{-\infty}^{h}\int_{-\infty}^{k} \exp\left[-\tfrac{1}{2}(x^2 - 2\rho xy + y^2)/(1-\rho^2)\right] dx\, dy.\end{aligned}$$

The conditional distribution of x and y given z being a bivariate normal with appropriate means and covariance matrix leads to (25). Steck [40] expresses the trivariate integral of (25) in terms of the univariate normal integral, the T function of Owen and the $S(h, a, b)$ function defined by

$$(27)\qquad S(h, a, b) = \int_{-\infty}^{h} T(as, b)f(s)\, ds.$$

For $h > 0$, $a > 0$, $b > 0$, $S(h, a, b) - (1/4\pi)\arctan\,(b(1 + a^2 + b^2)^{-\frac{1}{2}})$ is the probability that three independent standardized normal variables will lie in the region between the planes $x = 0$, $x - bz = 0$, $y = 0$ and $y = h$ and beyond the planes $z - ay = 0 (z \geqq ay)$. Steck discussed the evaluation of the trivariate normal integral and gave properties of and relations among the functions used. Steck's function $S(m, a, b)$ is tabulated to seven decimal places for $a = 0(.1)2(.2)5(.5)8$, $b = .1(.1)1$. The range of values of m decreases from $0(.1)1.5, \infty$ for $a = 0(.1)1.2$ to $0(.1).3, \infty$ for $a = 6(.5)8$.

The result for the trivariate integral over the infinite cube $[0 \leqq x < \infty, 0 \leqq y < \infty, 0 \leqq z < \infty]$ with faces parallel to the coordinate planes is given by David [53] as follows:

$$\begin{aligned}(28)\quad &\int_{0}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(x, y, z; \{\rho_{ij}\})\, dx\, dy\, dz \\ &= \frac{2\pi - \arccos\rho_{12} - \arccos\rho_{13} - \arccos\rho_{23}}{4\pi} \\ &= \frac{2\pi - 3\arccos\rho}{4\pi} \quad \text{if} \quad \rho_{12} = \rho_{23} = \rho_{13} = \rho.\end{aligned}$$

The latter function corresponding to the case when all the correlations are equal is tabulated by Ruben [63] as $\bar{u}_3(x)$, with $x = 1/\rho = 2(1)12$. Ruben's excellent table (Table I, [63]) gives $\bar{u}_n(x)$, the relative surface content of regular hyperspherical simplices, with dimensionality $n = 1(1)50$ and primary bounding angles $\cos^{-1}(-1/x)$ from which we can obtain the probability that all the n equally correlated variables are positive.

Plackett [32] obtained a reduction formula for $\Phi_n(a_1, a_2, \cdots, a_n; \{\rho_{ij}\}) = \Pr\{x_1 > a_1, x_2 > a_2, \cdots, x_n > a_n\}$ which allows Φ_3 to be expressed as finite sums of single integrals of tabulated functions. Plackett's method for $n = 3$ gives

$$\Phi_3(P) = \Pr\{x_1 > a_1, x_2 > a_2, x_3^* > a_3\}$$

$$+ \frac{1}{2\pi} \int_{\arccos \rho_{23}}^{\arccos \rho_{23}^*} \exp\{-(a_2^2 + a_3^2 - 2a_2 a_3 \cos\theta)/2\sin^2\theta\} \tag{29}$$

$$\cdot \Phi_1 \left\{ \frac{(a_1 - \rho_{12} a_2 - \rho_{13} a_3) + (\rho_{13} a_2 + \rho_{12} a_3)\cos\theta - a_1 \cos^2\theta}{\sin\theta(1 - \rho_{12}^2 - \rho_{13}^2 + 2\rho_{12}\rho_{13}\cos\theta - \cos^2\theta)^{\frac{1}{2}}}; 1 \right\} d\theta$$

where $x_3^* = \{(\rho_{13} - \rho_{12}\rho_{23}^*)x_1 + (\rho_{23}^* - \rho_{12}\rho_{13})x_2\}/(1 - \rho_{12}^2)$ and $\rho_{23}^* = \rho_{12}\rho_{13} \pm \{(1 - \rho_{12}^2)(1 - \rho_{13}^2)\}^{\frac{1}{2}}$.

Das's [15] reduction is applicable to the trivariate case if the correlations are such that their joint product is positive and each one is numerically greater than the product of the other two. In this case Das's reduction gives

$$\Phi_3 = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[\prod_{i=1}^{3} \Phi_1 \left\{ a_i + z \sum_{i=1}^{3} b_i \right\} \right] f(z)\, dz \tag{30}$$

where b_i's are the solutions of

$$b_i b_j / ((1 + b_i^2)^{\frac{1}{2}} (1 + b_j^2)^{\frac{1}{2}}) = \rho_{ij} \qquad (i, j = 1, 2, 3, i \neq j) \tag{31}$$

and $\Phi_1(x) = 1 - F(x)$.

Moran's [27] reduction formula covers the special case when $a_1 = a_2 = \cdots a_n = 0$. The formulae of Ihm [19] and John [20] do not lead to any computationally simple results for the trivariate case.

5. Probability integral for special cases of the correlation matrix. Let

$$F_n(h_1, h_2, \cdots, h_n; \{\rho_{ij}\}) = \int_{-\infty}^{h_1} \int_{-\infty}^{h_2} \cdots \int_{-\infty}^{h_n} f(x_1, x_2, \cdots, x_n; \{\rho_{ij}\})\, dx_1 \cdots dx_n. \tag{32}$$

If the correlation matrix $\{\rho_{ij}\}$ of the x_i's has the structure $\rho_{ij} = \alpha_i \alpha_j (i \neq j)$ where $-1 \leqq \alpha_i \leqq +1$, then these variates X_i can be generated from $n + 1$ independent standard normal variates $Z_1, Z_2, \cdots, Z_n; Y$ by the transformation

$$X_i = (1 - \alpha_i)^{\frac{1}{2}} Z_i + \alpha_i Y \tag{33}$$

and it follows that

$$(34)\qquad F_n(h_1, h_2, \cdots, h_n; \{\rho_{ij}\}) = \int_{-\infty}^{\infty}\left[\prod_{i=1}^{n} F\left(\frac{h_i - \alpha_i y}{(1-\alpha_i^2)^{\frac{1}{2}}}\right)\right] f(y)\, dy.$$

The relation (34) was obtained by Dunnett and Sobel [169] and in a more generalized form by Das [15]. For some special cases it is also given by Moran [27], Ruben [63], Gupta [17], Stuart [43] and Ihm [19]. It appears to be periodically and independently rediscovered. If $\rho_{ij} = \rho$ for all i, j, then (34) reduces to

$$(35)\qquad F_n(h, h, \cdots, h; \{\rho\}) = \int_{-\infty}^{\infty} F^n[(h + \rho^{\frac{1}{2}} y)/(1-\rho)^{\frac{1}{2}}] f(y)\, dy.$$

For given n, h and ρ, a table of the above integral has been completed by Gupta [18] and parts of it are given as Table II at the end of this paper. It should be noted that

$$(36)\qquad \begin{aligned} F_n(0, 0, \cdots, 0; \{\rho\}) &= \int_{-\infty}^{\infty} F^n\{y[\rho/(1-\rho)]^{\frac{1}{2}}\} f(y)\, dy \\ &= 1/(n+1) \quad \text{if} \quad \rho = \tfrac{1}{2}. \end{aligned}$$

The integrals in (36) can be evaluated in closed form for $n = 1$, 2, and 3 and have been given above in Equations (6) and (28) for $n = 2$ and 3, respectively. For $n = 1$, the value of the integral is equal to one-half.

The integral in (35) has also been tabulated by Owen [30]. Owen's tables [Section 8.10 of the above reference] give this integral to $3D$ for $n = 2(1)8$, $h = -2.50(.10)3.00$ and $\rho = 0, \frac{1}{4}, \frac{1}{2}$ while in Section 8.11 the correlation ρ is equal to $1/(1 + n^{\frac{1}{2}})$ and the integral is tabulated to $5D$ for $n = 2(1)8$ and $h = -3.00(.10)3.00$. Table II at the end of the present paper gives the integral to $5D$ for $\rho = .1(.1).9$; $\rho = \frac{1}{8}(\frac{1}{8})\frac{7}{8}$ and $\rho = \frac{1}{3}, \frac{2}{3}$; $n = 1(1)12$ and $h = -3.50(.10)3.50$.

Integrals (34) and (35) can be evaluated easily by Gauss-Hermite and other methods of quadrature. Gupta [17] tabulated and discussed the evaluation of (35) for $\rho = \frac{1}{2}$ and also gave the solution in $h' = 2^{\frac{1}{2}} h$ for a given value $1 - \alpha$ of F_n. The percentage points h are given for $n = 1(1)50$ for $\alpha = .01, .025, .05, .10, .25$ in Table I. For large n the Cornish-Fisher method was used to derive the percentage points. The values of these percentage points are in agreement with the values of Bechhofer [71] for $n = 1(1)9$ which were based on some computations of the probability integral made by Teichroew [82]. The integral in (35) is also the probability integral of the largest of n correlated normal variates with equal correlation ρ. It is easy to show (see for example, [43]) that for the case of equal correlation $\rho \geqq -1/(n-1)$.

Removing the restriction on Y in (33) to be uncorrelated with Z_i's and writing (33) as

$$(37)\qquad Z_i = (X_i - \delta_i Y)/(1-\delta_i^2)^{\frac{1}{2}}, \qquad \delta_i = \operatorname{cov}(X_i, Y),$$

Curnow and Dunnett [14] have found some reduction formulae for the integral of the type (32). For the case $\rho_{ij} = \gamma_i/\gamma_j$ where $|\gamma_i| < |\gamma_j|$ $(i < j)$, the repeated application of their result reduces an n-dimensional integral to one over either $n/2$ (n even) or $(n - 1)/2$ (n odd) dimensions. We quote their results for $n = 4$. The reduced integral is

$$(38) \qquad \int_{-\infty}^{h_2} F\left(\frac{h_1 - \gamma_1 y/\gamma_2}{(1 - \gamma_1^2/\gamma_2^2)^{\frac{1}{2}}}\right)\left[\int_{-\infty}^{h_4'} F\left(\frac{h_3' - \gamma_3' z/\gamma_4'}{(1 - \gamma_3'^2/\gamma_4'^2)^{\frac{1}{2}}}\right) f(z)\, dz\right] f(y)\, dy,$$

where $\gamma_i' = (\gamma_i^2 - \gamma_2^2)^{\frac{1}{2}}$ and $h_i' = (h_i - \rho_{2i} y)/(1 - \rho_{2i}^2)^{\frac{1}{2}}$, $i = 3, 4$. Note that the outer integral in (38) does not have upper limit ∞.

Steck and Owen [41] also studied the case of equicorrelated multivariate normal distribution and showed that the relation (35) is also true for negative ρ even though the integral and on the right hand side is complex. But for computation purposes in this case Steck and Owen [41] suggest the following relation proved by them [41],

$$(38a) \qquad F_n(x; \rho) = \sum_{k=1}^{n} (-1)^{k+1} \binom{n}{k} F_k(\alpha_n x; \rho_n) F_{n-k}(x; \rho)$$

where $\alpha_j^2 = (1 - \rho)/[1 + (j - 1)\rho][1 + (j - 2)\rho]$, $(j = 1, 2, \cdots, n)$ and $\rho_n = -\rho/[1 + (n - 2)\rho]$.

6. Probability that all the variables have the same sign. Let

$$(39) \quad F_n^0 = \Pr\{x_1 \leqq 0, x_2 \leqq 0, \cdots, x_n \leqq 0\} = \Pr\{x_1 \geqq 0, x_2 \geqq 0, \cdots, x_n \geqq 0\}$$

where, as before, the x's follow a standardized multivariate normal distribution with correlation matrix $\{\rho_{ij}\}$. A recurrence relation (earlier given by Schläfli [65] in connection with the area of a simplex on the surface of a sphere in n dimensions) has been given for F_n^0 by David [53]. For n odd this relation is

$$(40) \qquad F_n^0 = \tfrac{1}{2}[1 - \textstyle\sum F_1^0 + \sum F_2^0 + \cdots + (-1)^{n-1} \sum F_{n-1}^0]$$

where F_j^0 represents the probability that j of the x's are positive and the summation in $\sum F_j^0$ extends over all $\binom{n}{j}$ combinations. Now it is known [11], [203] that

$$(41) \qquad F_2^0 = \tfrac{1}{4} + \arcsin \rho/2\pi$$

(this result can be obtained by actual integration, see remark after Equation (36)) so that

$$(42) \qquad F_3^0 = \tfrac{1}{8} + (1/4\pi)\,(\arcsin \rho_{12} + \arcsin \rho_{13} + \arcsin \rho_{23})$$

(an alternative form to that of (23)

If we know F_4^0, F_6^0, $\cdots$, we can get F_5^0, F_7^0, $\cdots$, by using the formula (40). However, no closed form expressions are available for F_4^0, F_6^0, $\cdots$. An expansion for F_4^0 [61] based on the generalization of the tetrachoric series and stated earlier is

$$F_4^0 = \tfrac{1}{16} + (1/8\pi) \sum_{i=j} \rho_{ij} + (1/4\pi^2) \sum_{i,j,k,l} \rho_{ij}\rho_{kl} + \cdots . \tag{43}$$

The series in (43) converges only slowly if ρ_{ij} are not small. Also the terms are not all positive. Thus there is a need for developing (see David [53]) quickly converging series for F_n^0 for n even and ρ large. Using the relationship between $\Phi_4^0(\equiv F_4^0)$ and the functions $f(\alpha, \beta, \gamma)$ of Schläfli which were computed by him, Plackett [32] gives a table of Φ_4^0 for the case when $\rho_{13} = \rho_{14} = \rho_{24} = 0$. Plackett's approximation for F_4^0 is

$$\begin{aligned} F_4^0 \simeq -\tfrac{1}{8} + (1/4\pi^2)\{&\arccos(-\rho_{12}) \arccos(-\rho_{34}) \\ &+ \arccos(-\rho_{13}) \arccos(-\rho_{24}) + \arccos(-\rho_{14}) \arccos(-\rho_{23})\}. \end{aligned} \tag{44}$$

On expanding and ignoring terms of order ρ^3 and higher, (44) gives the first three terms of (43).

Ruben derived expressions for and tabulated the relative surface content $\bar{u}_\beta(x) = \bar{v}_{\beta\beta}(x)$ of regular hyperspherical simplices with dimensionality β and primary bounding angles $\cos^{-1}(-1/x)$. If all the correlations are equal to $1/x$, the tables of $\bar{u}_\beta(x)$ given by Ruben for $x = 2(1)12$ and $\beta = 1(1)49$ provide the probabilities F_n^0. Explicitly

$$F_n^0 = \bar{u}_n(x) \quad \text{if} \quad \rho_{ij} = 1/x \quad \text{for all} \quad i, j. \tag{45}$$

It should be pointed out here that the moments of the extreme order statistc in a sample from a normal population can be expressed in terms of the surface contents of regular hyperspherical simplices (see Ruben [63]).

McFadden [25] suggests an approximation based on a generalization of Pólya's urn scheme. He shows that it is not very satisfactory unless the ρ_{ij} are very small or very nearly equal. When all the correlations are equal to ρ, then the urn scheme is precisely that of Pólya and McFadden's approximation becomes

$$F_n^0 = ((1 - r)/2r)_n/((1 - r)/r)_n, \tag{46}$$

where $r = (2/\pi) \arcsin \rho$ and $(a)_n = \Gamma(a + n)/\Gamma(a)$. Using formula (46) for $n = 4$, McFadden constructed a table for $1/\rho = 2(1)12$ and found that the best agreement takes place when ρ is small. He also found that for a given ρ the approximation (46) grows worse as n increases.

McFadden also discussed the problem of evaluation of the quadrivariate ntegral in greater detail in [59], [60]. By a special transformation of Moran's tetrachoric series for F_4^0, McFadden gave an approximation for F_4^0 for the case of equal correlations ρ which is

$$F_4^0 \simeq \tfrac{1}{16} + \frac{3}{4\pi}\phi + \frac{1}{4\pi^2}\frac{\phi^2(3 + 5\phi)}{(1 + \phi)(1 + 2\phi)} \tag{47}$$

where $\phi = \arcsin \rho$, $(-\arcsin \tfrac{1}{3} \leqq \phi \leqq \pi/2)$, which is more accurate than the approximation for this case obtained by the urn scheme. The error in this approximation increases with ρ and equals 0.000008 at $\rho = \tfrac{1}{2}$ and 0.001 at $\rho = 1$.

In another paper [60] McFadden derives two new expansions for the quadrivariate normal integral F_4^0, the first one for the case where $\rho_{13} = \rho_{12}\rho_{23}$; $\rho_{14} = \rho_{12}\rho_{23}\rho_{34}$; $\rho_{24} = \rho_{23}\rho_{34}$ and the second one being applicable when $\rho_{13} = \rho_{14} = \rho_{24} = 0$. These expansions are series in powers of ρ_{23}, the coefficients being products of polynomials in ρ_{12} and in ρ_{34}. They are well suited for computation according to the author.

Sondhi [67] has derived an approximate closed expression for F_4^0 when all the correlations are equal. His formula gives 5 decimal place accuracy for $.5 \leqq \rho \leqq 1$, thus providing a complement to McFadden's (47). This approximation is

$$(47a) \qquad F_4^0(\rho) \simeq \tfrac{1}{2} - \frac{3\phi}{2\pi^2}\left(\frac{\pi}{2} + \sin^{-1}\tfrac{1}{3}\right) + \frac{3\phi^3}{\pi^2 8^{\frac{1}{2}}}\left(\tfrac{1}{36} + \phi^2\,\frac{(ac - b)\phi^2 - ab}{c\phi^2 - b}\right)$$

where $\phi = \cos^{-1}\rho$, $a = 23/(5!48)$, $b = 3727/(7!1152)$, $c = 3320309/(9!82944)$. [On recomputing the constants Steck [68] found that the numerator of c should be 3,364,929.]

Sondhi [63] also derived the relation

$$(47b) \qquad \begin{aligned} F_4^0\left(\frac{-\rho}{1 + 2\rho}\right) + F_4^0(\rho) &= \tfrac{1}{8} + \frac{3}{4\pi}\left[\sin^{-1}\rho - \sin^{-1}\left(\frac{\rho}{1 + 2\rho}\right)\right] \\ &\quad + \frac{3}{2\pi^2}\sin^{-1}\rho\,\sin^{-1}\left(\frac{\rho}{1 + 2\rho}\right) \end{aligned}$$

from which one can obtain $F_4^0(\rho)$ for the range $-\frac{1}{3} \leqq \rho \leqq 0$ as well.

In a recent paper [68] Steck has given a very good review of results pertaining to orthant probabilities in the equicorrelated case. Steck has proved the following results:

$$(47c) \qquad \tilde{F}_n(\rho) = F_n^0(-\rho/\{1 + (n - 2)\rho\}) = \sum_{k=0}^{n} \beta_k(n)\,(\cos^{-1}\rho)^k$$

where $\beta_k(n)$ are tabulated for some values of k and n. This result can be used for computing the orthant probabilities for negative values of ρ.

$$(47d) \qquad F_4^0(\rho) \simeq \frac{1}{16} + \frac{3}{4\pi}\left(1 + \frac{1}{\pi}\right)\sin^{-1}\rho - \frac{3^{\frac{1}{2}}}{4\pi^2}\,V(\rho)^{-1},$$

$$(47e) \qquad \begin{aligned} F_4^0(\rho) &\simeq \frac{1}{16} + \frac{3}{4\pi}\left(1 + \frac{25}{24\pi}\right)\sin^{-1}\rho \\ &\quad - \frac{3^{\frac{1}{2}}}{4\pi^2}\left(\frac{85}{72}\right)V(\rho) + \frac{1}{16\pi^2}\left(1 - U\,(\rho) - \frac{\rho U(\rho)}{3(1 + 2\rho)}\right), \end{aligned}$$

where $V(\rho) = \cosh^{-1}2 - \cosh^{-1}\{(2 + \rho)/(1 + 2\rho)\}$ and $U(\rho) = (1 - \rho^2)^{\frac{1}{2}}/(1 + 2\rho)$. The errors in using (47d) and (47e) have been discussed by Steck.

Steck [68] has also derived a Gram-Charlier Type A Series for $F_n(x; \rho)$ and computed the coefficients $\Psi_k(n)k = 0(1)4$, $n = 2(1)50$, in the following series for the special case of orthant probabilities at $x = 0$.

$$(47f) \qquad F_n^0(\rho) = \tfrac{1}{2} - ((1 - \rho)/2\pi)^{\frac{1}{2}}\sum_{k=0}^{\infty}(-1)^k(1 - \rho)^k\Psi_k(n).$$

This series is useful for ρ near its positive and negative extremes. Using the above series Steck [68] has computed $F_n^0(\rho)$ for $\rho = .5(.1).9$ and $n = 2(1)50$ and has made some comparisons with the general tables of Gupta [18] parts of which appear in this paper.

In the course of some work on the variance of Spearman's ρ in normal samples, David and Mallows [54] made a comparative study of the many methods proposed for the evaluation of the positive quadrant of the quadrivariate normal integral and found Plackett's method [32] as the most useful one. David and Mallows [54] have given twenty-four evaluations of the special cases, most of them new and have demonstrated how Plackett's method can be used to obtain quickly convergent series.

Another special case is where the inverse of the covariance matrix, $A = (a_{ij})$ is such that

$$a_{ij} = \begin{cases} 2 & \text{if} \quad i = j \\ -1 & \text{if} \quad i - j = \pm 1 \\ 0 & \text{otherwise.} \end{cases}$$

In this case Anis and Lloyd [46] have proved that

$$F_n^0 = 1/(n+1)^{3/2}. \tag{48}$$

7. Integrals in higher dimensions. Not very much has been done on the evaluation of multivariate normal integrals in higher dimensions over domains of the type $[-\infty < x_1 \leqq h_1, -\infty < x_2 \leqq h_2, \cdots, -\infty < x_n \leqq h_n]$. Even for the case of four variables the results are applicable only to special cases. As n, the number of variables, increases the number of correlations, which is equal to $\binom{n}{2}$, increases rapidly. The following remarks may summarize the situation for the general case.

(1) The integral can be expressed as an infinite series in powers of the correlations (generalization of tetrachoric series of bivariate). This is done by expressing (formally) the density as the inverse of the characteristic function and then carrying out the integration. This has been discussed by Aitken [13], Kendall [57] and Moran [61] among others. This series expansion, however, is not very suitable for computation if ρ_{ij} are large because the series then converges only very slowly.

(2) Plackett [32] gives a method of reduction which is as follows. For fixed $a_1, a_2, \cdots, a_n$, the function $\Phi_n = \Pr\{x_1 > a_1, x_2 > a_2, \cdots, x_n > a_n\}$ can be regarded as a function of the point P with coordinates $\{\rho_{ij}\}$ in a space of $\binom{n}{2}$ dimensions and denoted by $\Phi_n(P)$. By integrating the differential element $d\Phi_n$ along the line KP where K is a point with coordinates $\{\kappa_{ij}\}$ and the value of $\Phi(K)$ is available or easily computable, Plackett [32] obtains

$$\Phi_n(P) = \Phi_n(K) + \sum_{i<j} \int_{\kappa_{ij}}^{\rho_{ij}} \frac{\partial \Phi_n}{\partial \lambda_{ij}} (L)\, d\lambda_{ij} \tag{49}$$

where L with coordinates $\{\lambda_{ij}(t)\}$ divides KP in the ratio $t:1 - t$ so that $\lambda_{ij}(t) = t\rho_{ij} + (1 - t)\kappa_{ij}$. The line integrals on the right hand side of (49) have to be evaluated numerically. By setting $\lambda_{ij} = \cos\theta_{ij}$, the singularity at the lower limit is avoided and the labor of computations is shortened. If $R(t)$ is the matrix with unit diagonal elements and non-diagonal elements $\{\lambda_{ij}(t)\}$, then by choosing $R(0)$ to be a matrix of rank $(n - 1)$ with all but one of its correlation coefficients the same as those of $R(1)$ ($R(0)$ corresponds to K and $R(1)$ corresponds to P), the second term on the right hand side of (49) is only a single integral. Sometimes this may be a desirable way of choosing K. Plackett shows that it is always possible to do this. Plackett's method of reduction is theoretically applicable to all n dimensional normal integrals but it is quite laborious to apply it for large n.

(3) Methods of reduction for many special cases have already been mentioned. Ihm [19] discusses the numerical evaluation of an n-fold integral of a multivariate normal with covariance matrix as the sum of a diagonal matrix D and the product of a row vector with its transpose. The integral I can be written as

$$I = [c/(2\pi)^{\frac{1}{2}}] \int_{-\infty}^{\infty} e^{-c^2\tau^2/2}\, I(\tau)\, d\tau \tag{50}$$

when $I(\tau)$ (with covariance matrix D) is easy to compute. John [20] obtains a formula for evaluating probability integrals in n dimensions if those of $(n - 1)$ are available. His method depends on evaluating the density of the minimum of the random variables $\mathbf{X}$ which are distributed $N(\boldsymbol{\mu}, \Sigma)$. This gives a simple result only if all the correlations are equal.

(4) Ruben [33], [37] discusses the evaluation of the probability contents of regions R under a spherical normal distribution. The transformation that carries the ellipsoidal normal into the spherical normal changes this type of regions (intervals) into polyhedral cones.

A result concerning $\Phi_n(a_1, a_2, \cdots, a_n; \{\rho_{ij}\})$.

THEOREM (Slepian). *Let $X_1, X_2, \cdots, X_n$ be normal with zero means and covariance matrix $\{\rho_{ij}\}$ and let $Y_1, Y_2, \cdots, Y_n$ be normal with zero means and covariance matrix $\{\kappa_{ij}\}$. Let $\rho_{ii} = \kappa_{ii} = 1$, $i = 1, 2, \cdots, n$. If $\rho_{ij} \geqq \kappa_{ij}$, $i, j = 1, 2, \cdots, n$, then*

$$\Pr[X_1 > a_1, X_2 > a_2, \cdots, X_n > a_n] \geqq \Pr[Y_1 > a_1, Y_2 > a_2, \cdots, Y_n > a_n],$$

i.e.

$$\Phi_n(a_1, a_2, \cdots, a_n; \{\rho_{ij}\}) \geqq \Phi_n(a_1, a_2, \cdots, a_n; \{\kappa_{ij}\}). \tag{51}$$

PROOF. By writing the density function f_n as the inverse of the characteristic function, it is easy to verify that if $\{\rho_{ij}\}$ is non-singular

$$\partial f_n/\partial\rho_{ij} = \partial^2 f_n/\partial x_i \partial x_j \tag{52}$$

$$\begin{aligned} \frac{\partial\Phi_n}{\partial\rho_{12}} &= \int_{a_1}^{\infty} dx_1 \int_{a_2}^{\infty} dx_2 \cdots \int_{a_n}^{\infty} dx_n \frac{\partial^2 f_n}{\partial x_1\, \partial x_2} \\ &= \int_{a_3}^{\infty} dx_3 \int_{a_4}^{\infty} dx_4 \cdots \int_{a_n}^{\infty} dx_n\, f_n(a_1, a_2, x_3, \cdots, x_n; \{\rho_{ij}\}) \geqq 0, \end{aligned} \tag{53}$$

and similarly

$$\partial\Phi_n/\partial\rho_{ij} \geqq 0, \qquad i \neq j. \tag{54}$$

If $\{\kappa_{ij}\}$ is non-singular, set

$$\gamma_{ij} = \lambda\rho_{ij} + (1 - \lambda)\kappa_{ij} \tag{55}$$

then, $\{\gamma_{ij}\}$ is also positive definite for all λ satisfying $0 \leqq \lambda \leqq 1$.

$$d\Phi_n/d\lambda = \sum_{i<j} (\partial\Phi_n/\partial\gamma_{ij})\,(d\gamma_{ij}/d\lambda) = \sum_{i<j} (\partial\Phi_n/\partial\gamma_{ij})(\rho_{ij} - \kappa_{ij}) \tag{56}$$

and if $\rho_{ij} \geqq \kappa_{ij}$, $\partial\Phi_n/d\lambda \geqq 0$ since $\partial\Phi_n/\partial\rho_{ij} \geqq 0$.
Integrating on λ from 0 to 1, we get the required result.

Bounds on Φ_n. Bounds on Φ_n may be set by using the above result. For example, if $\rho_{\max}$ and $\rho_{\min}$ denote the maximum and minimum values of the correlations, and if $\rho_{\min}$ is non-negative, then both the upper and lower bounds on the function Φ_n can be obtained in terms of integrals of the type given in Equation (34). If $a_1 = a_2 = \cdots = a_n = a$, we have

$$\int_{-\infty}^{\infty} F^n\left(\frac{(\rho_{\min})^{\frac{1}{2}}x - a}{(1 - \rho_{\min})^{\frac{1}{2}}}\right) f(x)\,dx \leqq \Phi_n \leqq \int_{-\infty}^{\infty} F_n\left(\frac{(\rho_{\max})^{\frac{1}{2}}x - a}{(1 - \rho_{\max})^{\frac{1}{2}}}\right) f(x)\,dx. \tag{57}$$

If at least one of the $\rho_{\min}$ and $\rho_{\max}$ is non-negative, we can get easily computable one-sided bound on Φ_n. The integrals of the type in (57) have been evaluated for selected values of ρ, a and n and are given in Table II. This table is excerpted from a bigger table of the probability integrals of the equally correlated normal variables [18].

8. Multivariate analogue of Student's *t*-distribution. Consider the joint distribution of n variates $t_i = z_i/s(i = 1, 2, \cdots, n)$, the z_i having a non-singular multivariate normal distribution with zero means, common unknown variance σ^2 and known correlation matrix $\{\rho_{ij}\}$ while $\nu s^2/\sigma^2$ has a χ^2-distribution with ν degrees of freedom and is distributed independently of the z_i's.

The joint density of the t_i can be obtained from the joint density of the $z_i(i = 1, 2, \cdots, n)$ and s by integrating out s. This joint density function is (See Dunnett and Sobel [168])

$$g_\nu(t_1, t_2, \cdots, t_n; \{\rho_{ij}\}) = \frac{A^{\frac{1}{2}}\,\Gamma((n+\nu)/2)}{(\nu\pi)^{n/2}\,\Gamma(\nu/2)}\left[1 + \frac{1}{\nu}\sum_{i,j} a_{ij}\,t_i\,t_j\right]^{-(n+\nu)/2} \tag{58}$$

where A is the determinant of the positive definite matrix $\{a_{ij}\} = \{\rho_{ij}\}^{-1}$. As $\nu \to \infty$, (58) approaches the standardized n-variate normal density function with correlation matrix $\{\rho_{ij}\}$. For $n = 1$, it reduces to Student's t-distribution. Thus it may be considered as a multivariate analogue of Student's t-distribution.

The multivariate analogue of t-distribution arises very naturally in multiple decision problems concerned with the selection and ranking of population means of the $(n + 1)$ normal populations. The correlations of the normal variables in this case are all equal, one important case being that when they are all equal to half.

Dunnett and Sobel [168] considered the bivariate t and derived series expansions for the integral and equi-coordinate percentage points of this distribution. They tabulated the probability integral and the percentage points of bivariate t for $\rho = .5$ and $-.5$ for $\nu = 1(1)30(3)60(15)120, 150, 300, 600, \infty$. In the case $\rho = \pm\frac{1}{2}$, they evaluated the integral

$$(59) \qquad P = \int_{-\infty}^{h}\int_{-\infty}^{h} \frac{1}{\pi(3)^{\frac{1}{2}}}\left\{1 + \frac{4(x^2 \mp xy + y^2)}{3\nu}\right\}^{-\frac{1}{2}(\nu+2)} dx\, dy, \qquad \begin{matrix} - \text{ if } \rho = +\frac{1}{2} \\ + \text{ if } \rho = -\frac{1}{2} \end{matrix}$$

and for given P solved for the equi-coordinate percentage point h. In a subsequent paper Dunnett and Sobel [169] derived approximations (which are also lower bounds) to the probability integral of the t for the special structure of the correlation matrix with $\rho_{ij} = b_i b_j$.

In general we are interested in evaluating the probability integral

$$(60) \quad G_\nu(h_1, h_2, \cdots, h_n; \{\rho_{ij}\}) = \int_{-\infty}^{h_1}\int_{-\infty}^{h_2}\cdots\int_{-\infty}^{h_n} g_\nu(t_1, t_2, \cdots, t_n; \{\rho_{ij}\})\, dt_1\, dt_2 \cdots dt_n$$

or in obtaining an inverse solution for a given value of G_ν.

The evaluation of G_ν in the special case when $\rho_{ij} = \rho = \frac{1}{2}(i \neq j)$ has been studied by Gupta [17] and by Gupta and Sobel [172]. It was shown by Gupta [17] that for the case where $h_1 = h_2 = \cdots = h_n = h$ the probability integral of $y = (x_{[n]} - x)/s$ where $x_{[n]}$ is the largest of n independent identical normal chance variables $x_1, x_2, \cdots, x_n$; x is another normal chance variable with the same mean μ and same variance σ^2 as $x_1, x_2, \cdots, x_n$ and s^2 is distributed as $\chi^2\sigma^2/\nu$ independent of $x, x_1, x_2, \cdots, x_n$, is related to the probability integral of t.

The relationship between the distribution of the multivariate t and the statistic y is

$$(61) \quad \int_{-\infty}^{h}\int_{-\infty}^{h}\cdots\int_{-\infty}^{h} g_\nu(t_1, t_2, \cdots, t_n; \{\tfrac{1}{2}\})\, dt_1\, dt_2 \cdots dt_n = \Pr\{y \leqq 2^{\frac{1}{2}} h\}.$$

Some percentage points h of the multivariate t are also tabulated by Dunnett [170] for $n = 2(1)10$. The correlation matrix $\{\frac{1}{2}\}$ has a simple inverse and the density function g_ν for this case is

$$(62) \quad \begin{aligned} g_\nu(t_1, t_2, \cdots, t_n; \{\tfrac{1}{2}\}) &= \left[2^{n/2}\,\Gamma\left(\frac{\nu+n}{2}\right)\Big/\Gamma\left(\frac{\nu}{2}\right)\left(\frac{\pi\nu}{2}\right)^{n/2}(n+1)^{\frac{1}{2}}\right] \\ &\quad \cdot\left[1 + \frac{2n}{\nu(n+1)}\left\{\Sigma t_i^2 - \frac{2}{n}\sum_{i<j} t_i t_j\right\}\right]^{-\frac{1}{2}(\nu+n)}. \end{aligned}$$

For a fixed n let $G_\nu(u)$ denote the probability integral

$$\int\int\cdots\int g_\nu(t_1, t_2, \cdots, t_n; \{\tfrac{1}{2}\})\, dt_1\, dt_2 \cdots dt_n,$$

where each integral runs from $-\infty$ to $u(\nu/2)^{\frac{1}{2}}$, then it can be shown that $G_\nu(u)$ satisfies

$$u(dG_\nu(u)/du) + \nu[G_\nu(u) - G_{\nu+2}(u)] = 0 \tag{63}$$

which is Hartley's differential-difference equation [201], for the probability integral of a general class of statistics known as Studentized statistics. We then desire a solution of (63) in terms of powers of $1/\nu$ and such that when ν goes to infinity $G_\nu(u)$ reduces to the corresponding integral for the known σ case. Using Hartley's solution, an expansion for $G_\nu(u)$ in powers of ν^{-1} can be obtained as follows.

$$\begin{aligned} G_\nu(u/\nu^{\frac{1}{2}}) &= I_0(u) + (1/4\nu)[u^2\{I_2(u) - I_0(u)\} - uI_1(u)] \\ &\quad + (1/16\nu^2)[(u^4/2)\{3I_0(u) - 6I_2(u) + I_4(u)\} - (u^3/3) \\ &\quad \{I_3(u) - 3I_1(u)\} - (u^2/2)\{I_2(u) - I_0(u)\} + (u/2)I_1(u)] + O(\nu^{-3}), \end{aligned} \tag{64}$$

where

$$I_j(u) = \int_{-\infty}^{\infty} F^n(x+u)f(x)\,x^j\,dx. \tag{65}$$

As remarked earlier, the integrals of the type (65) can be easily computed by Gauss-Hermite quadrature so that the probability integral of this special multivariate t can be computed. Edgeworth type expansions and Cornish-Fisher expansion can also be applied with reasonable accuracy. For some further details and tables of the percentage points see [17] and [172].

We shall now discuss the evaluation of the multivariate t integral when $\rho_{ij} = \rho > 0 (i \neq j)$. In this case the multivariate t density function can be written as

$$\begin{aligned} &g_\nu(t_1, t_2, \cdots, t_n; \{\rho\}) \\ &= [1 + (n-1)\rho]^{-\frac{1}{2}}\Gamma(\tfrac{1}{2}(\nu+n))/[(\nu\pi)^{\frac{1}{2}n}\Gamma(\tfrac{1}{2}\nu)(1-\rho)^{\frac{1}{2}(n-1)}] \\ &\cdot[1 + \{\nu[\{1+(n-1)\rho\}(1-\rho)]\}^{-1}\{[1+(n-2)\rho]\Sigma t_i^2 - 2\rho \textstyle\sum_{i<j} t_it_j\}]^{-\frac{1}{2}(\nu+n)}. \end{aligned} \tag{66}$$

The integral $G_\nu(h, h, \cdots, h; \{\rho\})$ corresponding to the above density can also be looked on as the probability integral of the statistic $[(1-\rho)^{\frac{1}{2}}x_{[n]} - \rho^{\frac{1}{2}}x]/s$ where $x_{[n]}$, x and s have the same meaning as given earlier in this Section. This relationship is

$$\begin{aligned} G_\nu(h, h, \cdots, h; \{\rho\}) &= \Pr\{[(1-\rho)^{\frac{1}{2}}x_{[n]} - \rho^{\frac{1}{2}}x]/s \leqq h\} = \int_0^\infty h_\nu(\chi) \\ &\quad \left[\int_{-\infty}^{\infty} F^n\left(\frac{h\chi}{[\nu(1-\rho)]^{\frac{1}{2}}} + \left(\frac{\rho}{1-\rho}\right)^{\frac{1}{2}} x\right) f(x)\,dx\right] d\chi \\ &= \left(\frac{1-\rho}{\rho}\right)^{\frac{1}{2}} \int_0^\infty h_\nu(\chi)\left[\int_{-\infty}^{\infty} F^n(u) f\left(\left(\frac{1-\rho}{\rho}\right)^{\frac{1}{2}} u - \frac{h\chi}{(\nu\rho)^{\frac{1}{2}}}\right) du\right] d\chi \end{aligned} \tag{67}$$

where $h_\nu(\chi)$ is the chi-density corresponding to ν degrees of freedom for the χ^2-distribution. Equation (67) is thus a generalization of the case $\rho = \frac{1}{2}$ discussed in [17], [172]. The evaluation of the double integral in (67) can be carried out by first evaluating the integral in square brackets for those χ which are the zeros of Laguerre polynomials (and using a Gauss-Hermite method) and then evaluating the integral with respect to χ by using the Gauss-Laguerre method of integration. For $\rho = \frac{1}{2}$, the integral in the square bracket in (67) can be evaluated by expanding the function f about $h = 0$. This leads to a power series in χ in which the coefficients are the moments of the largest order statistic in a sample from a standardized normal population. A term by term integration of this series gives the required expansion which is good if h is small (see reference [172]). It should be remarked, however, that in the general case the expansion of the function f about $h = 0$ does not give the coefficients of the series as moments of the largest order statistic from a standard normal population.

Another expansion is derived by expanding the function f in (67) about $s = \sigma$ (i.e., about $\chi = \nu^{\frac{1}{2}}$), obtaining

$$
\begin{aligned}
(68)\qquad G_\nu(h, h, \cdots, h; \{\rho\}) &= \left(\frac{1-\rho}{\rho}\right)^{\frac{1}{2}} \int_{-\infty}^{\infty} h_\nu(\chi) \Bigg[\int_0^{\infty} F^n(u) f\left(\left(\frac{1-\rho}{\rho}\right)^{\frac{1}{2}} u \right. \\
&\quad \left. - \frac{h}{\rho^{\frac{1}{2}}}\right) du + \sum_{j=1}^{\infty} \frac{h^j(\chi - \nu^{\frac{1}{2}})^j}{j!\,\nu^{j/2}\,\rho^{j/2}} \int_{-\infty}^{\infty} F^n(u) f\left(\left(\frac{1-\rho}{\rho}\right)^{\frac{1}{2}} u - \frac{h}{\rho^{\frac{1}{2}}}\right) \\
&\quad \cdot H_j\left(\left(\frac{1-\rho}{\rho}\right)^{\frac{1}{2}} u - \frac{h}{\rho^{\frac{1}{2}}}\right) du \Bigg] d\chi = \left(\frac{1-\rho}{\rho}\right)^{\frac{1}{2}} \int_{-\infty}^{\infty} F^n(u) f\left(\left(\frac{1-\rho}{\rho}\right)^{\frac{1}{2}} u - \frac{h}{\rho^{\frac{1}{2}}}\right) du \\
&\quad + \sum_j \frac{h^j E(\chi - \nu^{\frac{1}{2}})^j}{j!\,\nu^{j/2}\,\rho^{j/2}} \int_{-\infty}^{\infty} F^n\left(\left(\frac{\rho}{1-\rho}\right)^{\frac{1}{2}} y + \frac{h}{(1-\rho)^{\frac{1}{2}}}\right) f(y) H_j(y)\, dy.
\end{aligned}
$$

By collecting terms containing $\nu^{-\frac{1}{2}}$, $\nu^{-1} \cdots$, we get an expansion for the probability integral of t in powers of $\nu^{-\frac{1}{2}}$. If integrals of the type occurring inside the summation sign in (68) are evaluated, then using the known values $E(\chi - \nu^{\frac{1}{2}})^j$, we can compute the integral by using a suitable number of terms in the above series.

A discussion of the evaluation of the probability integrals of the multivariate t with some useful recurrence relations and some statistical applications of the distribution is given by John [179].

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Table I. For given values of α and n this table gives the values of h (equal

TABLE I

Upper 100 α percentage points of the maximum of n equally correlated standard normal variates with correlation $\rho = \frac{1}{2}$

n	α .01	.025	.05	.10	.25
1	2.326	1.960	1.645	1.282	.675
2	2.558	2.212	1.916	1.577	1.014
3	2.685	2.350	2.064	1.735	1.189
4	2.772	2.442	2.160	1.838	1.306
5	2.837	2.511	2.233	1.916	1.391
6	2.889	2.567	2.290	1.978	1.458
7	2.933	2.613	2.340	2.029	1.514
8	2.970	2.652	2.381	2.072	1.560
9	3.002	2.686	2.417	2.109	1.601
10	3.031	2.716	2.448	2.142	1.636
11	3.057	2.743	2.477	2.172	1.667
12	3.079	2.768	2.502	2.180	1.696
13	3.100	2.790	2.525	2.222	1.724
14	3.120	2.810	2.546	2.244	1.745
15	3.138	2.829	2.565	2.264	1.767
16	3.154	2.846	2.583	2.283	1.787
17	3.170	2.863	2.600	2.301	1.805
18	3.185	2.878	2.616	2.317	1.823
19	3.198	2.892	2.631	2.332	1.839
20	3.211	2.906	2.645	2.347	1.854
21	3.223	2.918	2.658	2.361	1.869
22	3.235	2.930	2.671	2.374	1.883
23	3.246	2.942	2.683	2.386	1.896
24	3.257	2.953	2.694	2.392	1.908
25	3.268	2.964	2.705	2.409	1.920
26	3.276	2.973	2.715	2.420	1.931
27	3.286	2.983	2.725	2.430	1.942
28	3.295	2.993	2.735	2.440	1.953
29	3.303	3.001	2.744	2.450	1.963
30	3.312	3.010	2.753	2.459	1.972
31	3.319	3.018	2.761	2.467	1.982
32	3.327	3.026	2.770	2.476	1.990
33	3.335	3.034	2.777	2.484	1.999
34	3.342	3.041	2.785	2.492	2.007
35	3.349	3.048	2.792	2.500	2.015
36	3.355	3.055	2.800	2.507	2.023
37	3.362	3.062	2.807	2.514	2.031
38	3.368	3.069	2.813	2.521	2.038
39	3.374	3.075	2.820	2.528	2.045
40	3.380	3.081	2.826	2.535	2.052
41	3.386	3.087	2.833	2.541	2.059
42	3.392	3.093	2.839	2.547	2.065
43	3.398	3.099	2.845	2.553	2.072
44	3.403	3.105	2.850	2.559	2.078
45	3.408	3.110	2.856	2.565	2.084
46	3.413	3.115	2.861	2.571	2.090
47	3.417	3.120	2.867	2.576	2.095
48	3.423	3.126	2.872	2.582	2.101
49	3.428	3.131	2.877	2.587	2.107
50	3.433	3.135	2.882	2.592	2.112

coordinate 100 α percentage points) for which

$$1 - \alpha = \frac{1}{(2\pi)^{n/2} |\Sigma|^{\frac{1}{2}}} \int_{-\infty}^{h} \int_{-\infty}^{h} \cdots \int_{-\infty}^{h} \exp\left[-\tfrac{1}{2}\mathbf{x}' \Sigma^{-1} \mathbf{x}\right] dx_1\, dx_2 \cdots dx_n$$

$$= \int_{-\infty}^{\infty} F^n(z + h2^{\frac{1}{2}}) f(z)\, dz$$

where

$\mathbf{X}' = (X_1, X_2, \cdots, X_n)$ follow a joint n-variate normal distribution with zero means and covariance matrix Σ,

$$\Sigma = \begin{Bmatrix} 1 & \frac{1}{2} & \cdots & \frac{1}{2} \\ \frac{1}{2} & 1 & \cdots & \frac{1}{2} \\ & & \cdots & \\ \frac{1}{2} & \frac{1}{2} & \cdots & 1 \end{Bmatrix}, \qquad \Sigma^{-1} \doteq \frac{2n}{(n+1)} \begin{Bmatrix} 1 & -\frac{1}{n} & \cdots & -\frac{1}{n} \\ -\frac{1}{n} & 1 & \cdots & -\frac{1}{n} \\ & & \cdots & \\ -\frac{1}{n} & -\frac{1}{n} & \cdots & 1 \end{Bmatrix}$$

and where F and f refer to the cumulative distribution function and the density of a standard normal chance variable, respectively.

Table II. For given values of ($\rho \geqq 0$) N and H this table gives the values of the integral

$$F_N(H; \rho) = \int_{-\infty}^{\infty} F^N \left(\frac{x\rho^{\frac{1}{2}} + H}{(1 - \rho)^{\frac{1}{2}}} \right) f(x)\, dx$$

which is the probability that each of the N standardized normal random variables with equal correlations ρ will not exceed H. In the integrand above, F and f denote the cumulative distribution function and the density of a standard normal random variable, respectively.

Note that $F_N(H; \rho)$ also gives the probability that the maximum of a set of n equally correlated standardized normal random variables does not exceed H or the probability that the minimum of this set exceeds $-H$.

Table II

Probability that N standard normal random variables with common correlation ρ are simultaneously less than or equal to H.

$\rho = .100$

N / −H	1	2	3	4	5	6	7	8	9	10	11	12
3.50	.00023	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.40	.00034	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.30	.00048	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.20	.00069	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.10	.00097	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.00	.00135	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.90	.00187	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.80	.00256	.00002	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.70	.00347	.00003	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.60	.00466	.00005	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.50	.00621	.00008	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.40	.00820	.00013	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.30	.01072	.00022	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.20	.01390	.00035	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.10	.01786	.00056	.00003	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.00	.02275	.00087	.00005	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
1.90	.02872	.00134	.00009	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
1.80	.03593	.00202	.00016	.00002	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
1.70	.04457	.00300	.00028	.00003	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000
1.60	.05480	.00440	.00048	.00007	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000
1.50	.06681	.00633	.00079	.00012	.00002	.00000	.00000	.00000	.00000	.00000	.00000	.00000
1.40	.08076	.00899	.00129	.00023	.00005	.00001	.00000	.00000	.00000	.00000	.00000	.00000
1.30	.09680	.01256	.00205	.00040	.00009	.00002	.00001	.00000	.00000	.00000	.00000	.00000
1.20	.11507	.01729	.00321	.00071	.00018	.00005	.00002	.00001	.00000	.00000	.00000	.00000
1.10	.13567	.02344	.00491	.00120	.00033	.00010	.00004	.00001	.00001	.00000	.00000	.00000
1.00	.15866	.03132	.00736	.00199	.00061	.00020	.00007	.00003	.00001	.00001	.00000	.00000
0.90	.18406	.04125	.01082	.00322	.00107	.00039	.00015	.00006	.00003	.00001	.00001	.00000
0.80	.21186	.05355	.01559	.00510	.00183	.00072	.00030	.00013	.00006	.00003	.00002	.00001
0.70	.24196	.06854	.02204	.00786	.00306	.00128	.00057	.00027	.00013	.00007	.00004	.00002
0.60	.27425	.08653	.03057	.01186	.00497	.00223	.00106	.00053	.00027	.00015	.00008	.00005
0.50	.30854	.10776	.04162	.01746	.00786	.00375	.00189	.00099	.00054	.00031	.00018	.00011
0.40	.34458	.13242	.05562	.02515	.01210	.00614	.00326	.00181	.00103	.00061	.00037	.00023
0.30	.38209	.16062	.07301	.03543	.01816	.00976	.00547	.00317	.00190	.00117	.00074	.00048
0.20	.42074	.19237	.09418	.04883	.02658	.01508	.00888	.00540	.00337	.00216	.00142	.00095
0.10	.46017	.22755	.11942	.06588	.03794	.02266	.01398	.00887	.00577	.00384	.00261	.00180
0.00	.50000	.26594	.14891	.08709	.05286	.03314	.02137	.01413	.00955	.00659	.00462	.00330

N / H	1	2	3	4	5	6	7	8	9	10	11	12
0.00	.50000	.26594	.14891	.08709	.05286	.03314	.02137	.01413	.00955	.00659	.00462	.00330
0.10	.53983	.30720	.18271	.11283	.07196	.04720	.03173	.02180	.01528	.01089	.00789	.00580
0.20	.57926	.35089	.22070	.14336	.09574	.06551	.04580	.03263	.02365	.01741	.01299	.00982
0.30	.61791	.39645	.26259	.17875	.12461	.08870	.06431	.04741	.03547	.02689	.02063	.01601
0.40	.65542	.44327	.30791	.21889	.15877	.11724	.08795	.06692	.05157	.04020	.03167	.02519
0.50	.69146	.49068	.35605	.26340	.19820	.15141	.11722	.09186	.07277	.05823	.04701	.03827
0.60	.72575	.53802	.40625	.31173	.24261	.19122	.15244	.12276	.09979	.08180	.06758	.05622
0.70	.75804	.58461	.45769	.36310	.29145	.23642	.19360	.15990	.13310	.11159	.09417	.07995
0.80	.78814	.62984	.50948	.41659	.34393	.28641	.24039	.20320	.17289	.14799	.12737	.11019
0.90	.81594	.67313	.56074	.47120	.39906	.34035	.29216	.25226	.21899	.19105	.16744	.14738
1.00	.84134	.71401	.61064	.52586	.45570	.39715	.34794	.30628	.27080	.24041	.21424	.19159
1.10	.86433	.75211	.65841	.57955	.51267	.45557	.40653	.36416	.32738	.29528	.26716	.24241
1.20	.88493	.78715	.70344	.63131	.56879	.51430	.46657	.42456	.38744	.35451	.32518	.29898
1.30	.90320	.81896	.74522	.68034	.62298	.57205	.52664	.48600	.44951	.41664	.38693	.36002
1.40	.91924	.84747	.78340	.72597	.67430	.62764	.58538	.54698	.51200	.48004	.45078	.42393
1.50	.93319	.87272	.81779	.76774	.72199	.68007	.64155	.60608	.57335	.54307	.51501	.48895
1.60	.94520	.89480	.84831	.80534	.76552	.72855	.69415	.66209	.63215	.60415	.57792	.55331
1.70	.95543	.91387	.87503	.83868	.80458	.77255	.74241	.71402	.68723	.66193	.63800	.61534
1.80	.96407	.93016	.89811	.86777	.83902	.81174	.78583	.76118	.73772	.71535	.69402	.67365
1.90	.97128	.94390	.91777	.89281	.86893	.84607	.82417	.80317	.78302	.76367	.74507	.72718
2.00	.97725	.95537	.93431	.91403	.89449	.87563	.85743	.83986	.82288	.80646	.79058	.77520
2.10	.98214	.96483	.94806	.93179	.91601	.90069	.88580	.87135	.85729	.84362	.83032	.81738
2.20	.98610	.97255	.95933	.94645	.93387	.92160	.90961	.89791	.88647	.87529	.86436	.85367
2.30	.98928	.97877	.96848	.95839	.94850	.93880	.92929	.91995	.91079	.90180	.89298	.88431
2.40	.99180	.98374	.97580	.96800	.96031	.95275	.94530	.93797	.93074	.92363	.91662	.90971
2.50	.99379	.98766	.98161	.97564	.96974	.96391	.95816	.95247	.94686	.94131	.93583	.93042
2.60	.99534	.99072	.98616	.98164	.97716	.97273	.96834	.96399	.95969	.95542	.95120	.94702
2.70	.99653	.99309	.98968	.98630	.98294	.97960	.97630	.97301	.96976	.96652	.96332	.96013
2.80	.99744	.99491	.99238	.98987	.98738	.98490	.98244	.97999	.97756	.97514	.97273	.97034
2.90	.99813	.99628	.99443	.99259	.99076	.98894	.98712	.98532	.98352	.98173	.97995	.97818
3.00	.99865	.99730	.99596	.99463	.99330	.99197	.99065	.98934	.98802	.98672	.98541	.98412
3.10	.99903	.99807	.99710	.99614	.99519	.99423	.99328	.99233	.99139	.99044	.98950	.98856
3.20	.99931	.99863	.99794	.99726	.99658	.99590	.99522	.99454	.99387	.99319	.99252	.99185
3.30	.99952	.99903	.99855	.99807	.99759	.99711	.99663	.99615	.99568	.99520	.99472	.99425
3.40	.99966	.99933	.99899	.99865	.99832	.99798	.99765	.99731	.99698	.99665	.99631	.99598
3.50	.99977	.99953	.99930	.99907	.99884	.99861	.99838	.99814	.99791	.99768	.99745	.99722

Table II

Probability that N standard normal random variables with common correlation ρ are simultaneously less than or equal to H.

$\rho = .125$

N / -H	1	2	3	4	5	6	7	8	9	10	11	12
3.50	.00023	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.40	.00034	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.30	.00048	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.20	.00069	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.10	.00097	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.00	.00135	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.90	.00187	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.80	.00256	.00002	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.70	.00347	.00003	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.60	.00466	.00006	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.50	.00621	.00009	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.40	.00820	.00016	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.30	.01072	.00025	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.20	.01390	.00040	.00002	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.10	.01786	.00064	.00004	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.00	.02275	.00098	.00007	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
1.90	.02872	.00149	.00012	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
1.80	.03593	.00224	.00021	.00003	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
1.70	.04457	.00330	.00036	.00005	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000
1.60	.05480	.00480	.00059	.00010	.00002	.00000	.00000	.00000	.00000	.00000	.00000	.00000
1.50	.06681	.00686	.00097	.00018	.00004	.00001	.00000	.00000	.00000	.00000	.00000	.00000
1.40	.08076	.00967	.00155	.00031	.00008	.00002	.00001	.00000	.00000	.00000	.00000	.00000
1.30	.09680	.01343	.00244	.00054	.00014	.00004	.00001	.00001	.00000	.00000	.00000	.00000
1.20	.11507	.01838	.00375	.00093	.00027	.00009	.00003	.00001	.00001	.00000	.00000	.00000
1.10	.13567	.02479	.00566	.00154	.00048	.00017	.00006	.00003	.00001	.00001	.00000	.00000
1.00	.15866	.03295	.00838	.00250	.00085	.00032	.00013	.00006	.00003	.00001	.00001	.00000
0.90	.18406	.04318	.01217	.00396	.00145	.00058	.00025	.00012	.00006	.00003	.00002	.00001
0.80	.21186	.05580	.01734	.00614	.00241	.00103	.00047	.00023	.00012	.00006	.00004	.00002
0.70	.24196	.07112	.02425	.00930	.00392	.00178	.00087	.00045	.00024	.00013	.00008	.00005
0.60	.27425	.08942	.03330	.01377	.00620	.00300	.00154	.00083	.00047	.00027	.00016	.00010
0.50	.30854	.11096	.04491	.01997	.00958	.00490	.00264	.00149	.00088	.00053	.00033	.00021
0.40	.34458	.13589	.05951	.02833	.01444	.00779	.00441	.00260	.00159	.00100	.00065	.00043
0.30	.38209	.16432	.07749	.03936	.02124	.01205	.00714	.00439	.00279	.00182	.00122	.00083
0.20	.42074	.19623	.09923	.05356	.03050	.01817	.01125	.00720	.00474	.00320	.00221	.00155
0.10	.46017	.23152	.12499	.07144	.04281	.02670	.01722	.01144	.00780	.00544	.00387	.00280
0.00	.50000	.26995	.15492	.09344	.05875	.03825	.02566	.01766	.01244	.00894	.00654	.00486

N / H	1	2	3	4	5	6	7	8	9	10	11	12
0.00	.50000	.26995	.15492	.09344	.05875	.03825	.02566	.01766	.01244	.00894	.00654	.00486
0.10	.53983	.31117	.18905	.11992	.07885	.05346	.03721	.02650	.01925	.01424	.01070	.00815
0.20	.57926	.35475	.22724	.15106	.10360	.07296	.05258	.03865	.02892	.02198	.01694	.01322
0.30	.61791	.40014	.26919	.18692	.13332	.09729	.07242	.05486	.04220	.03291	.02598	.02074
0.40	.65542	.44673	.31443	.22733	.16815	.12685	.09734	.07583	.05987	.04783	.03863	.03150
0.50	.69146	.49388	.36235	.27192	.20804	.16185	.12777	.10217	.08265	.06755	.05573	.04636
0.60	.72575	.54092	.41221	.32011	.25264	.20223	.16390	.13430	.11114	.09279	.07809	.06619
0.70	.75804	.58719	.46321	.37115	.30142	.24769	.20568	.17239	.14570	.12407	.10637	.09177
0.80	.78814	.63209	.51449	.42414	.35357	.29764	.25274	.21629	.18641	.16167	.14103	.12368
0.90	.81594	.67506	.56519	.47811	.40814	.35121	.30441	.26555	.23300	.20554	.18220	.16223
1.00	.84134	.71564	.61450	.53204	.46404	.40737	.35973	.31935	.28487	.25524	.22961	.20733
1.10	.86433	.75346	.66170	.58495	.52013	.46493	.41756	.37663	.34105	.30995	.28263	.25850
1.20	.88493	.78824	.70618	.63593	.57531	.52264	.47659	.43610	.40032	.36856	.34023	.31487
1.30	.90320	.81983	.74746	.68419	.62853	.57929	.53549	.49637	.46127	.42966	.40108	.37517
1.40	.91924	.84816	.78520	.72912	.67891	.63376	.59299	.55603	.52241	.49174	.46366	.43790
1.50	.93319	.87325	.81920	.77025	.72574	.68512	.64792	.61377	.58230	.55325	.52636	.50141
1.60	.94520	.89520	.84940	.80731	.76850	.73261	.69935	.66843	.63963	.61276	.58762	.56407
1.70	.95543	.91417	.87585	.84018	.80688	.77573	.74653	.71911	.69331	.66900	.64605	.62435
1.80	.96407	.93038	.89871	.86890	.84077	.81418	.78902	.76516	.74252	.72099	.70050	.68097
1.90	.97128	.94406	.91821	.89363	.87022	.84790	.82658	.80621	.78671	.76804	.75013	.73295
2.00	.97725	.95548	.93463	.91463	.89542	.87697	.85921	.84212	.82565	.80976	.79443	.77962
2.10	.98214	.96491	.94827	.93221	.91667	.90164	.88708	.87298	.85931	.84605	.83317	.82067
2.20	.98610	.97260	.95948	.94673	.93433	.92226	.91051	.89907	.88791	.87703	.86642	.85606
2.30	.98928	.97880	.96858	.95858	.94881	.93925	.92990	.92075	.91179	.90302	.89442	.88600
2.40	.99180	.98376	.97587	.96813	.96052	.95305	.94572	.93851	.93142	.92446	.91761	.91087
2.50	.99379	.98768	.98165	.97572	.96987	.96411	.95843	.95283	.94731	.94187	.93649	.93119
2.60	.99534	.99073	.98618	.98169	.97725	.97286	.96852	.96422	.95998	.95578	.95163	.94753
2.70	.99653	.99310	.98970	.98633	.98299	.97968	.97641	.97316	.96994	.96675	.96359	.96046
2.80	.99744	.99491	.99239	.98989	.98741	.98495	.98251	.98008	.97767	.97528	.97290	.97054
2.90	.99813	.99628	.99443	.99260	.99078	.98897	.98716	.98537	.98359	.98182	.98005	.97830
3.00	.99865	.99731	.99597	.99464	.99331	.99199	.99068	.98937	.98807	.98677	.98548	.98419
3.10	.99903	.99807	.99711	.99615	.99519	.99424	.99330	.99235	.99141	.99047	.98954	.98860
3.20	.99931	.99863	.99794	.99726	.99658	.99590	.99523	.99455	.99388	.99321	.99254	.99187
3.30	.99952	.99903	.99855	.99807	.99759	.99711	.99664	.99616	.99568	.99521	.99473	.99426
3.40	.99966	.99933	.99899	.99865	.99832	.99799	.99765	.99732	.99699	.99665	.99632	.99599
3.50	.99977	.99953	.99930	.99907	.99884	.99861	.99838	.99815	.99792	.99768	.99745	.99722

Table II

Probability that N standard normal random variables with common correlation ρ are simultaneously less than or equal to H.

$\rho = .200$

−H \ N	1	2	3	4	5	6	7	8	9	10	11	12
3.50	.00023	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.40	.00034	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.30	.00048	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.20	.00069	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.10	.00097	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.00	.00135	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.90	.00187	.00002	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.80	.00256	.00003	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.70	.00347	.00006	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.60	.00466	.00009	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.50	.00621	.00015	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.40	.00820	.00024	.00002	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.30	.01072	.00038	.00003	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.20	.01390	.00059	.00005	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.10	.01786	.00091	.00009	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.00	.02275	.00137	.00015	.00002	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000
1.90	.02872	.00204	.00025	.00005	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000
1.80	.03593	.00298	.00042	.00008	.00002	.00001	.00000	.00000	.00000	.00000	.00000	.00000
1.70	.04457	.00431	.00067	.00015	.00004	.00001	.00000	.00000	.00000	.00000	.00000	.00000
1.60	.05480	.00614	.00107	.00025	.00007	.00003	.00001	.00000	.00000	.00000	.00000	.00000
1.50	.06681	.00861	.00168	.00043	.00014	.00005	.00002	.00001	.00000	.00000	.00000	.00000
1.40	.08076	.01192	.00257	.00072	.00024	.00009	.00004	.00002	.00001	.00001	.00000	.00000
1.30	.09680	.01626	.00387	.00117	.00042	.00017	.00008	.00004	.00002	.00001	.00001	.00000
1.20	.11507	.02189	.00573	.00187	.00072	.00031	.00015	.00008	.00004	.00002	.00001	.00001
1.10	.13567	.02906	.00834	.00293	.00119	.00054	.00027	.00014	.00008	.00005	.00003	.00002
1.00	.15866	.03807	.01192	.00449	.00194	.00093	.00048	.00027	.00016	.00010	.00006	.00004
0.90	.18406	.04921	.01676	.00676	.00308	.00155	.00084	.00048	.00029	.00018	.00012	.00008
0.80	.21186	.06278	.02315	.00996	.00480	.00252	.00142	.00084	.00053	.00034	.00023	.00016
0.70	.24196	.07906	.03146	.01439	.00730	.00401	.00234	.00144	.00093	.00062	.00042	.00030
0.60	.27425	.09830	.04206	.02041	.01087	.00623	.00378	.00240	.00159	.00109	.00076	.00055
0.50	.30854	.12072	.05533	.02840	.01586	.00946	.00595	.00390	.00265	.00186	.00134	.00098
0.40	.34458	.14643	.07165	.03879	.02267	.01405	.00914	.00618	.00432	.00310	.00228	.00171
0.30	.38209	.17552	.09135	.05204	.03174	.02042	.01372	.00954	.00684	.00503	.00378	.00289
0.20	.42074	.20792	.11471	.06856	.04356	.02903	.02011	.01439	.01057	.00794	.00609	.00475
0.10	.46017	.24351	.14192	.08878	.05863	.04041	.02883	.02117	.01593	.01223	.00956	.00759
0.00	.50000	.28205	.17307	.11301	.07741	.05508	.04043	.03044	.02343	.01836	.01463	.01182

H \ N	1	2	3	4	5	6	7	8	9	10	11	12
0.00	.50000	.28205	.17307	.11301	.07741	.05508	.04043	.03044	.02343	.01836	.01463	.01182
0.10	.53983	.32317	.20810	.14149	.10033	.07358	.05546	.04277	.03363	.02689	.02181	.01791
0.20	.57926	.36644	.24684	.17430	.12769	.09634	.07448	.05876	.04716	.03842	.03170	.02646
0.30	.61791	.41134	.28893	.21138	.15966	.12375	.09797	.07897	.06463	.05360	.04496	.03810
0.40	.65542	.45728	.33392	.25248	.19624	.15599	.12631	.10390	.08662	.07305	.06224	.05351
0.50	.69146	.50364	.38120	.29720	.23725	.19309	.15970	.13389	.11358	.09734	.08417	.07336
0.60	.72575	.54980	.43008	.34496	.28229	.23486	.19813	.16912	.14584	.12689	.11126	.09824
0.70	.75804	.59513	.47982	.39502	.33077	.28089	.24137	.20952	.18348	.16191	.14385	.12859
0.80	.78814	.63906	.52961	.44658	.38194	.33054	.28895	.25477	.22633	.20240	.18206	.16462
0.90	.81594	.68109	.57869	.49874	.43491	.38301	.34016	.30431	.27398	.24806	.22572	.20632
1.00	.84134	.72076	.62632	.55059	.48871	.43733	.39411	.35733	.32572	.29832	.27439	.25334
1.10	.86433	.75773	.67185	.60127	.54234	.49246	.44975	.41283	.38063	.35234	.32732	.30505
1.20	.88493	.79175	.71472	.64999	.59484	.54732	.50596	.46968	.43760	.40905	.38351	.36052
1.30	.90320	.82266	.75451	.69605	.64532	.60088	.56162	.52669	.49542	.46726	.44177	.41861
1.40	.91924	.85040	.79091	.73891	.69302	.65221	.61565	.58270	.55285	.52568	.50083	.47802
1.50	.93319	.87500	.82373	.77816	.73733	.70051	.66710	.63663	.60872	.58305	.55935	.53740
1.60	.94520	.89654	.85293	.81357	.77782	.74515	.71517	.68753	.66196	.63821	.61609	.59543
1.70	.95543	.91518	.87856	.84504	.81421	.78572	.75929	.73468	.71170	.69017	.66995	.65092
1.80	.96407	.93112	.90074	.87260	.84641	.82197	.79906	.77755	.75728	.73814	.72004	.70287
1.90	.97128	.94460	.91971	.89639	.87448	.85382	.83431	.81582	.79827	.78159	.76569	.75051
2.00	.97725	.95587	.93571	.91664	.89856	.88138	.86502	.84940	.83448	.82019	.80649	.79334
2.10	.98214	.96518	.94904	.93365	.91894	.90486	.89135	.87838	.86590	.85388	.84230	.83111
2.20	.98610	.97279	.96002	.94775	.93594	.92456	.91358	.90297	.89271	.88278	.87315	.86381
2.30	.98928	.97893	.96894	.95928	.94993	.94086	.93206	.92352	.91521	.90714	.89927	.89161
2.40	.99180	.98385	.97612	.96860	.96128	.95416	.94721	.94043	.93381	.92734	.92102	.91484
2.50	.99379	.98773	.98182	.97603	.97038	.96485	.95944	.95413	.94893	.94384	.93884	.93393
2.60	.99534	.99077	.98629	.98189	.97758	.97334	.96918	.96509	.96106	.95710	.95321	.94937
2.70	.99653	.99312	.98977	.98646	.98321	.98000	.97684	.97372	.97065	.96762	.96463	.96168
2.80	.99744	.99492	.99243	.98998	.98755	.98515	.98278	.98044	.97812	.97583	.97357	.97133
2.90	.99813	.99629	.99446	.99265	.99086	.98909	.98733	.98560	.98387	.98217	.98047	.97880
3.00	.99865	.99731	.99598	.99467	.99336	.99207	.99078	.98950	.98824	.98698	.98574	.98450
3.10	.99903	.99807	.99712	.99617	.99523	.99429	.99336	.99243	.99151	.99060	.98969	.98879
3.20	.99931	.99863	.99795	.99727	.99660	.99593	.99526	.99460	.99394	.99329	.99263	.99198
3.30	.99952	.99903	.99856	.99808	.99760	.99713	.99666	.99619	.99572	.99525	.99479	.99433
3.40	.99966	.99933	.99899	.99866	.99833	.99799	.99766	.99733	.99701	.99668	.99635	.99603
3.50	.99977	.99953	.99930	.99907	.99884	.99861	.99838	.99815	.99793	.99770	.99747	.99724

Table II

Probability that N standard normal random variables with common correlation ρ are simultaneously less than or equal to H.

$\rho = .250$

N / −H	1	2	3	4	5	6	7	8	9	10	11	12
3.50	.00023	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.40	.00034	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.30	.00048	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.20	.00069	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.10	.00097	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.00	.00135	.00002	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.90	.00187	.00003	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.80	.00256	.00005	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.70	.00347	.00008	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.60	.00466	.00013	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.50	.00621	.00020	.00002	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.40	.00820	.00032	.00003	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.30	.01072	.00049	.00005	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.20	.01390	.00075	.00008	.00002	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.10	.01786	.00113	.00014	.00003	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.00	.02275	.00168	.00024	.00005	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000
1.90	.02872	.00247	.00039	.00009	.00003	.00001	.00000	.00000	.00000	.00000	.00000	.00000
1.80	.03593	.00356	.00062	.00015	.00005	.00002	.00001	.00000	.00000	.00000	.00000	.00000
1.70	.04457	.00508	.00098	.00026	.00009	.00003	.00002	.00001	.00000	.00000	.00000	.00000
1.60	.05480	.00715	.00151	.00044	.00015	.00006	.00003	.00002	.00001	.00000	.00000	.00000
1.50	.06681	.00991	.00230	.00071	.00027	.00012	.00006	.00003	.00002	.00001	.00001	.00000
1.40	.08076	.01357	.00345	.00114	.00045	.00021	.00010	.00006	.00003	.00002	.00001	.00001
1.30	.09680	.01832	.00508	.00180	.00075	.00036	.00019	.00010	.00006	.00004	.00003	.00002
1.20	.11507	.02441	.00735	.00278	.00123	.00061	.00033	.00019	.00012	.00007	.00005	.00003
1.10	.13567	.03210	.01047	.00421	.00195	.00101	.00056	.00034	.00021	.00014	.00009	.00007
1.00	.15866	.04168	.01468	.00626	.00305	.00164	.00095	.00058	.00038	.00025	.00017	.00012
0.90	.18406	.05342	.02025	.00914	.00466	.00260	.00156	.00098	.00065	.00045	.00032	.00023
0.80	.21186	.06762	.02749	.01310	.00699	.00405	.00250	.00163	.00110	.00077	.00056	.00041
0.70	.24196	.08453	.03675	.01846	.01028	.00617	.00394	.00263	.00182	.00131	.00096	.00072
0.60	.27425	.10439	.04838	.02556	.01483	.00922	.00606	.00415	.00295	.00216	.00162	.00124
0.50	.30854	.12738	.06274	.03478	.02099	.01350	.00912	.00641	.00465	.00347	.00265	.00206
0.40	.34458	.15360	.08016	.04653	.02916	.01936	.01344	.00968	.00718	.00546	.00424	.00335
0.30	.38209	.18311	.10095	.06121	.03977	.02722	.01941	.01431	.01084	.00839	.00663	.00532
0.20	.42074	.21583	.12532	.07922	.05327	.03755	.02746	.02070	.01599	.01261	.01012	.00825
0.10	.46017	.25162	.15344	.10089	.07010	.05081	.03808	.02932	.02308	.01852	.01510	.01248
0.00	.50000	.29022	.18532	.12648	.09066	.06748	.05176	.04067	.03262	.02660	.02202	.01845

N / H	1	2	3	4	5	6	7	8	9	10	11	12
0.00	.50000	.29022	.18532	.12648	.09066	.06748	.05176	.04067	.03262	.02660	.02202	.01845
0.10	.53983	.33127	.22090	.15615	.11528	.08800	.06900	.05530	.04513	.03739	.03140	.02666
0.20	.57926	.37435	.25994	.18994	.14418	.11274	.09027	.07370	.06116	.05145	.04380	.03768
0.30	.61791	.41893	.30210	.22771	.17745	.14193	.11593	.09634	.08123	.06933	.05981	.05208
0.40	.65542	.46445	.34691	.26918	.21500	.17569	.14623	.12358	.10578	.09154	.07997	.07045
0.50	.69146	.51030	.39377	.31393	.25660	.21393	.18125	.15563	.13514	.11849	.10477	.09331
0.60	.72575	.55588	.44203	.36137	.30181	.25641	.22089	.19252	.16946	.15044	.13453	.12109
0.70	.75804	.60060	.49095	.41080	.35003	.30265	.26484	.23410	.20870	.18745	.16945	.15405
0.80	.78814	.64391	.53980	.46144	.40054	.35202	.31256	.27994	.25259	.22937	.20946	.19223
0.90	.81594	.68530	.58784	.51245	.45248	.40372	.36337	.32946	.30062	.27582	.25429	.23545
1.00	.84134	.72437	.63439	.56299	.50495	.45686	.41638	.38185	.35208	.32616	.30339	.28326
1.10	.86433	.76077	.67884	.61227	.55704	.51046	.47063	.43617	.40607	.37954	.35600	.33497
1.20	.88493	.79427	.72067	.65954	.60787	.56355	.52509	.49137	.46155	.43498	.41115	.38965
1.30	.90320	.82472	.75947	.70419	.65663	.61520	.57874	.54637	.51742	.49135	.46774	.44625
1.40	.91924	.85205	.79498	.74571	.70263	.66457	.63063	.60014	.57258	.54751	.52460	.50357
1.50	.93319	.87630	.82701	.78374	.74533	.71094	.67991	.65173	.62599	.60236	.58058	.56042
1.60	.94520	.89755	.85552	.81806	.78434	.75378	.72590	.70031	.67672	.65489	.63459	.61566
1.70	.95543	.91595	.88057	.84858	.81943	.79270	.76807	.74525	.72403	.70422	.68568	.66826
1.80	.96407	.93170	.90228	.87533	.85050	.82750	.80609	.78609	.76734	.74971	.73308	.71736
1.90	.97128	.94503	.92086	.89847	.87761	.85812	.83982	.82258	.80630	.79089	.77626	.76234
2.00	.97725	.95618	.93656	.91819	.90093	.88465	.86925	.85464	.84074	.82750	.81486	.80276
2.10	.98214	.96540	.94966	.93478	.92069	.90729	.89453	.88234	.87068	.85950	.84877	.83844
2.20	.98610	.97294	.96046	.94856	.93721	.92634	.91592	.90591	.89628	.88699	.87804	.86938
2.30	.98928	.97904	.96925	.95986	.95083	.94214	.93375	.92565	.91782	.91024	.90288	.89575
2.40	.99180	.98392	.97633	.96900	.96191	.95505	.94840	.94194	.93567	.92957	.92363	.91784
2.50	.99379	.98778	.98196	.97630	.97081	.96547	.96026	.95519	.95024	.94541	.94068	.93607
2.60	.99534	.99080	.98638	.98208	.97787	.97376	.96974	.96581	.96196	.95819	.95449	.95086
2.70	.99653	.99314	.98983	.98658	.98340	.98027	.97721	.97420	.97125	.96835	.96549	.96269
2.80	.99744	.99494	.99247	.99005	.98767	.98533	.98302	.98076	.97852	.97632	.97414	.97200
2.90	.99813	.99630	.99449	.99270	.99094	.98920	.98749	.98580	.98413	.98248	.98085	.97924
3.00	.99865	.99732	.99600	.99470	.99341	.99214	.99088	.98963	.98840	.98718	.98597	.98478
3.10	.99903	.99807	.99713	.99619	.99526	.99433	.99342	.99251	.99162	.99072	.98984	.98897
3.20	.99931	.99863	.99795	.99728	.99662	.99596	.99530	.99465	.99400	.99336	.99272	.99209
3.30	.99952	.99904	.99856	.99808	.99761	.99714	.99668	.99622	.99576	.99530	.99484	.99439
3.40	.99966	.99933	.99899	.99866	.99833	.99800	.99768	.99735	.99703	.99670	.99638	.99606
3.50	.99977	.99954	.99930	.99907	.99885	.99862	.99839	.99816	.99794	.99771	.99749	.99727

Table II

Probability that N standard normal random variables with common correlation ρ are simultaneously less than or equal to H.

$\rho = .300$

−H \ N	1	2	3	4	5	6	7	8	9	10	11	12
3.50	.00023	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.40	.00034	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.30	.00048	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.20	.00069	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.10	.00097	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.00	.00135	.00002	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.90	.00187	.00004	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.80	.00256	.00006	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.70	.00347	.00010	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.60	.00466	.00017	.00002	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.50	.00621	.00026	.00003	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.40	.00820	.00041	.00005	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.30	.01072	.00062	.00008	.00002	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.20	.01390	.00094	.00014	.00003	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.10	.01786	.00139	.00022	.00005	.00002	.00001	.00000	.00000	.00000	.00000	.00000	.00000
2.00	.02275	.00204	.00036	.00009	.00003	.00001	.00001	.00000	.00000	.00000	.00000	.00000
1.90	.02872	.00295	.00057	.00016	.00006	.00002	.00001	.00001	.00000	.00000	.00000	.00000
1.80	.03593	.00422	.00089	.00027	.00010	.00004	.00002	.00001	.00001	.00000	.00000	.00000
1.70	.04457	.00594	.00137	.00044	.00017	.00008	.00004	.00002	.00001	.00001	.00001	.00000
1.60	.05480	.00826	.00207	.00070	.00029	.00014	.00007	.00004	.00002	.00002	.00001	.00001
1.50	.06681	.01133	.00308	.00111	.00048	.00024	.00013	.00008	.00005	.00003	.00002	.00001
1.40	.08076	.01535	.00451	.00172	.00078	.00040	.00022	.00013	.00009	.00006	.00004	.00003
1.30	.09680	.02052	.00650	.00262	.00124	.00066	.00038	.00024	.00015	.00010	.00007	.00005
1.20	.11507	.02709	.00923	.00393	.00195	.00107	.00064	.00040	.00027	.00019	.00013	.00010
1.10	.13567	.03531	.01290	.00580	.00299	.00170	.00104	.00068	.00046	.00032	.00024	.00018
1.00	.15866	.04546	.01777	.00840	.00451	.00265	.00167	.00111	.00077	.00055	.00041	.00031
0.90	.18406	.05781	.02410	.01196	.00668	.00406	.00263	.00179	.00126	.00092	.00069	.00053
0.80	.21186	.07263	.03221	.01676	.00971	.00609	.00405	.00282	.00203	.00151	.00115	.00090
0.70	.24196	.09017	.04243	.02309	.01388	.00896	.00611	.00434	.00320	.00242	.00188	.00148
0.60	.27425	.11063	.05509	.03131	.01948	.01295	.00904	.00657	.00493	.00379	.00298	.00239
0.50	.30854	.13418	.07052	.04177	.02687	.01836	.01314	.00974	.00744	.00582	.00464	.00377
0.40	.34458	.16090	.08902	.05487	.03645	.02558	.01872	.01416	.01100	.00874	.00708	.00582
0.30	.38209	.19082	.11085	.07096	.04861	.03500	.02618	.02019	.01596	.01287	.01056	.00879
0.20	.42074	.22385	.13620	.09039	.06376	.04705	.03595	.02824	.02268	.01856	.01543	.01300
0.10	.46017	.25983	.16516	.11344	.08229	.06217	.04847	.03875	.03162	.02624	.02210	.01883
0.00	.50000	.29849	.19774	.14031	.10453	.08077	.06421	.05221	.04325	.03638	.03101	.02673

H \ N	1	2	3	4	5	6	7	8	9	10	11	12
0.00	.50000	.29849	.19774	.14031	.10453	.08077	.06421	.05221	.04325	.03638	.03101	.02673
0.10	.53983	.33949	.23381	.17109	.13073	.10321	.08358	.06907	.05805	.04948	.04267	.03718
0.20	.57926	.38237	.27314	.20575	.16104	.12975	.10694	.08977	.07650	.06602	.05759	.05070
0.30	.61791	.42664	.31534	.24413	.19547	.16056	.13458	.11466	.09901	.08647	.07625	.06781
0.40	.65542	.47175	.35996	.28591	.23387	.19565	.16662	.14397	.12590	.11122	.09911	.08898
0.50	.69146	.51710	.40641	.33064	.27594	.23487	.20306	.17782	.15738	.14055	.12649	.11461
0.60	.72575	.56213	.45405	.37774	.32124	.27789	.24370	.21613	.19348	.17458	.15861	.14496
0.70	.75804	.60624	.50219	.42653	.36916	.32423	.28817	.25863	.23403	.21325	.19549	.18015
0.80	.78814	.64892	.55011	.47627	.41898	.37325	.33592	.30488	.27869	.25630	.23696	.22008
0.90	.81594	.68969	.59715	.52617	.46991	.42417	.38623	.35424	.32690	.30327	.28263	.26446
1.00	.84134	.72815	.64264	.57545	.52111	.47615	.43829	.40593	.37795	.35349	.33191	.31275
1.10	.86433	.76398	.68602	.62337	.57172	.52828	.49117	.45905	.43095	.40613	.38404	.36423
1.20	.88493	.79695	.72682	.66925	.62093	.57967	.54394	.51264	.48495	.46026	.43807	.41801
1.30	.90320	.82692	.76465	.71252	.66803	.62949	.59568	.56573	.53895	.51484	.49299	.47308
1.40	.91924	.85383	.79926	.75273	.71240	.67698	.64554	.61737	.59195	.56886	.54776	.52838
1.50	.93319	.87771	.83049	.78954	.75354	.72151	.69275	.66674	.64304	.62132	.60132	.58283
1.60	.94520	.89866	.85830	.82277	.79110	.76260	.73674	.71311	.69140	.67135	.65276	.63544
1.70	.95543	.91681	.88275	.85234	.82489	.79991	.77703	.75594	.73640	.71822	.70124	.68532
1.80	.96407	.93236	.90397	.87828	.85483	.83328	.81335	.79483	.77754	.76135	.74612	.73175
1.90	.97128	.94552	.92214	.90073	.88099	.86267	.84558	.82958	.81454	.80035	.78693	.77420
2.00	.97725	.95654	.93751	.91990	.90350	.88816	.87373	.86013	.84726	.83504	.82342	.81233
2.10	.98214	.96566	.95036	.93606	.92262	.90995	.89795	.88657	.87572	.86538	.85548	.84600
2.20	.98610	.97313	.96096	.94949	.93863	.92831	.91848	.90909	.90010	.89148	.88319	.87521
2.30	.98928	.97917	.96961	.96052	.95186	.94357	.93563	.92800	.92067	.91359	.90676	.90015
2.40	.99180	.98401	.97658	.96947	.96264	.95608	.94975	.94365	.93774	.93203	.92648	.92111
2.50	.99379	.98784	.98213	.97663	.97132	.96619	.96122	.95640	.95172	.94717	.94274	.93843
2.60	.99534	.99084	.98650	.98230	.97822	.97426	.97040	.96665	.96299	.95943	.95594	.95254
2.70	.99653	.99317	.98990	.98673	.98363	.98061	.97766	.97478	.97196	.96920	.96650	.96385
2.80	.99744	.99495	.99252	.99015	.98783	.98555	.98333	.98114	.97900	.97690	.97483	.97280
2.90	.99813	.99631	.99452	.99276	.99104	.98935	.98769	.98605	.98445	.98286	.98130	.97977
3.00	.99865	.99732	.99602	.99474	.99347	.99223	.99101	.98980	.98861	.98743	.98627	.98513
3.10	.99903	.99808	.99714	.99621	.99530	.99439	.99350	.99262	.99175	.99088	.99003	.98919
3.20	.99931	.99863	.99796	.99730	.99664	.99599	.99535	.99471	.99408	.99346	.99284	.99223
3.30	.99952	.99904	.99856	.99809	.99763	.99717	.99671	.99625	.99581	.99536	.99492	.99448
3.40	.99966	.99933	.99900	.99867	.99834	.99802	.99769	.99737	.99706	.99674	.99643	.99612
3.50	.99977	.99953	.99930	.99908	.99885	.99862	.99840	.99818	.99796	.99773	.99752	.99730

Table II

Probability that N standard normal random variables with common correlation ρ are simultaneously less than or equal to H.

$$\rho = 1/3$$

$-H$ \ N	1	2	3	4	5	6	7	8	9	10	11	12
3.50	.00023	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.40	.00034	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.30	.00048	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.20	.00069	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.10	.00097	.00002	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.00	.00135	.00003	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.90	.00187	.00005	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.80	.00256	.00008	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.70	.00347	.00013	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.60	.00466	.00020	.00002	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.50	.00621	.00031	.00004	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.40	.00820	.00048	.00007	.00002	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.30	.01072	.00072	.00011	.00003	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.20	.01390	.00108	.00018	.00005	.00002	.00001	.00000	.00000	.00000	.00000	.00000	.00000
2.10	.01786	.00159	.00029	.00008	.00003	.00001	.00001	.00000	.00000	.00000	.00000	.00000
2.00	.02275	.00231	.00046	.00014	.00005	.00002	.00001	.00001	.00000	.00000	.00000	.00000
1.90	.02872	.00331	.00072	.00023	.00009	.00004	.00002	.00001	.00001	.00000	.00000	.00000
1.80	.03593	.00469	.00111	.00037	.00015	.00007	.00004	.00002	.00001	.00001	.00001	.00000
1.70	.04457	.00656	.00168	.00059	.00026	.00013	.00007	.00004	.00003	.00002	.00001	.00001
1.60	.05480	.00906	.00251	.00093	.00042	.00022	.00012	.00007	.00005	.00003	.00002	.00002
1.50	.06681	.01234	.00368	.00145	.00068	.00036	.00021	.00013	.00008	.00006	.00004	.00003
1.40	.08076	.01661	.00532	.00220	.00108	.00059	.00035	.00022	.00015	.00010	.00007	.00006
1.30	.09680	.02207	.00758	.00330	.00168	.00095	.00058	.00038	.00026	.00018	.00013	.00010
1.20	.11507	.02896	.01063	.00486	.00257	.00150	.00094	.00063	.00043	.00031	.00023	.00018
1.10	.13567	.03754	.01469	.00705	.00386	.00232	.00150	.00102	.00072	.00053	.00040	.00031
1.00	.15866	.04807	.02002	.01005	.00571	.00353	.00233	.00162	.00117	.00087	.00066	.00052
0.90	.18406	.06083	.02688	.01411	.00829	.00528	.00357	.00253	.00186	.00141	.00109	.00086
0.80	.21186	.07607	.03558	.01948	.01185	.00776	.00537	.00388	.00290	.00223	.00175	.00140
0.70	.24196	.09402	.04644	.02649	.01664	.01119	.00792	.00583	.00443	.00346	.00275	.00223
0.60	.27425	.11489	.05978	.03547	.02298	.01586	.01147	.00860	.00665	.00526	.00425	.00349
0.50	.30854	.13881	.07591	.04678	.03123	.02209	.01631	.01246	.00978	.00785	.00642	.00533
0.40	.34458	.16586	.09512	.06076	.04176	.03024	.02280	.01772	.01413	.01150	.00952	.00799
0.30	.38209	.19604	.11763	.07778	.05495	.04072	.03130	.02475	.02003	.01651	.01383	.01174
0.20	.42074	.22928	.14360	.09813	.07119	.05393	.04224	.03395	.02787	.02328	.01972	.01692
0.10	.46017	.26538	.17311	.12206	.09082	.07028	.05604	.04576	.03808	.03221	.02760	.02392
0.00	.50000	.30409	.20613	.14974	.11413	.09012	.07311	.06061	.05113	.04375	.03790	.03318

H \ N	1	2	3	4	5	6	7	8	9	10	11	12
0.00	.50000	.30409	.20613	.14974	.11413	.09012	.07311	.06061	.05113	.04375	.03790	.03318
0.10	.53983	.34504	.24251	.18121	.14133	.11377	.09384	.07893	.06744	.05838	.05111	.04516
0.20	.57926	.38780	.28201	.21642	.17250	.14144	.11853	.10109	.08744	.07654	.06768	.06036
0.30	.61791	.43187	.32423	.25516	.20763	.17324	.14739	.12737	.11149	.09863	.08805	.07922
0.40	.65542	.47670	.36871	.29711	.24653	.20913	.18049	.15796	.13983	.12497	.11260	.10217
0.50	.69146	.52173	.41489	.34180	.28887	.24891	.21777	.19288	.17258	.15574	.14157	.12950
0.60	.72575	.56638	.46212	.38866	.33418	.29223	.25897	.23200	.20971	.19100	.17508	.16140
0.70	.75804	.61010	.50974	.43703	.38187	.33858	.30371	.27501	.25100	.23061	.21310	.19789
0.80	.78814	.65236	.55707	.48618	.43124	.38733	.35140	.32143	.29605	.27426	.25536	.23880
0.90	.81594	.69271	.60344	.53536	.48150	.43771	.40135	.37062	.34429	.32145	.30144	.28377
1.00	.84134	.73076	.64824	.58381	.53186	.48892	.45275	.42180	.39499	.37150	.35073	.33223
1.10	.86433	.76621	.69093	.63084	.58150	.54009	.50473	.47412	.44731	.42360	.40245	.38346
1.20	.88493	.79882	.73104	.67581	.62968	.59039	.55641	.52665	.50033	.47683	.45570	.43657
1.30	.90320	.82847	.76822	.71818	.67570	.63902	.60691	.57849	.55310	.53024	.50951	.49062
1.40	.91924	.85509	.80223	.75753	.71900	.68529	.65545	.62876	.60471	.58286	.56291	.54458
1.50	.93319	.87873	.83292	.79354	.75912	.72863	.70134	.67670	.65430	.63380	.61493	.59748
1.60	.94520	.89946	.86026	.82604	.79573	.76858	.74403	.72166	.70115	.68224	.66471	.64841
1.70	.95543	.91743	.88431	.85497	.82866	.80484	.78310	.76312	.74466	.72752	.71153	.69656
1.80	.96407	.93283	.90518	.88036	.85785	.83727	.81831	.80076	.78441	.76913	.75479	.74129
1.90	.97128	.94588	.92307	.90235	.88336	.86583	.84956	.83437	.82013	.80674	.79409	.78213
2.00	.97725	.95681	.93821	.92114	.90533	.89062	.87686	.86392	.85172	.84017	.82921	.81878
2.10	.98214	.96586	.95088	.93698	.92401	.91184	.90037	.88952	.87922	.86942	.86007	.85113
2.20	.98610	.97327	.96134	.95017	.93966	.92973	.92031	.91134	.90279	.89460	.88676	.87922
2.30	.98928	.97927	.96988	.96102	.95261	.94462	.93699	.92969	.92269	.91595	.90947	.90322
2.40	.99180	.98408	.97677	.96982	.96319	.95684	.95074	.94488	.93923	.93378	.92851	.92341
2.50	.99379	.98789	.98226	.97688	.97171	.96673	.96193	.95729	.95280	.94845	.94423	.94012
2.60	.99534	.99088	.98659	.98247	.97849	.97463	.97090	.96728	.96376	.96034	.95701	.95376
2.70	.99653	.99319	.98997	.98684	.98381	.98087	.97801	.97522	.97250	.96984	.96725	.96471
2.80	.99744	.99497	.99256	.99023	.98795	.98573	.98356	.98144	.97937	.97734	.97535	.97339
2.90	.99813	.99632	.99454	.99281	.99112	.98947	.98784	.98625	.98469	.98316	.98165	.98017
3.00	.99865	.99733	.99604	.99477	.99353	.99231	.99111	.98993	.98877	.98763	.98650	.98540
3.10	.99903	.99808	.99715	.99623	.99533	.99444	.99357	.99270	.99185	.99101	.99018	.98937
3.20	.99931	.99863	.99797	.99731	.99666	.99602	.99539	.99477	.99415	.99354	.99294	.99234
3.30	.99951	.99904	.99857	.99810	.99764	.99718	.99673	.99629	.99585	.99541	.99498	.99455
3.40	.99966	.99933	.99900	.99867	.99835	.99803	.99771	.99739	.99708	.99677	.99647	.99616
3.50	.99977	.99953	.99931	.99908	.99885	.99863	.99841	.99819	.99797	.99775	.99754	.99732

Table II

Probability that N standard normal random variables with common correlation ρ are simultaneously less than or equal to H.

$\rho = .375$

$-H$ \ N	1	2	3	4	5	6	7	8	9	10	11	12
3.50	.00023	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.40	.00034	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.30	.00048	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.20	.00069	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.10	.00097	.00002	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.00	.00135	.00004	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.90	.00187	.00006	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.80	.00256	.00010	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.70	.00347	.00016	.00002	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.60	.00466	.00025	.00004	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.50	.00621	.00038	.00006	.00001	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.40	.00820	.00058	.00010	.00003	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.30	.01072	.00086	.00016	.00005	.00002	.00001	.00000	.00000	.00000	.00000	.00000	.00000
2.20	.01390	.00128	.00025	.00008	.00003	.00001	.00001	.00000	.00000	.00000	.00000	.00000
2.10	.01786	.00186	.00040	.00013	.00005	.00002	.00001	.00001	.00000	.00000	.00000	.00000
2.00	.02275	.00268	.00063	.00021	.00009	.00004	.00002	.00001	.00001	.00001	.00000	.00000
1.90	.02872	.00381	.00096	.00034	.00015	.00008	.00004	.00003	.00002	.00001	.00001	.00001
1.80	.03593	.00534	.00145	.00054	.00025	.00013	.00007	.00005	.00003	.00002	.00001	.00001
1.70	.04457	.00740	.00215	.00085	.00040	.00022	.00013	.00008	.00005	.00004	.00003	.00002
1.60	.05480	.01013	.00315	.00130	.00064	.00036	.00022	.00014	.00009	.00007	.00005	.00004
1.50	.06681	.01369	.00455	.00197	.00101	.00058	.00036	.00024	.00016	.00012	.00009	.00007
1.40	.08076	.01828	.00647	.00293	.00155	.00091	.00058	.00039	.00028	.00020	.00015	.00012
1.30	.09680	.02410	.00908	.00431	.00236	.00143	.00093	.00064	.00046	.00034	.00026	.00020
1.20	.11507	.03141	.01256	.00622	.00352	.00219	.00146	.00102	.00074	.00056	.00043	.00034
1.10	.13567	.04044	.01714	.00885	.00518	.00330	.00224	.00160	.00118	.00090	.00071	.00056
1.00	.15866	.05146	.02305	.01239	.00748	.00490	.00340	.00247	.00185	.00144	.00114	.00092
0.90	.18406	.06473	.03059	.01709	.01064	.00714	.00506	.00374	.00285	.00224	.00179	.00146
0.80	.21186	.08049	.04004	.02323	.01489	.01023	.00739	.00556	.00431	.00342	.00278	.00229
0.70	.24196	.09896	.05171	.03111	.02051	.01442	.01063	.00812	.00639	.00514	.00422	.00352
0.60	.27425	.12032	.06591	.04106	.02783	.02000	.01502	.01166	.00930	.00758	.00629	.00530
0.50	.30854	.14470	.08292	.05342	.03717	.02730	.02088	.01647	.01331	.01098	.00921	.00784
0.40	.34458	.17216	.10299	.06852	.04890	.03667	.02854	.02286	.01873	.01563	.01325	.01138
0.30	.38209	.20268	.12633	.08668	.06338	.04850	.03839	.03120	.02589	.02186	.01872	.01622
0.20	.42074	.23616	.15306	.10815	.08096	.06315	.05081	.04188	.03519	.03004	.02598	.02272
0.10	.46017	.27242	.18323	.13315	.10193	.08100	.06620	.05531	.04704	.04058	.03544	.03127
0.00	.50000	.31118	.21677	.16179	.12653	.10235	.08493	.07189	.06185	.05392	.04753	.04229

H \ N	1	2	3	4	5	6	7	8	9	10	11	12
0.00	.50000	.31118	.21677	.16179	.12653	.10235	.08493	.07189	.06185	.05392	.04753	.04229
0.10	.53983	.35208	.25352	.19408	.15491	.12745	.10730	.09199	.08002	.07046	.06267	.05622
0.20	.57926	.39468	.29320	.22992	.18711	.15645	.13356	.11590	.10191	.09059	.08128	.07349
0.30	.61791	.43850	.33544	.26907	.22304	.18940	.16385	.14383	.12777	.11464	.10371	.09450
0.40	.65542	.48300	.37975	.31120	.26249	.22620	.19816	.17589	.15779	.14282	.13025	.11955
0.50	.69146	.52763	.42558	.35582	.30511	.26660	.23637	.21202	.19200	.17526	.16105	.14886
0.60	.72575	.57182	.47230	.40237	.35040	.31021	.27817	.25203	.23027	.21189	.19615	.18252
0.70	.75804	.61503	.51929	.45020	.39778	.35652	.32315	.29555	.27234	.25252	.23540	.22045
0.80	.78814	.65678	.56587	.49862	.44655	.40489	.37070	.34209	.31775	.29677	.27849	.26239
0.90	.81594	.69661	.61143	.54690	.49598	.45458	.42015	.39099	.36592	.34411	.32493	.30792
1.00	.84134	.73415	.65537	.59434	.54530	.50483	.47072	.44150	.41613	.39385	.37411	.35646
1.10	.86433	.76911	.69719	.64028	.59376	.55481	.52158	.49279	.46756	.44521	.42524	.40726
1.20	.88493	.80127	.73645	.68413	.64066	.60376	.57190	.54402	.51934	.49730	.47746	.45948
1.30	.90320	.83050	.77282	.72539	.68537	.65095	.62089	.59433	.57061	.54925	.52988	.51221
1.40	.91924	.85676	.80608	.76367	.72736	.69574	.66783	.64293	.62051	.60017	.58159	.56453
1.50	.93319	.88007	.83610	.79868	.76622	.73761	.71211	.68914	.66829	.64924	.63172	.61554
1.60	.94520	.90053	.86284	.83028	.80166	.77616	.75321	.73237	.71330	.69575	.67951	.66442
1.70	.95543	.91827	.88636	.85840	.83352	.81113	.79079	.77217	.75501	.73912	.72431	.71048
1.80	.96407	.93348	.90679	.88309	.86177	.84239	.82464	.80826	.79306	.77889	.76562	.75315
1.90	.97128	.94637	.92431	.90449	.88646	.86993	.85466	.84047	.82722	.81479	.80309	.79204
2.00	.97725	.95718	.93916	.92278	.90775	.89385	.88091	.86880	.85743	.84670	.83654	.82690
2.10	.98214	.96613	.95159	.93823	.92586	.91433	.90352	.89334	.88372	.87460	.86593	.85765
2.20	.98610	.97347	.96186	.95110	.94106	.93162	.92272	.91429	.90628	.89864	.89135	.88436
2.30	.98928	.97942	.97026	.96170	.95364	.94603	.93880	.93192	.92534	.91904	.91300	.90719
2.40	.99180	.98418	.97704	.97031	.96393	.95787	.95208	.94654	.94122	.93610	.93117	.92642
2.50	.99379	.98796	.98246	.97723	.97224	.96747	.96290	.95850	.95425	.95016	.94620	.94236
2.60	.99534	.99093	.98673	.98271	.97886	.97516	.97159	.96815	.96481	.96158	.95844	.95539
2.70	.99653	.99322	.99006	.98701	.98407	.98124	.97849	.97583	.97324	.97072	.96827	.96588
2.80	.99744	.99499	.99263	.99034	.98813	.98598	.98389	.98186	.97988	.97795	.97606	.97422
2.90	.99813	.99633	.99458	.99289	.99124	.98964	.98807	.98654	.98504	.98358	.98215	.98074
3.00	.99865	.99734	.99606	.99482	.99361	.99242	.99126	.99012	.98901	.98791	.98684	.98578
3.10	.99903	.99809	.99717	.99626	.99538	.99451	.99366	.99283	.99201	.99120	.99041	.98962
3.20	.99931	.99864	.99798	.99733	.99670	.99607	.99545	.99485	.99425	.99366	.99308	.99251
3.30	.99952	.99904	.99857	.99811	.99766	.99721	.99677	.99634	.99591	.99549	.99507	.99466
3.40	.99966	.99933	.99900	.99868	.99836	.99805	.99773	.99743	.99712	.99682	.99652	.99623
3.50	.99977	.99954	.99931	.99908	.99886	.99864	.99843	.99821	.99800	.99778	.99758	.99737

Table II

Probability that N standard normal random variables with common correlation ρ are simultaneously less than or equal to H.

$\rho = .400$

N / −H	1	2	3	4	5	6	7	8	9	10	11	12
3.50	.00023	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.40	.00034	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.30	.00048	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.20	.00069	.00002	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.10	.00097	.00003	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.00	.00135	.00005	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.90	.00187	.00007	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.80	.00256	.00012	.00002	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.70	.00347	.00018	.00003	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.60	.00466	.00028	.00004	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.50	.00621	.00043	.00007	.00002	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.40	.00820	.00065	.00012	.00004	.00001	.00001	.00000	.00000	.00000	.00000	.00000	.00000
2.30	.01072	.00096	.00020	.00006	.00002	.00001	.00001	.00000	.00000	.00000	.00000	.00000
2.20	.01390	.00141	.00031	.00010	.00004	.00002	.00001	.00001	.00000	.00000	.00000	.00000
2.10	.01786	.00204	.00048	.00017	.00007	.00004	.00002	.00001	.00001	.00001	.00000	.00000
2.00	.02275	.00292	.00074	.00027	.00012	.00006	.00004	.00002	.00001	.00001	.00001	.00001
1.90	.02872	.00413	.00112	.00043	.00020	.00011	.00006	.00004	.00003	.00002	.00001	.00001
1.80	.03593	.00576	.00168	.00067	.00032	.00018	.00011	.00007	.00005	.00003	.00002	.00002
1.70	.04457	.00794	.00247	.00103	.00052	.00029	.00018	.00012	.00008	.00006	.00004	.00003
1.60	.05480	.01081	.00358	.00157	.00081	.00047	.00030	.00020	.00014	.00010	.00008	.00006
1.50	.06681	.01454	.00513	.00234	.00125	.00075	.00048	.00033	.00023	.00017	.00013	.00010
1.40	.08076	.01933	.00724	.00345	.00191	.00117	.00077	.00053	.00039	.00029	.00022	.00018
1.30	.09680	.02538	.01007	.00500	.00285	.00179	.00120	.00085	.00062	.00047	.00037	.00029
1.20	.11507	.03294	.01382	.00714	.00420	.00270	.00185	.00133	.00099	.00076	.00060	.00049
1.10	.13567	.04225	.01872	.01006	.00610	.00401	.00281	.00205	.00155	.00121	.00097	.00079
1.00	.15866	.05356	.02500	.01394	.00870	.00587	.00418	.00311	.00239	.00189	.00152	.00125
0.90	.18406	.06714	.03295	.01906	.01223	.00843	.00612	.00463	.00361	.00288	.00235	.00195
0.80	.21186	.08321	.04285	.02567	.01693	.01193	.00882	.00677	.00535	.00433	.00357	.00299
0.70	.24196	.10199	.05501	.03409	.02308	.01661	.01251	.00975	.00780	.00638	.00532	.00450
0.60	.27425	.12365	.06973	.04463	.03099	.02277	.01744	.01380	.01119	.00926	.00779	.00665
0.50	.30854	.14831	.08726	.05763	.04100	.03074	.02394	.01921	.01577	.01320	.01122	.00966
0.40	.34458	.17600	.10785	.07339	.05347	.04086	.03234	.02631	.02187	.01850	.01588	.01380
0.30	.38209	.20672	.13168	.09222	.06872	.05350	.04302	.03547	.02983	.02550	.02209	.01936
0.20	.42074	.24036	.15885	.11436	.08709	.06902	.05634	.04706	.04004	.03457	.03023	.02671
0.10	.46017	.27671	.18940	.13998	.10884	.08775	.07268	.06148	.05288	.04611	.04068	.03623
0.00	.50000	.31549	.22324	.16917	.13419	.10998	.09238	.07909	.06876	.06053	.05385	.04833

N / H	1	2	3	4	5	6	7	8	9	10	11	12
0.00	.50000	.31549	.22324	.16917	.13419	.10998	.09238	.07909	.06876	.06053	.05385	.04833
0.10	.53983	.35636	.26020	.20193	.16325	.13592	.11570	.10022	.08803	.07821	.07015	.06344
0.20	.57926	.39887	.29999	.23812	.19603	.16569	.14287	.12514	.11100	.09949	.08996	.08194
0.30	.61791	.44255	.34223	.27750	.23241	.19928	.17396	.15401	.13791	.12466	.11358	.10419
0.40	.65542	.48685	.38643	.31971	.27216	.23657	.20894	.18689	.16887	.15390	.14126	.13044
0.50	.69146	.53123	.43205	.36428	.31491	.27730	.24765	.22368	.20388	.18725	.17309	.16087
0.60	.72575	.57515	.47847	.41063	.36017	.32105	.28977	.26416	.24277	.22463	.20904	.19550
0.70	.75804	.61807	.52508	.45814	.40734	.36730	.33484	.30794	.28523	.26579	.24895	.23419
0.80	.78814	.65950	.57122	.50612	.45575	.41542	.38229	.35450	.33080	.31033	.29244	.27666
0.90	.81594	.69902	.61629	.55387	.50468	.46470	.43141	.40319	.37888	.35770	.33904	.32245
1.00	.84134	.73625	.65972	.60071	.55339	.51436	.48147	.45327	.42876	.40721	.38808	.37095
1.10	.86433	.77091	.70102	.64600	.60115	.56364	.53165	.50394	.47963	.45808	.43881	.42144
1.20	.88493	.80280	.73978	.68919	.64729	.61179	.58117	.55438	.53067	.50949	.49040	.47309
1.30	.90320	.83178	.77567	.72979	.69122	.65813	.62927	.60378	.58104	.56056	.54198	.52502
1.40	.91924	.85781	.80847	.76743	.73243	.70205	.67527	.65141	.62994	.61047	.59269	.57636
1.50	.93319	.88093	.83807	.80185	.77055	.74305	.71859	.69660	.67666	.65845	.64171	.62625
1.60	.94520	.90121	.86445	.83290	.80529	.78078	.75876	.73881	.72058	.70383	.68833	.67394
1.70	.95543	.91881	.88765	.86053	.83652	.81498	.79547	.77764	.76124	.74607	.73195	.71877
1.80	.96407	.93390	.90781	.88480	.86420	.84554	.82850	.81281	.79829	.78476	.77212	.76024
1.90	.97128	.94669	.92511	.90583	.88840	.87247	.85780	.84420	.83153	.81967	.80851	.79798
2.00	.97725	.95742	.93977	.92383	.90927	.89585	.88341	.87180	.86091	.85066	.84098	.83180
2.10	.98214	.96631	.95205	.93903	.92703	.91589	.90548	.89571	.88649	.87777	.86949	.86161
2.20	.98610	.97360	.96221	.95170	.94194	.93282	.92423	.91613	.90844	.90113	.89416	.88750
2.30	.98928	.97951	.97051	.96214	.95431	.94693	.93995	.93332	.92700	.92097	.91519	.90964
2.40	.99180	.98425	.97722	.97063	.96442	.95853	.95293	.94759	.94247	.93756	.93284	.92829
2.50	.99379	.98801	.98258	.97746	.97259	.96796	.96352	.95927	.95518	.95124	.94744	.94376
2.60	.99534	.99096	.98681	.98287	.97911	.97551	.97204	.96871	.96548	.96237	.95935	.95642
2.70	.99653	.99325	.99012	.98712	.98425	.98148	.97881	.97623	.97372	.97129	.96893	.96663
2.80	.99744	.99501	.99267	.99042	.98825	.98615	.98411	.98214	.98022	.97835	.97653	.97475
2.90	.99813	.99634	.99461	.99294	.99132	.98975	.98822	.98673	.98528	.98386	.98247	.98111
3.00	.99865	.99734	.99608	.99485	.99366	.99250	.99136	.99025	.98916	.98810	.98706	.98604
3.10	.99903	.99809	.99718	.99629	.99542	.99456	.99373	.99291	.99211	.99133	.99055	.98979
3.20	.99931	.99864	.99799	.99735	.99672	.99610	.99550	.99491	.99432	.99375	.99318	.99263
3.30	.99952	.99904	.99858	.99812	.99768	.99724	.99680	.99638	.99596	.99554	.99513	.99473
3.40	.99966	.99933	.99901	.99869	.99837	.99806	.99775	.99745	.99715	.99686	.99656	.99628
3.50	.99977	.99954	.99931	.99909	.99887	.99865	.99844	.99822	.99801	.99781	.99760	.99740

Table II

Probability that N standard normal random variables with common correlation ρ are simultaneously less than or equal to H.

$\rho = .500$

-H \ N	1	2	3	4	5	6	7	8	9	10	11	12
3.50	.00023	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.40	.00034	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.30	.00048	.00002	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.20	.00069	.00003	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.10	.00097	.00005	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.00	.00135	.00008	.00002	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.90	.00187	.00013	.00003	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.80	.00256	.00020	.00004	.00001	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.70	.00347	.00030	.00007	.00002	.00001	.00001	.00000	.00000	.00000	.00000	.00000	.00000
2.60	.00466	.00045	.00011	.00004	.00002	.00001	.00001	.00000	.00000	.00000	.00000	.00000
2.50	.00621	.00067	.00017	.00006	.00003	.00002	.00001	.00001	.00000	.00000	.00000	.00000
2.40	.00820	.00098	.00027	.00011	.00005	.00003	.00002	.00001	.00001	.00001	.00000	.00000
2.30	.01072	.00143	.00041	.00017	.00009	.00005	.00003	.00002	.00001	.00001	.00001	.00001
2.20	.01390	.00204	.00062	.00027	.00014	.00008	.00005	.00004	.00003	.00002	.00001	.00001
2.10	.01786	.00289	.00093	.00041	.00022	.00013	.00009	.00006	.00004	.00003	.00003	.00002
2.00	.02275	.00405	.00137	.00063	.00035	.00021	.00014	.00010	.00007	.00006	.00004	.00004
1.90	.02872	.00561	.00201	.00096	.00054	.00034	.00023	.00016	.00012	.00009	.00007	.00006
1.80	.03593	.00767	.00289	.00143	.00083	.00053	.00037	.00026	.00020	.00016	.00012	.00010
1.70	.04457	.01037	.00411	.00210	.00125	.00082	.00057	.00042	.00032	.00025	.00020	.00017
1.60	.05480	.01386	.00576	.00305	.00185	.00124	.00088	.00066	.00051	.00040	.00033	.00027
1.50	.06681	.01832	.00799	.00436	.00272	.00185	.00133	.00101	.00079	.00063	.00052	.00043
1.40	.08076	.02394	.01093	.00616	.00393	.00272	.00200	.00153	.00121	.00098	.00081	.00068
1.30	.09680	.03094	.01476	.00858	.00560	.00395	.00294	.00228	.00182	.00149	.00124	.00105
1.20	.11507	.03955	.01970	.01179	.00788	.00565	.00427	.00335	.00270	.00223	.00188	.00160
1.10	.13567	.04999	.02596	.01600	.01092	.00797	.00611	.00485	.00395	.00329	.00279	.00241
1.00	.15866	.06251	.03380	.02142	.01494	.01109	.00861	.00692	.00570	.00479	.00410	.00355
0.90	.18406	.07734	.04347	.02832	.02015	.01521	.01197	.00973	.00809	.00686	.00591	.00516
0.80	.21186	.09469	.05526	.03695	.02683	.02058	.01641	.01348	.01133	.00969	.00841	.00739
0.70	.24196	.11472	.06941	.04762	.03525	.02746	.02218	.01842	.01562	.01348	.01179	.01043
0.60	.27425	.13757	.08619	.06060	.04570	.03614	.02957	.02481	.02124	.01847	.01627	.01449
0.50	.30854	.16332	.10580	.07617	.05850	.04694	.03887	.03296	.02847	.02495	.02214	.01984
0.40	.34458	.19198	.12841	.09458	.07393	.06017	.05041	.04318	.03762	.03323	.02969	.02677
0.30	.38209	.22349	.15414	.11606	.09227	.07612	.06451	.05579	.04902	.04363	.03924	.03561
0.20	.42074	.25771	.18303	.14075	.11374	.09508	.08147	.07113	.06302	.05650	.05115	.04668
0.10	.46017	.29442	.21503	.16873	.13850	.11727	.10156	.08948	.07991	.07215	.06574	.06034
0.00	.50000	.33333	.25000	.20000	.16667	.14286	.12500	.11111	.10000	.09091	.08333	.07692

H \ N	1	2	3	4	5	6	7	8	9	10	11	12
0.00	.50000	.33333	.25000	.20000	.16667	.14286	.12500	.11111	.10000	.09091	.08333	.07692
0.10	.53983	.37408	.28772	.23446	.19823	.17192	.15193	.13620	.12350	.11302	.10422	.09673
0.20	.57926	.41623	.32788	.27192	.23308	.20445	.18240	.16487	.15058	.13869	.12863	.12000
0.30	.61791	.45931	.37006	.31206	.27104	.24032	.21637	.19713	.18129	.16801	.15669	.14692
0.40	.65542	.50282	.41379	.35450	.31177	.27930	.25368	.23287	.21559	.20099	.18845	.17757
0.50	.69146	.54624	.45855	.39874	.35486	.32104	.29403	.27187	.25332	.23750	.22384	.21190
0.60	.72575	.58906	.50376	.44425	.39981	.36509	.33704	.31380	.29417	.27732	.26266	.24977
0.70	.75804	.63079	.54885	.49041	.44605	.41091	.38220	.35820	.33775	.32006	.30458	.29089
0.80	.78814	.67098	.59323	.53661	.49293	.45788	.42894	.40451	.38353	.36525	.34915	.33483
0.90	.81594	.70922	.63638	.58224	.53983	.50536	.47660	.45211	.43090	.41231	.39582	.38107
1.00	.84134	.74520	.67778	.62670	.58608	.55257	.52450	.50030	.47920	.46056	.44394	.42898
1.10	.86433	.77866	.71701	.66944	.63107	.59913	.57195	.54839	.52770	.50930	.49279	.47786
1.20	.88493	.80941	.75373	.71000	.67424	.64414	.61828	.59568	.57569	.55780	.54166	.52698
1.30	.90320	.83734	.78766	.74799	.71510	.68713	.66287	.64151	.62248	.60535	.58980	.57559
1.40	.91924	.86243	.81864	.78309	.75326	.72753	.70519	.68529	.66744	.65127	.63652	.62297
1.50	.93319	.88471	.84656	.81513	.78843	.76525	.74480	.72652	.71001	.69498	.68119	.66847
1.60	.94520	.90427	.87143	.84398	.82040	.79973	.78134	.76479	.74975	.73598	.72328	.71151
1.70	.95543	.92124	.89331	.86964	.84908	.83090	.81459	.79982	.78631	.77388	.76236	.75163
1.80	.96407	.93581	.91233	.89218	.87448	.85870	.84444	.83144	.81948	.80841	.79811	.78848
1.90	.97128	.94817	.92867	.91172	.89669	.88317	.87087	.85958	.84914	.83943	.83036	.82183
2.00	.97725	.95855	.94253	.92845	.91585	.90442	.89395	.88429	.87531	.86691	.85902	.85159
2.10	.98214	.96717	.95416	.94260	.93217	.92264	.91385	.90569	.89806	.89090	.88415	.87776
2.20	.98610	.97424	.96380	.95443	.94590	.93805	.93077	.92397	.91758	.91156	.90586	.90045
2.30	.98928	.97998	.97169	.96419	.95730	.95092	.94497	.93938	.93411	.92911	.92437	.91984
2.40	.99180	.98459	.97809	.97215	.96665	.96153	.95673	.95220	.94790	.94382	.93992	.93619
2.50	.99379	.98825	.98321	.97856	.97423	.97017	.96635	.96272	.95927	.95597	.95282	.94979
2.60	.99534	.99113	.98726	.98366	.98030	.97712	.97411	.97125	.96851	.96588	.96337	.96094
2.70	.99653	.99336	.99043	.98768	.98509	.98264	.98030	.97807	.97592	.97386	.97188	.96996
2.80	.99744	.99509	.99288	.99081	.98884	.98697	.98517	.98345	.98180	.98020	.97865	.97716
2.90	.99813	.99640	.99476	.99321	.99173	.99032	.98896	.98765	.98639	.98517	.98398	.98283
3.00	.99865	.99738	.99618	.99504	.99394	.99288	.99187	.99089	.98994	.98901	.98812	.98724
3.10	.99903	.99812	.99724	.99641	.99560	.99483	.99408	.99335	.99254	.99195	.99128	.99063
3.20	.99931	.99866	.99803	.99743	.99684	.99628	.99573	.99520	.99468	.99417	.99367	.99319
3.30	.99952	.99905	.99861	.99817	.99776	.99735	.99695	.99657	.99619	.99582	.99546	.99511
3.40	.99966	.99934	.99902	.99872	.99842	.99813	.99785	.99757	.99730	.99704	.99678	.99652
3.50	.99977	.99954	.99932	.99911	.99890	.99870	.99850	.99830	.99811	.99792	.99774	.99756

Table II

Probability that N standard normal random variables with common correlation ρ are simultaneously less than or equal to H.

$$\rho = .600$$

-H \ N	1	2	3	4	5	6	7	8	9	10	11	12
3.50	.00023	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.40	.00034	.00002	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.30	.00048	.00004	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.20	.00069	.00006	.00001	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.10	.00097	.00009	.00002	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.00	.00135	.00014	.00004	.00002	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000
2.90	.00187	.00021	.00006	.00003	.00001	.00001	.00001	.00000	.00000	.00000	.00000	.00000
2.80	.00256	.00032	.00010	.00004	.00002	.00001	.00001	.00001	.00001	.00000	.00000	.00000
2.70	.00347	.00047	.00015	.00007	.00004	.00002	.00002	.00001	.00001	.00001	.00001	.00000
2.60	.00466	.00069	.00023	.00011	.00006	.00004	.00003	.00002	.00001	.00001	.00001	.00001
2.50	.00621	.00101	.00035	.00017	.00010	.00006	.00004	.00003	.00002	.00002	.00002	.00001
2.40	.00820	.00145	.00053	.00026	.00016	.00010	.00007	.00005	.00004	.00003	.00003	.00002
2.30	.01072	.00206	.00078	.00040	.00024	.00016	.00012	.00009	.00007	.00005	.00004	.00004
2.20	.01390	.00289	.00115	.00060	.00037	.00025	.00018	.00014	.00011	.00009	.00007	.00006
2.10	.01786	.00401	.00166	.00090	.00056	.00039	.00028	.00022	.00017	.00014	.00012	.00010
2.00	.02275	.00550	.00237	.00132	.00084	.00059	.00044	.00034	.00027	.00022	.00019	.00016
1.90	.02872	.00747	.00335	.00191	.00124	.00088	.00066	.00052	.00042	.00035	.00029	.00025
1.80	.03593	.01003	.00468	.00274	.00182	.00131	.00099	.00078	.00064	.00053	.00045	.00039
1.70	.04457	.01333	.00645	.00387	.00262	.00191	.00147	.00117	.00096	.00081	.00069	.00050
1.60	.05480	.01753	.00880	.00541	.00372	.00275	.00214	.00172	.00143	.00121	.00104	.00091
1.50	.06681	.02279	.01186	.00747	.00523	.00392	.00308	.00251	.00209	.00178	.00154	.00135
1.40	.08076	.02933	.01581	.01018	.00725	.00551	.00438	.00359	.00302	.00259	.00226	.00199
1.30	.09680	.03736	.02082	.01372	.00994	.00765	.00614	.00509	.00431	.00372	.00326	.00290
1.20	.11507	.04708	.02711	.01826	.01345	.01049	.00851	.00711	.00607	.00528	.00465	.00415
1.10	.13567	.05873	.03492	.02403	.01798	.01420	.01164	.00981	.00844	.00738	.00655	.00587
1.00	.15866	.07253	.04447	.03125	.02375	.01898	.01572	.01336	.01158	.01019	.00909	.00819
0.90	.18406	.08866	.05601	.04017	.03099	.02507	.02096	.01796	.01568	.01389	.01246	.01128
0.80	.21186	.10731	.06978	.05104	.03996	.03271	.02761	.02385	.02097	.01869	.01685	.01533
0.70	.24196	.12863	.08600	.06412	.05093	.04216	.03593	.03128	.02769	.02483	.02250	.02057
0.60	.27425	.15270	.10488	.07965	.06416	.05370	.04618	.04052	.03610	.03256	.02966	.02724
0.50	.30854	.17956	.12657	.09786	.07989	.06759	.05864	.05184	.04649	.04217	.03862	.03563
0.40	.34458	.20919	.15118	.11893	.09837	.08410	.07360	.06553	.05915	.05395	.04965	.04602
0.30	.38209	.24150	.17876	.14299	.11978	.10344	.09128	.08187	.07434	.06819	.06305	.05869
0.20	.42074	.27631	.20927	.17012	.14425	.12580	.11192	.10107	.09233	.08514	.07910	.07394
0.10	.46017	.31338	.24262	.20030	.17186	.15130	.13567	.12334	.11334	.10505	.09804	.09204
0.00	.50000	.35242	.27862	.23345	.20259	.17999	.16263	.14882	.13753	.12810	.12009	.11320

H \ N	1	2	3	4	5	6	7	8	9	10	11	12
0.00	.50000	.35242	.27862	.23345	.20259	.17999	.16263	.14882	.13753	.12810	.12009	.11320
0.10	.53983	.39304	.31701	.26941	.23636	.21185	.19282	.17756	.16499	.15442	.14540	.13759
0.20	.57926	.43482	.35742	.30790	.27297	.24674	.22618	.20954	.19573	.18406	.17404	.16532
0.30	.61791	.47732	.39946	.34859	.31214	.28445	.26252	.24463	.22969	.21698	.20600	.19639
0.40	.65542	.52003	.44266	.39105	.35352	.32466	.30160	.28263	.26668	.25302	.24116	.23073
0.50	.69146	.56248	.48650	.43480	.39665	.36699	.34305	.32321	.30641	.29194	.27930	.26815
0.60	.72575	.60419	.53046	.47931	.44104	.41094	.38644	.36596	.34851	.33339	.32012	.30835
0.70	.75804	.64470	.57400	.52403	.48612	.45599	.43123	.41039	.39251	.37693	.36319	.35095
0.80	.78814	.68360	.61660	.56840	.53133	.50155	.47688	.45595	.43788	.42204	.40801	.39545
0.90	.81594	.72054	.65780	.61187	.57608	.54704	.52277	.50204	.48402	.46815	.45402	.44131
1.00	.84134	.75521	.69715	.65392	.61982	.59186	.56830	.54804	.53032	.51463	.50059	.48791
1.10	.86433	.78740	.73429	.69410	.66202	.63545	.61289	.59334	.57616	.56086	.54710	.53462
1.20	.88493	.81694	.76893	.73203	.70222	.67730	.65598	.63738	.62093	.60621	.59291	.58080
1.30	.90320	.84376	.80085	.76739	.74004	.71697	.69707	.67961	.66407	.65010	.63743	.62584
1.40	.91924	.86782	.82993	.79994	.77516	.75408	.73576	.71958	.70510	.69202	.68010	.66916
1.50	.93319	.88919	.85610	.82955	.80739	.78836	.77171	.75691	.74361	.73152	.72046	.71027
1.60	.94520	.90793	.87938	.85616	.83658	.81963	.80469	.79133	.77926	.76825	.75813	.74876
1.70	.95543	.92420	.89984	.87979	.86269	.84778	.83455	.82266	.81185	.80194	.79281	.78432
1.80	.96407	.93817	.91763	.90050	.88577	.87282	.86125	.85079	.84124	.83245	.82431	.81673
1.90	.97128	.95004	.93291	.91846	.90591	.89481	.88482	.87574	.86742	.85972	.85256	.84587
2.00	.97725	.96000	.94588	.93383	.92328	.91387	.90536	.89759	.89042	.88377	.87756	.87173
2.10	.98213	.96828	.95677	.94684	.93808	.93020	.92304	.91647	.91038	.90470	.89938	.89438
2.20	.98610	.97508	.96580	.95772	.95052	.94402	.93807	.93258	.92747	.92269	.91820	.91396
2.30	.98928	.98061	.97321	.96671	.96087	.95556	.95068	.94615	.94192	.93795	.93421	.93066
2.40	.99180	.98505	.97922	.97405	.96937	.96509	.96114	.95745	.95399	.95073	.94765	.94472
2.50	.99379	.98859	.98405	.97998	.97628	.97287	.96970	.96673	.96394	.96130	.95880	.95641
2.60	.99534	.99137	.98787	.98471	.98181	.97913	.97662	.97427	.97204	.96993	.96792	.96600
2.70	.99653	.99354	.99087	.98844	.98620	.98411	.98216	.98031	.97856	.97689	.97530	.97378
2.80	.99744	.99521	.99319	.99135	.98964	.98803	.98652	.98509	.98373	.98243	.98119	.97999
2.90	.99813	.99648	.99498	.99359	.99230	.99108	.98993	.98884	.98779	.98679	.98583	.98490
3.00	.99865	.99744	.99633	.99530	.99433	.99342	.99255	.99173	.99094	.99017	.98944	.98873
3.10	.99903	.99816	.99735	.99659	.99588	.99520	.99455	.99394	.99334	.99277	.99222	.99169
3.20	.99931	.99868	.99810	.99755	.99703	.99653	.99606	.99560	.99516	.99474	.99433	.99393
3.30	.99952	.99907	.99865	.99826	.99788	.99752	.99718	.99685	.99652	.99621	.99591	.99562
3.40	.99966	.99935	.99905	.99877	.99851	.99825	.99800	.99776	.99753	.99730	.99708	.99687
3.50	.99977	.99955	.99934	.99915	.99896	.99877	.99860	.99843	.99826	.99810	.99794	.99779

Table II

Probability that N standard normal random variables with common correlation ρ are simultaneously less than or equal to H.

$\rho = .625$

N / − H	1	2	3	4	5	6	7	8	9	10	11	12
3.50	.00023	.00002	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.40	.00034	.00003	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.30	.00048	.00004	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.20	.00069	.00007	.00002	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.10	.00097	.00010	.00003	.00001	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.00	.00135	.00016	.00005	.00002	.00001	.00001	.00000	.00000	.00000	.00000	.00000	.00000
2.90	.00187	.00024	.00008	.00003	.00002	.00001	.00001	.00001	.00000	.00000	.00000	.00000
2.80	.00256	.00036	.00012	.00006	.00003	.00002	.00001	.00001	.00001	.00001	.00000	.00000
2.70	.00347	.00053	.00018	.00009	.00005	.00003	.00002	.00002	.00001	.00001	.00001	.00001
2.60	.00466	.00077	.00028	.00014	.00008	.00005	.00004	.00003	.00002	.00002	.00001	.00001
2.50	.00621	.00111	.00042	.00021	.00013	.00009	.00006	.00005	.00004	.00003	.00002	.00002
2.40	.00820	.00159	.00062	.00032	.00020	.00013	.00010	.00007	.00006	.00005	.00004	.00003
2.30	.01072	.00224	.00091	.00049	.00031	.00021	.00015	.00012	.00009	.00008	.00006	.00005
2.20	.01390	.00314	.00132	.00073	.00046	.00032	.00024	.00019	.00015	.00012	.00010	.00009
2.10	.01786	.00433	.00190	.00107	.00070	.00049	.00037	.00029	.00023	.00019	.00016	.00014
2.00	.02275	.00592	.00269	.00156	.00103	.00074	.00056	.00044	.00036	.00030	.00026	.00022
1.90	.02872	.00801	.00378	.00224	.00151	.00109	.00084	.00067	.00055	.00046	.00040	.00034
1.80	.03593	.01071	.00524	.00318	.00217	.00160	.00124	.00100	.00083	.00070	.00060	.00053
1.70	.04457	.01417	.00718	.00446	.00310	.00231	.00181	.00147	.00123	.00104	.00090	.00079
1.60	.05480	.01855	.00973	.00618	.00437	.00330	.00261	.00214	.00180	.00154	.00134	.00118
1.50	.06681	.02404	.01303	.00846	.00607	.00465	.00372	.00307	.00260	.00224	.00196	.00174
1.40	.08076	.03083	.01725	.01144	.00835	.00647	.00523	.00436	.00371	.00322	.00284	.00253
1.30	.09680	.03912	.02259	.01530	.01134	.00890	.00726	.00610	.00523	.00457	.00405	.00362
1.20	.11507	.04914	.02926	.02023	.01522	.01208	.00995	.00843	.00728	.00640	.00570	.00512
1.10	.13567	.06111	.03748	.02643	.02018	.01620	.01348	.01150	.01001	.00885	.00792	.00716
1.00	.15866	.07523	.04748	.03414	.02644	.02148	.01803	.01551	.01358	.01208	.01086	.00987
0.90	.18406	.09171	.05951	.04361	.03425	.02812	.02382	.02065	.01821	.01628	.01472	.01343
0.80	.21186	.11070	.07380	.05507	.04385	.03640	.03110	.02716	.02410	.02167	.01969	.01805
0.70	.24196	.13234	.09056	.06879	.05550	.04655	.04013	.03529	.03152	.02850	.02602	.02395
0.60	.27425	.15672	.10997	.08499	.06945	.05886	.05116	.04532	.04072	.03702	.03396	.03140
0.50	.30854	.18387	.13219	.10387	.08595	.07357	.06447	.05751	.05199	.04751	.04379	.04065
0.40	.34458	.21375	.15730	.12561	.10521	.09092	.08032	.07212	.06559	.06024	.05578	.05200
0.30	.38209	.24626	.18534	.15031	.12739	.11112	.09893	.08942	.08178	.07549	.07021	.06572
0.20	.42074	.28122	.21625	.17803	.15260	.13432	.12049	.10961	.10080	.09350	.08734	.08207
0.10	.46017	.31838	.24993	.20875	.18089	.16063	.14514	.13285	.12283	.11448	.10740	.10131
0.00	.50000	.35745	.28618	.24234	.21224	.19007	.17295	.15926	.14802	.13860	.13056	.12362

N / H	1	2	3	4	5	6	7	8	9	10	11	12
0.00	.50000	.35745	.28618	.24234	.21224	.19007	.17295	.15926	.14802	.13860	.13056	.12362
0.10	.53983	.39804	.32471	.27864	.24651	.22257	.20391	.18886	.17643	.16594	.15694	.14913
0.20	.57926	.43974	.36518	.31736	.28351	.25800	.23792	.22160	.20803	.19651	.18658	.17791
0.30	.61791	.48208	.40717	.35815	.32294	.29610	.27479	.25733	.24271	.23023	.21942	.20994
0.40	.65542	.54259	.45021	.50059	.36443	.33656	.31423	.29580	.28027	.26693	.25532	.24510
0.50	.69146	.56680	.49381	.44419	.40753	.37897	.35587	.33668	.32040	.30634	.29404	.28316
0.60	.72575	.60822	.53744	.48843	.45173	.42283	.39927	.37954	.36270	.34808	.33523	.32381
0.70	.75804	.64842	.58058	.53276	.49649	.46763	.44390	.42389	.40670	.39170	.37845	.36663
0.80	.78814	.68699	.62273	.57665	.54124	.51280	.48921	.46918	.45187	.43669	.42321	.41114
0.90	.81594	.72359	.66343	.61956	.58543	.55775	.53460	.51482	.49762	.48246	.46894	.45677
1.00	.84134	.75792	.70225	.66100	.62852	.60192	.57950	.56022	.54336	.52842	.51503	.50294
1.10	.86433	.78978	.73886	.70053	.67000	.64477	.62334	.60479	.58847	.57394	.56087	.54902
1.20	.88493	.81901	.77297	.73779	.70945	.68581	.66560	.64798	.63240	.61846	.60586	.59439
1.30	.90320	.84552	.80438	.77248	.74650	.72463	.70580	.68929	.67461	.66141	.64943	.63848
1.40	.91924	.86931	.83296	.80438	.78086	.76089	.74357	.72829	.71464	.70231	.69107	.68076
1.50	.93319	.89043	.85868	.83337	.81233	.79433	.77861	.76465	.75212	.74074	.73034	.72076
1.60	.94520	.90896	.88154	.85941	.84082	.82479	.81069	.79811	.78675	.77640	.76690	.75811
1.70	.95543	.92504	.90164	.88250	.86628	.85218	.83970	.82851	.81835	.80905	.80048	.79253
1.80	.96407	.93885	.91909	.90275	.88877	.87652	.86561	.85577	.84680	.83856	.83093	.82383
1.90	.97128	.95057	.93409	.92029	.90838	.89788	.88846	.87992	.87211	.86489	.85819	.85194
2.00	.97725	.96042	.94682	.93531	.92529	.91639	.90836	.90105	.89432	.88809	.88228	.87684
2.10	.98214	.96860	.95751	.94802	.93968	.93223	.92548	.91929	.91358	.90826	.90329	.89862
2.20	.98610	.97533	.96637	.95864	.95180	.94563	.94002	.93485	.93006	.92558	.92138	.91742
2.30	.98928	.98080	.97365	.96742	.96186	.95683	.95222	.94796	.94399	.94027	.93676	.93345
2.40	.99180	.98519	.97956	.97459	.97014	.96608	.96234	.95886	.95562	.95256	.94968	.94694
2.50	.99379	.98869	.98429	.98039	.97686	.97362	.97062	.96783	.96520	.96273	.96038	.95815
2.60	.99534	.98531	.98805	.98501	.98224	.97969	.97732	.97510	.97301	.97103	.96914	.96735
2.70	.99653	.99359	.99100	.98866	.98652	.98453	.98268	.98093	.97929	.97772	.97623	.97480
2.80	.99744	.99525	.99329	.99151	.98987	.98834	.98691	.98556	.98427	.98305	.98188	.98076
2.90	.99813	.99651	.99505	.99371	.99247	.99130	.99021	.98917	.98819	.98724	.98634	.98547
3.00	.99865	.99746	.99638	.99538	.99445	.99358	.99276	.99197	.99122	.99050	.98981	.98915
3.10	.99903	.99817	.99738	.99665	.99596	.99531	.99470	.99411	.99355	.99301	.99249	.99199
3.20	.99931	.99869	.99812	.99759	.99709	.99661	.99616	.99572	.99531	.99491	.99452	.99414
3.30	.99952	.99908	.99867	.99828	.99792	.99758	.99725	.99693	.99662	.99633	.99604	.99577
3.40	.99966	.99935	.99906	.99879	.99853	.99828	.99805	.99782	.99760	.99738	.99718	.99697
3.50	.99977	.99955	.99935	.99916	.99897	.99880	.99863	.99847	.99831	.99815	.99801	.99786

Table II

Probability that N standard normal random variables with common correlation ρ are simultaneously less than or equal to H.

$$\rho = 2/3$$

-H \ N	1	2	3	4	5	6	7	8	9	10	11	12
3.50	.00023	.00002	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.40	.00034	.00003	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.30	.00048	.00005	.00002	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.20	.00069	.00008	.00003	.00001	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.10	.00097	.00013	.00004	.00002	.00001	.00001	.00001	.00000	.00000	.00000	.00000	.00000
3.00	.00135	.00020	.00007	.00003	.00002	.00001	.00001	.00001	.00001	.00000	.00000	.00000
2.90	.00187	.00029	.00010	.00005	.00003	.00002	.00002	.00001	.00001	.00001	.00001	.00000
2.80	.00256	.00043	.00016	.00008	.00005	.00003	.00002	.00002	.00001	.00001	.00001	.00001
2.70	.00347	.00063	.00024	.00013	.00008	.00005	.00004	.00003	.00002	.00002	.00002	.00001
2.60	.00466	.00091	.00037	.00020	.00013	.00009	.00006	.00005	.00004	.00003	.00003	.00002
2.50	.00621	.00131	.00055	.00030	.00019	.00014	.00010	.00008	.00006	.00005	.00004	.00004
2.40	.00820	.00185	.00080	.00045	.00030	.00021	.00016	.00012	.00010	.00008	.00007	.00006
2.30	.01072	.00259	.00116	.00067	.00044	.00032	.00024	.00019	.00016	.00013	.00011	.00010
2.20	.01390	.00360	.00167	.00099	.00066	.00048	.00037	.00030	.00025	.00021	.00018	.00016
2.10	.01786	.00493	.00236	.00143	.00098	.00072	.00056	.00045	.00038	.00032	.00028	.00024
2.00	.02275	.00669	.00331	.00204	.00142	.00106	.00083	.00068	.00057	.00048	.00042	.00037
1.90	.02872	.00897	.00459	.00289	.00204	.00154	.00122	.00100	.00085	.00073	.00063	.00056
1.80	.03593	.01192	.00630	.00405	.00290	.00222	.00177	.00147	.00124	.00107	.00094	.00084
1.70	.04457	.01567	.00854	.00560	.00407	.00315	.00254	.00212	.00181	.00157	.00139	.00124
1.60	.05480	.02039	.01144	.00765	.00564	.00441	.00360	.00302	.00259	.00227	.00201	.00180
1.50	.06681	.02626	.01517	.01035	.00773	.00611	.00503	.00425	.00368	.00323	.00288	.00259
1.40	.08076	.03347	.01990	.01383	.01047	.00837	.00694	.00592	.00515	.00455	.00407	.00368
1.30	.09680	.04224	.02581	.01827	.01402	.01133	.00948	.00813	.00712	.00632	.00569	.00516
1.20	.11507	.05277	.03313	.02387	.01856	.01515	.01278	.01105	.00973	.00869	.00785	.00716
1.10	.13567	.06528	.04208	.03085	.02430	.02003	.01704	.01483	.01313	.01179	.01070	.00980
1.00	.15866	.07997	.05287	.03943	.03145	.02618	.02245	.01967	.01752	.01581	.01441	.01325
0.90	.18406	.09702	.06575	.04985	.04025	.03383	.02924	.02579	.02310	.02095	.01919	.01771
0.80	.21186	.11659	.08092	.06234	.05094	.04323	.03765	.03342	.03011	.02744	.02523	.02338
0.70	.24196	.13880	.09859	.07715	.06378	.05462	.04793	.04282	.03878	.03551	.03280	.03051
0.60	.27425	.16371	.11892	.09448	.07899	.06825	.06033	.05424	.04939	.04543	.04214	.03935
0.50	.30854	.19134	.14203	.11451	.09679	.08436	.07510	.06793	.06218	.05746	.05351	.05015
0.40	.34458	.22163	.16797	.13737	.11736	.10315	.09248	.08413	.07741	.07185	.06718	.06319
0.30	.38209	.25448	.19677	.16314	.14082	.12479	.11264	.10307	.09530	.08885	.08340	.07871
0.20	.42074	.28970	.22835	.19185	.16727	.14941	.13575	.12491	.11605	.10866	.10238	.09696
0.10	.46017	.32702	.26257	.22343	.19669	.17705	.16189	.14977	.13981	.13145	.12431	.11812
0.00	.50000	.36614	.29921	.25775	.22902	.20769	.19109	.17771	.16665	.15732	.14931	.14234

H \ N	1	2	3	4	5	6	7	8	9	10	11	12
0.00	.50000	.36614	.29921	.25775	.22902	.20769	.19109	.17771	.16665	.15732	.14931	.14234
0.10	.53983	.40668	.33798	.29459	.26411	.24124	.22328	.20871	.19659	.18630	.17744	.16969
0.20	.57926	.44822	.37852	.33366	.30173	.27751	.25833	.24266	.22955	.21837	.20868	.20018
0.30	.61791	.49031	.42041	.37459	.34154	.31622	.29601	.27937	.26537	.25336	.24292	.23372
0.40	.65542	.53248	.46319	.41696	.38317	.35703	.33599	.31856	.30380	.29108	.27996	.27014
0.50	.69146	.57426	.50637	.46027	.42616	.39950	.37788	.35985	.34449	.33119	.31952	.30916
0.60	.72575	.61520	.54944	.50403	.47001	.44317	.42123	.40281	.38704	.37331	.36121	.35043
0.70	.75804	.65487	.59190	.54770	.51418	.48750	.46551	.44694	.43095	.41697	.40459	.39351
0.80	.78814	.69288	.63328	.59076	.55815	.53194	.51020	.49171	.47570	.46164	.44913	.43790
0.90	.81594	.72890	.67313	.63272	.60137	.57596	.55471	.53654	.52072	.50676	.49429	.48305
1.00	.84134	.76266	.71106	.67311	.64335	.61900	.59851	.58087	.56544	.55175	.53948	.52839
1.10	.86433	.79394	.74676	.71154	.68362	.66059	.64106	.62416	.60929	.59605	.58414	.57332
1.20	.88493	.82263	.77996	.74766	.72179	.70026	.68189	.66590	.65176	.63911	.62769	.61728
1.30	.90320	.84863	.81049	.78122	.75753	.73766	.72059	.70565	.69237	.68044	.66963	.65974
1.40	.91924	.87196	.83824	.81202	.79059	.77248	.75681	.74302	.73072	.71961	.70950	.70023
1.50	.93319	.89264	.86317	.83997	.82081	.80450	.79030	.77774	.76647	.75626	.74693	.73835
1.60	.94520	.91079	.88533	.86502	.84810	.83359	.82088	.80957	.79939	.79012	.78163	.77378
1.70	.95543	.92654	.90478	.88722	.87246	.85971	.84847	.83842	.82933	.82102	.81338	.80630
1.80	.96407	.94006	.92168	.90667	.89394	.88287	.87305	.86423	.85621	.84886	.84207	.83577
1.90	.97128	.95154	.93618	.92350	.91266	.90316	.89469	.88704	.88006	.87363	.86768	.86213
2.00	.97725	.96119	.94849	.93790	.92877	.92072	.91350	.90695	.90095	.89541	.89025	.88543
2.10	.98213	.96920	.95883	.95009	.94249	.93574	.92967	.92413	.91903	.91430	.90990	.90576
2.20	.98610	.97579	.96741	.96027	.95402	.94844	.94339	.93876	.93448	.93050	.92678	.92328
2.30	.98928	.98115	.97445	.96869	.96361	.95905	.95489	.95107	.94753	.94422	.94112	.93819
2.40	.99180	.98546	.98016	.97557	.97149	.96780	.96443	.96131	.95841	.95570	.95314	.95072
2.50	.99379	.98889	.98475	.98113	.97789	.97494	.97224	.96973	.96738	.96518	.96310	.96113
2.60	.99534	.99159	.98839	.98557	.98302	.98070	.97855	.97656	.97468	.97292	.97125	.96966
2.70	.99653	.99370	.99125	.98907	.98710	.98529	.98361	.98203	.98056	.97916	.97783	.97657
2.80	.99744	.99532	.99347	.99181	.99029	.98890	.9876[illegible]	.98638	.98522	.98413	.98309	.98210
2.90	.99813	.99656	.99517	.99392	.99277	.99171	.99071	.98978	.98889	.98805	.98724	.98647
3.00	.99865	.99750	.99647	.99554	.99467	.99387	.99312	.99241	.99173	.99109	.99047	.98988
3.10	.99903	.99819	.99744	.99675	.99612	.99552	.99496	.99442	.99391	.99343	.99296	.99252
3.20	.99931	.99871	.99816	.99766	.99720	.99676	.99634	.99595	.99557	.99521	.99486	.99452
3.30	.99952	.99909	.99870	.99833	.99800	.99768	.99737	.99708	.99681	.99654	.99628	.99603
3.40	.99966	.99936	.99908	.99883	.99858	.99835	.99813	.99792	.99772	.99753	.99734	.99716
3.50	.99977	.99956	.99936	.99918	.99901	.99884	.99869	.99854	.99839	.99825	.99812	.99799

Table II

Probability that N standard normal random variables with common correlation ρ are simultaneously less than or equal to H.

$$\rho = .700$$

−H \ N	1	2	3	4	5	6	7	8	9	10	11	12
3.50	.00023	.00003	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.40	.00034	.00004	.00001	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.30	.00048	.00007	.00002	.00001	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.20	.00069	.00010	.00004	.00002	.00001	.00001	.00001	.00000	.00000	.00000	.00000	.00000
3.10	.00097	.00015	.00006	.00003	.00002	.00001	.00001	.00001	.00001	.00000	.00000	.00000
3.00	.00135	.00023	.00009	.00005	.00003	.00002	.00001	.00001	.00001	.00001	.00001	.00001
2.90	.00187	.00034	.00014	.00007	.00005	.00003	.00002	.00002	.00001	.00001	.00001	.00001
2.80	.00256	.00050	.00021	.00011	.00007	.00005	.00004	.00003	.00002	.00002	.00002	.00001
2.70	.00347	.00073	.00031	.00017	.00011	.00008	.00006	.00005	.00004	.00003	.00003	.00002
2.60	.00466	.00105	.00046	.00026	.00017	.00013	.00010	.00008	.00006	.00005	.00005	.00004
2.50	.00621	.00149	.00067	.00040	.00027	.00019	.00015	.00012	.00010	.00008	.00007	.00006
2.40	.00820	.00209	.00098	.00059	.00040	.00029	.00023	.00019	.00015	.00013	.00011	.00010
2.30	.01072	.00291	.00141	.00086	.00059	.00044	.00035	.00028	.00024	.00020	.00018	.00016
2.20	.01390	.00401	.00200	.00124	.00087	.00066	.00052	.00043	.00036	.00031	.00027	.00024
2.10	.01786	.00546	.00280	.00178	.00127	.00096	.00077	.00064	.00054	.00047	.00041	.00036
2.00	.02275	.00736	.00389	.00252	.00182	.00140	.00113	.00094	.00080	.00070	.00061	.00055
1.90	.02872	.00983	.00535	.00353	.00258	.00201	.00163	.00137	.00117	.00102	.00091	.00081
1.80	.03593	.01298	.00727	.00488	.00361	.00284	.00233	.00197	.00170	.00149	.00133	.00119
1.70	.04457	.01698	.00977	.00668	.00501	.00398	.00329	.00279	.00243	.00214	.00191	.00173
1.60	.05480	.02198	.01300	.00904	.00687	.00551	.00459	.00392	.00343	.00304	.00273	.00248
1.50	.06681	.02817	.01710	.01210	.00930	.00754	.00633	.00545	.00478	.00426	.00384	.00350
1.40	.08076	.03574	.02226	.01601	.01247	.01020	.00863	.00748	.00660	.00591	.00535	.00489
1.30	.09680	.04490	.02867	.02097	.01652	.01364	.01163	.01014	.00900	.00810	.00737	.00676
1.20	.11507	.05586	.03654	.02716	.02165	.01804	.01549	.01360	.01213	.01097	.01002	.00923
1.10	.13567	.06882	.04610	.03480	.02806	.02359	.02041	.01802	.01616	.01468	.01346	.01245
1.00	.15866	.08398	.05756	.04412	.03598	.03051	.02658	.02362	.02129	.01942	.01788	.01659
0.90	.18406	.10151	.07114	.05534	.04563	.03903	.03424	.03060	.02773	.02541	.02349	.02187
0.80	.21186	.12156	.08705	.06871	.05725	.04939	.04363	.03921	.03571	.03287	.03050	.02850
0.70	.24196	.14423	.10547	.08442	.07109	.06182	.05498	.04969	.04548	.04203	.03915	.03671
0.60	.27425	.16958	.12654	.10268	.08734	.07656	.06853	.06229	.05728	.05316	.04970	.04676
0.50	.30854	.19760	.15037	.12364	.10621	.09383	.08453	.07725	.07136	.06650	.06240	.05889
0.40	.34458	.22824	.17700	.14741	.12784	.11380	.10316	.09477	.08795	.08229	.07749	.07337
0.30	.38209	.26137	.20641	.17404	.15235	.13662	.12460	.11505	.10725	.10073	.09519	.09040
0.20	.42074	.29679	.23852	.20354	.17977	.16237	.14895	.13822	.12941	.12201	.11569	.11021
0.10	.46017	.33425	.27316	.23580	.21009	.19106	.17628	.16438	.15455	.14625	.13913	.13293
0.00	.50000	.37341	.31011	.27069	.24319	.22265	.20656	.19353	.18269	.17351	.16559	.15868

H \ N	1	2	3	4	5	6	7	8	9	10	11	12
0.00	.50000	.37341	.31011	.27069	.24319	.22265	.20656	.19353	.18269	.17351	.16559	.15868
0.10	.53983	.41390	.34906	.30794	.27891	.25700	.23971	.22561	.21382	.20378	.19509	.18747
0.20	.57926	.45531	.38964	.34727	.31697	.29389	.27554	.26048	.24782	.23698	.22756	.21927
0.30	.61791	.49720	.43144	.38828	.35705	.33304	.31380	.29790	.28448	.27293	.26285	.25395
0.40	.65542	.53909	.47400	.43056	.39875	.37407	.35414	.33758	.32352	.31137	.30072	.29128
0.50	.69146	.58053	.51682	.47362	.44161	.41655	.39617	.37913	.36458	.35196	.34085	.33096
0.60	.72575	.62107	.55943	.51695	.48513	.45999	.43941	.42209	.40724	.39428	.38283	.37261
0.70	.75804	.66030	.60133	.56006	.52880	.50389	.48335	.46598	.45099	.43786	.42622	.41579
0.80	.78814	.69765	.64206	.60244	.57210	.54772	.52747	.51025	.49532	.48218	.47049	.45997
0.90	.81594	.73339	.68121	.64361	.61451	.59093	.57122	.55436	.53967	.52669	.51509	.50463
1.00	.84134	.76667	.71842	.68315	.65557	.63304	.61409	.59778	.58350	.57084	.55948	.54920
1.10	.86433	.79749	.75336	.72067	.69484	.67358	.65557	.63999	.62629	.61409	.60310	.59313
1.20	.88493	.82572	.78582	.75586	.73196	.71213	.69523	.68053	.66755	.65593	.64544	.63588
1.30	.90320	.85130	.81563	.78849	.76663	.74836	.73269	.71899	.70684	.69592	.68602	.67698
1.40	.91924	.87422	.84269	.81839	.79863	.78200	.76765	.75504	.74380	.73366	.72444	.71599
1.50	.93319	.89455	.86698	.84547	.82783	.81287	.79988	.78841	.77815	.76885	.76037	.75257
1.60	.94520	.91238	.88854	.86972	.85415	.84085	.82924	.81894	.80968	.80126	.79355	.78644
1.70	.95543	.92785	.90746	.89119	.87760	.86592	.85567	.84652	.83827	.83074	.82382	.81742
1.80	.96407	.94112	.92388	.90997	.89826	.88812	.87917	.87115	.86388	.85723	.85110	.84541
1.90	.97128	.95239	.93798	.92622	.91624	.90754	.89982	.89288	.88656	.88075	.87538	.87039
2.00	.97725	.96186	.94994	.94011	.93170	.92433	.91776	.91181	.90638	.90138	.89673	.89240
2.10	.98214	.96973	.95998	.95186	.94486	.93868	.93315	.92812	.92351	.91925	.91528	.91157
2.20	.98610	.97620	.96831	.96167	.95591	.95080	.94620	.94200	.93813	.93454	.93120	.92806
2.30	.98928	.98146	.97515	.96979	.96510	.96092	.95713	.95367	.95046	.94748	.94469	.94207
2.40	.99180	.98569	.98070	.97642	.97265	.96927	.96619	.96336	.96074	.95829	.95599	.95383
2.50	.99379	.98907	.98515	.98177	.97878	.97608	.97361	.97132	.96920	.96722	.96535	.96358
2.60	.99534	.99172	.98869	.98605	.98370	.98156	.97960	.97779	.97609	.97450	.97300	.97157
2.70	.99653	.99379	.99147	.98943	.98760	.98594	.98440	.98297	.98163	.98037	.97918	.97804
2.80	.99744	.99539	.99363	.99208	.99067	.98938	.98819	.98708	.98603	.98505	.98411	.98322
2.90	.99813	.99661	.99529	.99412	.99305	.99207	.99115	.99030	.98949	.98873	.98800	.98731
3.00	.99865	.99753	.99655	.99567	.99487	.99413	.99344	.99279	.99217	.99159	.99103	.99050
3.10	.99903	.99822	.99750	.99685	.99626	.99570	.99519	.99470	.99423	.99379	.99337	.99297
3.20	.99931	.99873	.99821	.99773	.99729	.99689	.99650	.99614	.99580	.99547	.99515	.99485
3.30	.99952	.99910	.99872	.99838	.99806	.99777	.99749	.99722	.99697	.99672	.99649	.99627
3.40	.99966	.99937	.99910	.99886	.99863	.99842	.99821	.99802	.99783	.99766	.99749	.99732
3.50	.99977	.99956	.99937	.99920	.99904	.99889	.99874	.99860	.99847	.99834	.99822	.99810

Table II

Probability that N standard normal random variables with common correlation ρ are simultaneously less than or equal to H.

$\rho = .750$

-H \ N	1	2	3	4	5	6	7	8	9	10	11	12
3.50	.00023	.00004	.00001	.00001	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
3.40	.00034	.00006	.00002	.00001	.00001	.00001	.00000	.00000	.00000	.00000	.00000	.00000
3.30	.00048	.00009	.00004	.00002	.00001	.00001	.00001	.00001	.00000	.00000	.00000	.00000
3.20	.00069	.00013	.00006	.00003	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00000
3.10	.00097	.00020	.00009	.00005	.00003	.00002	.00002	.00001	.00001	.00001	.00001	.00001
3.00	.00135	.00029	.00013	.00008	.00005	.00004	.00003	.00002	.00002	.00002	.00001	.00001
2.90	.00187	.00043	.00020	.00012	.00008	.00006	.00005	.00004	.00003	.00003	.00002	.00002
2.80	.00256	.00062	.00029	.00018	.00012	.00009	.00007	.00006	.00005	.00004	.00004	.00003
2.70	.00347	.00090	.00043	.00027	.00019	.00014	.00011	.00009	.00008	.00007	.00006	.00005
2.60	.00466	.00128	.00063	.00040	.00028	.00021	.00017	.00014	.00012	.00010	.00009	.00008
2.50	.00621	.00179	.00092	.00059	.00042	.00032	.00026	.00022	.00018	.00016	.00014	.00013
2.40	.00820	.00250	.00131	.00085	.00062	.00048	.00039	.00033	.00028	.00024	.00022	.00019
2.30	.01072	.00345	.00186	.00123	.00090	.00070	.00057	.00048	.00042	.00037	.00033	.00029
2.20	.01390	.00470	.00260	.00174	.00129	.00102	.00084	.00071	.00062	.00055	.00049	.00044
2.10	.01786	.00636	.00360	.00245	.00184	.00147	.00122	.00104	.00090	.00080	.00072	.00065
2.00	.02275	.00850	.00494	.00342	.00259	.00209	.00174	.00149	.00131	.00116	.00105	.00095
1.90	.02872	.01125	.00670	.00471	.00361	.00293	.00246	.00213	.00187	.00167	.00151	.00138
1.80	.03593	.01475	.00899	.00641	.00498	.00407	.00345	.00299	.00264	.00237	.00215	.00197
1.70	.04457	.01914	.01193	.00864	.00678	.00559	.00477	.00416	.00370	.00333	.00303	.00279
1.60	.05480	.02460	.01569	.01153	.00914	.00760	.00652	.00572	.00511	.00462	.00422	.00390
1.50	.06681	.03130	.02041	.01521	.01219	.01021	.00882	.00778	.00698	.00634	.00582	.00538
1.40	.08076	.03945	.02627	.01987	.01608	.01358	.01180	.01047	.00943	.00860	.00792	.00735
1.30	.09680	.04924	.03349	.02567	.02099	.01786	.01562	.01393	.01261	.01155	.01067	.00993
1.20	.11507	.06088	.04226	.03283	.02711	.02325	.02046	.01834	.01667	.01533	.01421	.01327
1.10	.13567	.07455	.05280	.04156	.03464	.02993	.02650	.02388	.02180	.02012	.01872	.01753
1.00	.15866	.09046	.06532	.05207	.04382	.03814	.03396	.03076	.02821	.02612	.02439	.02291
0.90	.18406	.10874	.08002	.06459	.05485	.04808	.04307	.03920	.03610	.03356	.03143	.02962
0.80	.21186	.12954	.09709	.07933	.06797	.06000	.05406	.04943	.04571	.04265	.04007	.03787
0.70	.24196	.15293	.11668	.09647	.08338	.07411	.06715	.06169	.05728	.05363	.05055	.04790
0.60	.27425	.17896	.13891	.11618	.10127	.09062	.08255	.07620	.07103	.06674	.06309	.05996
0.50	.30854	.20761	.16386	.13859	.12181	.10970	.10047	.09315	.08718	.08218	.07794	.07427
0.40	.34458	.23878	.19153	.16375	.14509	.13150	.12107	.11274	.10591	.10018	.09528	.09104
0.30	.38209	.27235	.22187	.19170	.17118	.15611	.14445	.13510	.12738	.12088	.11531	.11046
0.20	.42074	.30809	.25478	.22237	.20008	.18356	.17069	.16031	.15170	.14441	.13814	.13267
0.10	.46017	.34575	.29006	.25565	.23172	.21382	.19979	.18840	.17891	.17084	.16388	.15777
0.00	.50000	.38497	.32746	.29135	.26594	.24679	.23167	.21932	.20899	.20017	.19252	.18580

H \ N	1	2	3	4	5	6	7	8	9	10	11	12
0.00	.50000	.38497	.32746	.29135	.26594	.24679	.23167	.21932	.20899	.20017	.19252	.18580
0.10	.53983	.42540	.36666	.32920	.30255	.28229	.26618	.25296	.24184	.23231	.22402	.21671
0.20	.57926	.46661	.40728	.36886	.34123	.32005	.30310	.28912	.27730	.26712	.25824	.25038
0.30	.61791	.50817	.44891	.40995	.38164	.35976	.34214	.32752	.31510	.30437	.29497	.28663
0.40	.65542	.54963	.49109	.45204	.42336	.40103	.38292	.36781	.35493	.34375	.33392	.32517
0.50	.69146	.59053	.53335	.49465	.46594	.44341	.42502	.40960	.39639	.38489	.37473	.36567
0.60	.72575	.63046	.57522	.53731	.50890	.48642	.46790	.45242	.43904	.42734	.41698	.40770
0.70	.75804	.66901	.61623	.57951	.55172	.52958	.51129	.49579	.48239	.47063	.46018	.45080
0.80	.78814	.70583	.65597	.62080	.59394	.57236	.55444	.53918	.52593	.51426	.50385	.49447
0.90	.81594	.74062	.69403	.66074	.63506	.61430	.59694	.58208	.56914	.55769	.54744	.53819
1.00	.84134	.77315	.73009	.69892	.67467	.65491	.63830	.62401	.61151	.60041	.59045	.58143
1.10	.86433	.80322	.76387	.73503	.71237	.69379	.67808	.66451	.65258	.64195	.63238	.62369
1.20	.88493	.83074	.79516	.76877	.74786	.73059	.71590	.70315	.69190	.68185	.67276	.66449
1.30	.90320	.85564	.82383	.79995	.78087	.76500	.75144	.73961	.72912	.71972	.71120	.70342
1.40	.91924	.87794	.84981	.82845	.81123	.79682	.78444	.77359	.76393	.75525	.74735	.74012
1.50	.93319	.89769	.87309	.85419	.83884	.82591	.81473	.80489	.79611	.78818	.78095	.77431
1.60	.94520	.91500	.89372	.87719	.86366	.85218	.84221	.83340	.82550	.81835	.81180	.80578
1.70	.95543	.93001	.91180	.89751	.88571	.87564	.86686	.85905	.85204	.84566	.83981	.83440
1.80	.96407	.94289	.92748	.91525	.90508	.89636	.88870	.88187	.87571	.87009	.86492	.86014
1.90	.97128	.95382	.94092	.93058	.92191	.91443	.90784	.90194	.89659	.89169	.88718	.88299
2.00	.97725	.96300	.95231	.94367	.93637	.93003	.92442	.91937	.91478	.91057	.90668	.90306
2.10	.98214	.97063	.96187	.95472	.94864	.94334	.93861	.93435	.93046	.92688	.92356	.92046
2.20	.98610	.97690	.96980	.96396	.95895	.95456	.95063	.94707	.94381	.94080	.93800	.93538
2.30	.98928	.98200	.97631	.97158	.96751	.96391	.96068	.95774	.95504	.95254	.95021	.94803
2.40	.99180	.98610	.98160	.97781	.97453	.97162	.96899	.96660	.96439	.96233	.96042	.95862
2.50	.99379	.98938	.98584	.98285	.98023	.97790	.97579	.97386	.97207	.97041	.96885	.96738
2.60	.99534	.99195	.98921	.98687	.98481	.98297	.98129	.97975	.97832	.97698	.97573	.97455
2.70	.99653	.99396	.99185	.99004	.98844	.98700	.98568	.98447	.98334	.98228	.98128	.98034
2.80	.99744	.99551	.99391	.99253	.99129	.99018	.98916	.98821	.98733	.98650	.98572	.98497
2.90	.99813	.99670	.99549	.99445	.99351	.99265	.99187	.99114	.99046	.98982	.98921	.98863
3.00	.99865	.99759	.99670	.99591	.99521	.99456	.99397	.99341	.99289	.99240	.99193	.99149
3.10	.99903	.99826	.99760	.99702	.99650	.99601	.99557	.99515	.99476	.99438	.99403	.99369
3.20	.99931	.99876	.99828	.99785	.99746	.99711	.99678	.99647	.99617	.99589	.99563	.99538
3.30	.99952	.99912	.99877	.99847	.99818	.99792	.99768	.99745	.99723	.99703	.99683	.99664
3.40	.99966	.99938	.99914	.99892	.99871	.99852	.99835	.99818	.99802	.99787	.99773	.99759
3.50	.99977	.99957	.99940	.99924	.99910	.99896	.99883	.99871	.99860	.99849	.99839	.99829

Table II

Probability that N standard normal random variables with common correlation ρ are simultaneously less than or equal to H.

$\rho = .800$

-H \ N	1	2	3	4	5	6	7	8	9	10	11	12
3.50	.00023	.00005	.00002	.00001	.00001	.00001	.00001	.00000	.00000	.00000	.00000	.00000
3.40	.00034	.00008	.00004	.00002	.00001	.00001	.00001	.00001	.00001	.00001	.00000	.00000
3.30	.00048	.00011	.00005	.00003	.00002	.00002	.00001	.00001	.00001	.00001	.00001	.00001
3.20	.00069	.00017	.00008	.00005	.00004	.00003	.00002	.00002	.00002	.00001	.00001	.00001
3.10	.00097	.00025	.00013	.00008	.00006	.00004	.00004	.00003	.00003	.00002	.00002	.00002
3.00	.00135	.00037	.00019	.00012	.00009	.00007	.00006	.00005	.00004	.00004	.00003	.00003
2.90	.00187	.00054	.00028	.00018	.00013	.00011	.00009	.00007	.00006	.00006	.00005	.00004
2.80	.00256	.00078	.00042	.00028	.00020	.00016	.00013	.00011	.00010	.00009	.00008	.00007
2.70	.00347	.00110	.00060	.00041	.00030	.00024	.00020	.00017	.00015	.00013	.00012	.00011
2.60	.00466	.00156	.00087	.00059	.00045	.00036	.00030	.00025	.00022	.00020	.00018	.00016
2.50	.00621	.00217	.00124	.00086	.00065	.00052	.00044	.00038	.00033	.00030	.00027	.00024
2.40	.00820	.00299	.00175	.00122	.00094	.00076	.00064	.00056	.00049	.00044	.00040	.00037
2.30	.01072	.00409	.00244	.00173	.00134	.00110	.00093	.00081	.00072	.00064	.00059	.00054
2.20	.01390	.00553	.00337	.00242	.00190	.00156	.00133	.00116	.00104	.00094	.00085	.00079
2.10	.01786	.00741	.00461	.00336	.00265	.00220	.00189	.00166	.00148	.00134	.00123	.00113
2.00	.02275	.00983	.00624	.00460	.00367	.00307	.00264	.00233	.00209	.00190	.00175	.00162
1.90	.02872	.01290	.00836	.00624	.00502	.00423	.00367	.00325	.00293	.00267	.00246	.00228
1.80	.03593	.01678	.01109	.00838	.00680	.00576	.00503	.00448	.00405	.00370	.00342	.00319
1.70	.04457	.02162	.01455	.01114	.00912	.00778	.00682	.00610	.00554	.00509	.00471	.00440
1.60	.05480	.02758	.01891	.01465	.01209	.01039	.00916	.00823	.00750	.00691	.00642	.00601
1.50	.06681	.03486	.02434	.01907	.01588	.01373	.01217	.01098	.01005	.00929	.00866	.00813
1.40	.08076	.04363	.03101	.02458	.02064	.01796	.01600	.01450	.01332	.01235	.01155	.01087
1.30	.09680	.05411	.03913	.03136	.02655	.02325	.02082	.01896	.01747	.01626	.01524	.01438
1.20	.11507	.06648	.04890	.03962	.03381	.02979	.02682	.02452	.02268	.02117	.01990	.01882
1.10	.13567	.08093	.06052	.04957	.04263	.03779	.03419	.03139	.02914	.02728	.02572	.02439
1.00	.15866	.09764	.07419	.06141	.05322	.04746	.04315	.03978	.03705	.03480	.03290	.03127
0.90	.18406	.11673	.09010	.07535	.06580	.05902	.05391	.04990	.04664	.04394	.04165	.03968
0.80	.21186	.13833	.10841	.09158	.08056	.07267	.06669	.06197	.05813	.05492	.05219	.04984
0.70	.24196	.16250	.12925	.11025	.09768	.08862	.08170	.07621	.07172	.06796	.06475	.06198
0.60	.27425	.18926	.15271	.13151	.11733	.10703	.09912	.09281	.08752	.08326	.07954	.07630
0.50	.30854	.21856	.17883	.15543	.13962	.12804	.11910	.11193	.10601	.10102	.09674	.09301
0.40	.34458	.25031	.20758	.18205	.16462	.15176	.14177	.13371	.12704	.12139	.11652	.11227
0.30	.38209	.28434	.23890	.21134	.19234	.17822	.16718	.15824	.15079	.14447	.13901	.13423
0.20	.42074	.32042	.27262	.24321	.22274	.20740	.19535	.18553	.17733	.17034	.16428	.15895
0.10	.46017	.35828	.30854	.27750	.25569	.23923	.22621	.21556	.20663	.19898	.19234	.18649
0.00	.50000	.39758	.34638	.31399	.29100	.27354	.25965	.24823	.23861	.23034	.22314	.21678

H \ N	1	2	3	4	5	6	7	8	9	10	11	12
0.00	.50000	.39758	.34638	.31399	.29100	.27354	.25965	.24823	.23861	.23034	.22314	.21678
0.10	.53983	.43794	.38580	.35238	.32844	.31011	.29545	.28335	.27311	.26428	.25657	.24973
0.20	.57926	.47894	.42643	.39232	.36766	.34866	.33337	.32068	.30990	.30059	.29242	.28516
0.30	.61791	.52016	.46785	.43342	.40831	.38882	.37305	.35990	.34870	.33898	.33042	.32281
0.40	.65542	.56115	.50960	.47524	.44995	.43019	.41411	.40065	.38913	.37910	.37025	.36235
0.50	.69146	.60149	.55124	.51732	.49214	.47233	.45613	.44250	.43079	.42057	.41151	.40342
0.60	.72575	.64075	.59230	.55920	.53440	.51477	.49862	.48498	.47322	.46292	.45377	.44556
0.70	.75804	.67858	.63236	.60040	.57626	.55702	.54112	.52763	.51594	.50568	.49653	.48831
0.80	.78814	.71462	.67102	.64051	.61727	.59863	.58314	.56994	.55847	.54836	.53933	.53119
0.90	.81594	.74861	.70792	.67911	.65698	.63912	.62421	.61145	.60032	.59047	.58166	.57369
1.00	.84134	.78033	.74275	.71585	.69502	.67811	.66391	.65171	.64103	.63156	.62305	.61534
1.10	.86433	.80960	.77528	.75044	.73105	.71521	.70185	.69032	.68019	.67117	.66306	.65568
1.20	.88493	.83634	.80534	.78264	.76480	.75012	.73769	.72692	.71742	.70894	.70129	.69432
1.30	.90320	.86051	.83279	.81229	.79605	.78261	.77117	.76122	.75242	.74454	.73740	.73089
1.40	.91924	.88212	.85761	.83930	.82467	.81251	.80210	.79302	.78495	.77770	.77113	.76511
1.50	.93319	.90124	.87981	.86362	.85060	.83971	.83035	.82215	.81484	.80826	.80227	.79677
1.60	.94520	.91798	.89943	.88529	.87383	.86419	.85587	.84854	.84199	.83608	.83069	.82573
1.70	.95543	.93249	.91661	.90438	.89440	.88597	.87865	.87218	.86638	.86113	.85633	.85191
1.80	.96407	.94492	.93147	.92102	.91243	.90512	.89876	.89312	.88804	.88343	.87921	.87530
1.90	.97128	.95547	.94420	.93536	.92805	.92179	.91632	.91145	.90706	.90306	.89938	.89598
2.00	.97725	.96432	.95498	.94759	.94143	.93614	.93149	.92733	.92357	.92014	.91698	.91404
2.10	.98214	.97168	.96402	.95790	.95277	.94834	.94443	.94093	.93775	.93483	.93214	.92964
2.20	.98610	.97772	.97151	.96650	.96227	.95861	.95536	.95244	.94978	.94733	.94507	.94296
2.30	.98928	.98264	.97765	.97359	.97015	.96715	.96448	.96207	.95987	.95785	.95597	.95421
2.40	.99180	.98660	.98263	.97939	.97661	.97418	.97201	.97005	.96825	.96659	.96504	.96360
2.50	.99379	.98975	.98663	.98406	.98185	.97991	.97816	.97658	.97512	.97378	.97252	.97135
2.60	.99534	.99223	.98981	.98780	.98605	.98451	.98313	.98186	.98070	.97962	.97861	.97767
2.70	.99653	.99417	.99231	.99074	.98939	.98818	.98709	.98610	.98518	.98432	.98352	.98277
2.80	.99744	.99567	.99425	.99305	.99200	.99107	.99022	.98945	.98873	.98806	.98743	.98684
2.90	.99813	.99681	.99574	.99483	.99403	.99332	.99267	.99207	.99151	.99099	.99051	.99004
3.00	.99865	.99767	.99688	.99619	.99559	.99505	.99456	.99410	.99367	.99328	.99290	.99255
3.10	.99903	.99832	.99773	.99723	.99678	.99637	.99600	.99565	.99533	.99503	.99475	.99448
3.20	.99931	.99880	.99837	.99800	.99767	.99736	.99709	.99683	.99659	.99637	.99615	.99595
3.30	.99952	.99915	.99884	.99857	.99833	.99811	.99790	.99771	.99754	.99737	.99721	.99706
3.40	.99966	.99940	.99918	.99899	.99881	.99865	.99850	.99837	.99824	.99811	.99800	.99789
3.50	.99977	.99958	.99943	.99929	.99916	.99905	.99894	.99884	.99875	.99866	.99858	.99850

Table II

Probability that N standard normal random variables with common correlation ρ are simultaneously less than or equal to H.

$\rho = .875$

-H \ N	1	2	3	4	5	6	7	8	9	10	11	12
3.50	.00023	.00008	.00004	.00003	.00002	.00002	.00002	.00001	.00001	.00001	.00001	.00001
3.40	.00034	.00012	.00007	.00005	.00004	.00003	.00003	.00002	.00002	.00002	.00002	.00002
3.30	.00048	.00017	.00010	.00007	.00006	.00005	.00004	.00004	.00003	.00003	.00003	.00002
3.20	.00069	.00026	.00015	.00011	.00009	.00007	.00006	.00006	.00005	.00004	.00004	.00004
3.10	.00097	.00037	.00023	.00017	.00013	.00011	.00010	.00008	.00008	.00007	.00006	.00006
3.00	.00135	.00054	.00034	.00025	.00020	.00017	.00014	.00013	.00012	.00011	.00010	.00009
2.90	.00187	.00077	.00049	.00036	.00029	.00025	.00021	.00019	.00017	.00016	.00015	.00014
2.80	.00256	.00109	.00070	.00053	.00043	.00036	.00032	.00028	.00026	.00024	.00022	.00020
2.70	.00347	.00152	.00100	.00076	.00062	.00053	.00046	.00041	.00038	.00035	.00032	.00030
2.60	.00466	.00211	.00141	.00108	.00088	.00076	.00067	.00060	.00055	.00051	.00047	.00044
2.50	.00621	.00290	.00196	.00151	.00125	.00108	.00096	.00086	.00079	.00073	.00068	.00064
2.40	.00820	.00395	.00271	.00211	.00176	.00152	.00135	.00122	.00112	.00104	.00097	.00092
2.30	.01072	.00533	.00370	.00291	.00244	.00212	.00190	.00172	.00158	.00147	.00138	.00130
2.20	.01390	.00711	.00502	.00398	.00336	.00294	.00263	.00240	.00221	.00206	.00194	.00183
2.10	.01786	.00941	.00673	.00539	.00457	.00402	.00361	.00330	.00306	.00286	.00269	.00255
2.00	.02275	.01232	.00893	.00722	.00616	.00544	.00491	.00451	.00418	.00392	.00370	.00351
1.90	.02872	.01599	.01175	.00957	.00823	.00730	.00662	.00609	.00567	.00532	.00503	.00478
1.80	.03593	.02056	.01531	.01258	.01087	.00969	.00882	.00814	.00760	.00715	.00677	.00645
1.70	.04457	.02619	.01975	.01637	.01423	.01275	.01164	.01078	.01009	.00951	.00903	.00862
1.60	.05480	.03305	.02525	.02109	.01845	.01660	.01522	.01413	.01326	.01253	.01192	.01139
1.50	.06681	.04132	.03197	.02692	.02369	.02141	.01970	.01835	.01726	.01635	.01558	.01492
1.40	.08076	.05120	.04010	.03404	.03012	.02734	.02525	.02359	.02225	.02113	.02017	.01935
1.30	.09680	.06288	.04984	.04263	.03794	.03459	.03205	.03004	.02840	.02703	.02586	.02485
1.20	.11507	.07653	.06138	.05290	.04734	.04335	.04030	.03788	.03590	.03424	.03283	.03160
1.10	.13567	.09232	.07491	.06504	.05852	.05381	.05020	.04732	.04496	.04297	.04127	.03979
1.00	.15866	.11040	.09059	.07924	.07167	.06617	.06194	.05856	.05577	.05342	.05140	.04964
0.90	.18406	.13089	.10859	.09566	.08698	.08063	.07572	.07179	.06853	.06578	.06341	.06134
0.80	.21186	.15386	.12902	.11446	.10460	.09735	.09173	.08719	.08343	.08024	.07749	.07509
0.70	.24196	.17935	.15198	.13574	.12467	.11649	.11010	.10494	.10064	.09699	.09383	.09107
0.60	.27425	.20734	.17749	.15959	.14729	.13814	.13098	.12516	.12031	.11617	.11259	.10944
0.50	.30854	.23775	.20554	.18602	.17250	.16239	.15444	.14796	.14253	.13790	.13387	.13033
0.40	.34458	.27046	.23607	.21500	.20030	.18925	.18051	.17337	.16737	.16224	.15777	.15382
0.30	.38209	.30527	.26894	.24644	.23063	.21867	.20919	.20140	.19484	.18921	.18430	.17996
0.20	.42074	.34194	.30396	.28018	.26336	.25057	.24038	.23198	.22489	.21878	.21344	.20872
0.10	.46017	.38015	.34086	.31601	.29830	.28476	.27393	.26498	.25739	.25084	.24511	.24001
0.00	.50000	.41957	.37935	.35365	.33521	.32104	.30965	.30020	.29218	.28523	.27913	.27370

H \ N	1	2	3	4	5	6	7	8	9	10	11	12
0.00	.50000	.41957	.37935	.35365	.33521	.32104	.30965	.30020	.29218	.28523	.27913	.27370
0.10	.53983	.45981	.41908	.39278	.37378	.35910	.34726	.33740	.32900	.32171	.31529	.30958
0.20	.57926	.50046	.45963	.43301	.41365	.39861	.38643	.37625	.36755	.35999	.35332	.34736
0.30	.61791	.54109	.50061	.47395	.45442	.43918	.42679	.41639	.40749	.39972	.39286	.38671
0.40	.65542	.58130	.54157	.51516	.49568	.48040	.46792	.45742	.44840	.44052	.43353	.42727
0.50	.69146	.62068	.58210	.55620	.53697	.52182	.50939	.49891	.48987	.48195	.47491	.46860
0.60	.72575	.65883	.62176	.59664	.57787	.56300	.55076	.54039	.53144	.52357	.51656	.51027
0.70	.75804	.69542	.66017	.63606	.61793	.60350	.59157	.58144	.57266	.56493	.55803	.55182
0.80	.78814	.73015	.69698	.67409	.65676	.64291	.63141	.62161	.61309	.60558	.59886	.59280
0.90	.81594	.76276	.73189	.71038	.69400	.68084	.66987	.66050	.65233	.64511	.63864	.63278
1.00	.84134	.79309	.76464	.74465	.72933	.71696	.70662	.69775	.69000	.68313	.67696	.67137
1.10	.86433	.82099	.79505	.77667	.76249	.75099	.74134	.73304	.72576	.71930	.71348	.70821
1.20	.88493	.84639	.82299	.80626	.79328	.78271	.77380	.76611	.75936	.75334	.74792	.74299
1.30	.90320	.86928	.84839	.83333	.82157	.81195	.80381	.79677	.79057	.78503	.78003	.77548
1.40	.91924	.88969	.87124	.85782	.84729	.83863	.83128	.82490	.81926	.81422	.80966	.80550
1.50	.93319	.90771	.89158	.87975	.87041	.86270	.85613	.85041	.84535	.84081	.83669	.83293
1.60	.94520	.92345	.90949	.89918	.89099	.88419	.87839	.87332	.86882	.86477	.86110	.85773
1.70	.95543	.93705	.92511	.91621	.90910	.90318	.89810	.89365	.88970	.88613	.88289	.87991
1.80	.96407	.94870	.93858	.93098	.92488	.91977	.91538	.91152	.90808	.90497	.90213	.89953
1.90	.97128	.95856	.95007	.94366	.93847	.93412	.93036	.92704	.92408	.92140	.91895	.91670
2.00	.97725	.96682	.95978	.95442	.95006	.94639	.94320	.94039	.93787	.93558	.93349	.93156
2.10	.98214	.97368	.96790	.96346	.95984	.95677	.95410	.95174	.94962	.94769	.94592	.94428
2.20	.98610	.97930	.97461	.97098	.96800	.96546	.96325	.96129	.95952	.95791	.95643	.95506
2.30	.98928	.98388	.98010	.97716	.97474	.97267	.97085	.96924	.96778	.96645	.96523	.96409
2.40	.99180	.98756	.98455	.98220	.98024	.97857	.97710	.97578	.97460	.97351	.97251	.97158
2.50	.99379	.99048	.98812	.98625	.98469	.98335	.98217	.98112	.98016	.97928	.97847	.97772
2.60	.99534	.99279	.99095	.98948	.98825	.98719	.98626	.98542	.98465	.98395	.98330	.98270
2.70	.99653	.99459	.99317	.99203	.99107	.99024	.98951	.98884	.98824	.98769	.98717	.98669
2.80	.99744	.99598	.99489	.99402	.99328	.99264	.99206	.99155	.99108	.99064	.99024	.98986
2.90	.99813	.99704	.99622	.99555	.99499	.99450	.99406	.99366	.99329	.99296	.99265	.99235
3.00	.99865	.99784	.99722	.99673	.99630	.99593	.99559	.99529	.99501	.99475	.99451	.99429
3.10	.99903	.99844	.99798	.99761	.99729	.99701	.99676	.99653	.99632	.99613	.99595	.99578
3.20	.99931	.99888	.99855	.99828	.99804	.99783	.99764	.99747	.99732	.99717	.99703	.99691
3.30	.99952	.99921	.99897	.99877	.99859	.99844	.99830	.99818	.99806	.99795	.99785	.99776
3.40	.99966	.99944	.99927	.99913	.99900	.99889	.99879	.99870	.99861	.99853	.99846	.99839
3.50	.99977	.99961	.99949	.99939	.99930	.99922	.99915	.99908	.99902	.99896	.99891	.99886

Table II

Probability that N standard normal random variables with common correlation ρ are simultaneously less than or equal to H.

$\rho = .900$

−H \ N	1	2	3	4	5	6	7	8	9	10	11	12
3.50	.00023	.00009	.00006	.00004	.00003	.00003	.00002	.00002	.00002	.00002	.00002	.00002
3.40	.00034	.00014	.00009	.00006	.00005	.00004	.00004	.00003	.00003	.00003	.00003	.00002
3.30	.00048	.00020	.00013	.00010	.00008	.00007	.00006	.00005	.00005	.00004	.00004	.00004
3.20	.00069	.00029	.00019	.00014	.00012	.00010	.00009	.00008	.00007	.00007	.00006	.00006
3.10	.00097	.00043	.00028	.00021	.00018	.00015	.00013	.00012	.00011	.00010	.00009	.00009
3.00	.00135	.00061	.00041	.00031	.00026	.00022	.00020	.00018	.00016	.00015	.00014	.00013
2.90	.00187	.00087	.00059	.00046	.00038	.00033	.00029	.00026	.00024	.00023	.00021	.00020
2.80	.00256	.00122	.00084	.00066	.00055	.00048	.00043	.00039	.00036	.00033	.00031	.00029
2.70	.00347	.00170	.00119	.00094	.00079	.00069	.00062	.00056	.00052	.00048	.00045	.00043
2.60	.00466	.00235	.00166	.00132	.00112	.00098	.00088	.00080	.00074	.00069	.00065	.00062
2.50	.00621	.00322	.00230	.00184	.00157	.00138	.00124	.00114	.00106	.00099	.00093	.00088
2.40	.00820	.00436	.00315	.00255	.00218	.00193	.00174	.00160	.00149	.00140	.00132	.00125
2.30	.01072	.00585	.00428	.00349	.00300	.00266	.00242	.00223	.00208	.00195	.00185	.00176
2.20	.01390	.00778	.00576	.00473	.00409	.00365	.00332	.00307	.00287	.00270	.00256	.00244
2.10	.01786	.01024	.00768	.00635	.00552	.00494	.00452	.00418	.00392	.00370	.00351	.00335
2.00	.02275	.01336	.01013	.00844	.00738	.00663	.00608	.00565	.00530	.00501	.00477	.00456
1.90	.02872	.01727	.01325	.01111	.00976	.00882	.00811	.00755	.00710	.00673	.00642	.00615
1.80	.03593	.02211	.01715	.01450	.01280	.01160	.01071	.01000	.00943	.00895	.00855	.00820
1.70	.04457	.02806	.02201	.01873	.01662	.01513	.01400	.01311	.01239	.01179	.01128	.01084
1.60	.05480	.03527	.02797	.02397	.02138	.01953	.01814	.01703	.01613	.01538	.01473	.01418
1.50	.06681	.04395	.03522	.03039	.02724	.02499	.02327	.02191	.02079	.01986	.01906	.01837
1.40	.08076	.05427	.04395	.03818	.03439	.03166	.02957	.02791	.02655	.02541	.02443	.02358
1.30	.09680	.06641	.05434	.04752	.04300	.03974	.03723	.03523	.03359	.03220	.03101	.02998
1.20	.11507	.08056	.06659	.05860	.05329	.04942	.04644	.04406	.04209	.04043	.03900	.03776
1.10	.13567	.09687	.08086	.07162	.06543	.06090	.05739	.05458	.05225	.05028	.04858	.04710
1.00	.15866	.11549	.09734	.08675	.07961	.07436	.07028	.06700	.06427	.06196	.05996	.05822
0.90	.18406	.13652	.11615	.10415	.09600	.08998	.08529	.08149	.07833	.07565	.07333	.07130
0.80	.21186	.16002	.13740	.12395	.11474	.10791	.10257	.09823	.09462	.09153	.08886	.08652
0.70	.24196	.18602	.16116	.14624	.13595	.12829	.12226	.11736	.11326	.10976	.10672	.10405
0.60	.27425	.21448	.18746	.17107	.15970	.15118	.14447	.13899	.13439	.13046	.12703	.12402
0.50	.30854	.24533	.21624	.19844	.18601	.17665	.16925	.16318	.15808	.15370	.14989	.14652
0.40	.34458	.27841	.24743	.22829	.21484	.20467	.19659	.18995	.18435	.17954	.17533	.17161
0.30	.38209	.31352	.28088	.26051	.24611	.23517	.22645	.21926	.21318	.20794	.20335	.19929
0.20	.42074	.35040	.31636	.29492	.27967	.26803	.25871	.25100	.24447	.23883	.23388	.22949
0.10	.46017	.38875	.35361	.33129	.31530	.30304	.29319	.28502	.27808	.27207	.26680	.26210
0.00	.50000	.42822	.39233	.36931	.35274	.33996	.32967	.32110	.31380	.30747	.30189	.29693

H \ N	1	2	3	4	5	6	7	8	9	10	11	12
0.00	.50000	.42822	.39233	.36931	.35274	.33996	.32967	.32110	.31380	.30747	.30189	.29693
0.10	.53983	.46841	.43213	.40866	.39165	.37848	.36783	.35894	.35135	.34475	.33892	.33373
0.20	.57926	.50892	.47263	.44894	.43168	.41825	.40734	.39822	.39040	.38359	.37758	.37220
0.30	.61791	.54934	.51341	.48976	.47241	.45886	.44782	.43855	.43059	.42364	.41749	.41198
0.40	.65542	.58925	.55405	.53068	.51344	.49991	.48884	.47953	.47151	.46450	.45827	.45269
0.50	.69146	.62825	.59412	.57127	.55431	.54095	.52998	.52072	.51273	.50572	.49950	.49390
0.60	.72575	.66597	.63323	.61112	.59462	.58155	.57079	.56168	.55381	.54689	.54072	.53517
0.70	.75804	.70209	.67099	.64982	.63393	.62130	.61086	.60200	.59431	.58754	.58151	.57607
0.80	.78814	.73631	.70708	.68703	.67189	.65980	.64978	.64124	.63383	.62728	.62143	.61615
0.90	.81594	.76840	.74122	.72241	.70814	.69670	.68718	.67905	.67197	.66570	.66009	.65502
1.00	.84134	.79818	.77317	.75572	.74241	.73169	.72274	.71508	.70839	.70246	.69714	.69232
1.10	.86433	.82554	.80276	.78674	.77446	.76452	.75620	.74905	.74280	.73725	.73226	.72773
1.20	.88493	.85042	.82988	.81533	.80411	.79500	.78734	.78075	.77497	.76982	.76519	.76098
1.30	.90320	.87281	.85449	.84141	.83127	.82300	.81603	.81001	.80472	.80000	.79575	.79188
1.40	.91924	.89275	.87658	.86494	.85587	.84845	.84217	.83674	.83195	.82767	.82380	.82028
1.50	.93319	.91033	.89620	.88596	.87793	.87134	.86574	.86089	.85660	.85276	.84929	.84611
1.60	.94520	.92568	.91345	.90453	.89751	.89171	.88678	.88248	.87868	.87527	.87218	.86936
1.70	.95543	.93893	.92847	.92078	.91469	.90965	.90534	.90159	.89826	.89526	.89254	.89005
1.80	.96407	.95025	.94139	.93483	.92962	.92528	.92156	.91831	.91542	.91282	.91045	.90828
1.90	.97128	.95984	.95241	.94687	.94245	.93875	.93558	.93280	.93032	.92808	.92604	.92417
2.00	.97725	.96786	.96170	.95708	.95336	.95025	.94757	.94521	.94310	.94120	.93946	.93786
2.10	.98214	.97451	.96946	.96563	.96255	.95995	.95771	.95573	.95396	.95236	.95090	.94955
2.20	.98610	.97997	.97586	.97273	.97020	.96806	.96620	.96456	.96309	.96176	.96054	.95941
2.30	.98928	.98440	.98110	.97857	.97650	.97476	.97324	.97189	.97068	.96958	.96858	.96764
2.40	.99180	.98796	.98533	.98331	.98165	.98024	.97901	.97791	.97693	.97603	.97521	.97445
2.50	.99379	.99080	.98873	.98712	.98580	.98467	.98368	.98281	.98201	.98129	.98063	.98001
2.60	.99534	.99303	.99142	.99015	.98911	.98822	.98744	.98674	.98611	.98553	.98500	.98451
2.70	.99653	.99477	.99352	.99254	.99173	.99103	.99042	.98987	.98937	.98892	.98850	.98811
2.80	.99744	.99611	.99516	.99441	.99378	.99324	.99276	.99234	.99195	.99159	.99127	.99096
2.90	.99813	.99714	.99642	.99585	.99537	.99495	.99459	.99426	.99396	.99368	.99343	.99319
3.00	.99865	.99791	.99737	.99694	.99658	.99627	.99599	.99574	.99551	.99530	.99510	.99492
3.10	.99903	.99849	.99809	.99777	.99750	.99727	.99706	.99687	.99669	.99654	.99639	.99625
3.20	.99931	.99892	.99863	.99839	.99819	.99802	.99786	.99772	.99759	.99747	.99736	.99726
3.30	.99952	.99923	.99902	.99885	.99870	.99857	.99846	.99836	.99826	.99817	.99809	.99801
3.40	.99966	.99946	.99931	.99919	.99908	.99899	.99890	.99883	.99876	.99869	.99863	.99858
3.50	.99977	.99963	.99952	.99943	.99935	.99929	.99923	.99917	.99912	.99907	.99903	.99899