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A. E. Sarhan and B. G. Greenberg, Editors, *Contributions to Order Statistics*, John Wiley and Sons, Inc., New York, London, 1962. $11.25 and £4/0/5. xxv + 482 pp.

Review by S. K. Mitra and C. R. Rao

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The twin purposes of bringing out this volume are, according to its editors, "to assemble scattered materials to help applied research workers learn how to use the tools of order statistics" and also "to provide materials and references that will facilitate further research in methodology". The volume starts with a foreword by H. O. Hartley, an excellent introduction (Chapter I) by H. A. David, and consists of eleven other chapters, with a total of twenty-nine sections written by sixteen authors. The most significant contributions of this volume are, however, its rich collection of tables relating to order statistics (covering nearly one-third of its printed pages) and the rather extensive bibliography (with more than 300 references) which follows at the end.

This book is divided into two parts. Part I, comprising 11 sections, is concerned with the general theory of order statistics, while in Part II, with 18 sections, the emphasis is on applications of the theory to several specific parent populations. This distinction is, however, only superficial for by applications is only meant evaluation of particular integrals. It is disappointing that there is no adequate discussion of "live examples" demonstrating the need for, and the advantages in, using methods based on order statistics.

Chapter 2 (Ogawa) gives the exact distribution of order statistics and also their asymptotic forms. Chapter 3 (Lloyd) provides the theoretical inspiration for at least 50% of the book. It indicates how for a location and scale parameter family of distributions, the knowledge of the variance covariance matrix of the standardised order statistics could be used to derive the best linear systematic estimates (linear function of order statistics) of these parameters. These computations of the Generalised Least Squares estimates involve inversion of matrices of order n (sample size). Useful approximations are therefore derived in Chapter 4. Jung (4A) considers linear systematic estimates with coefficients determined by a continuous differentiable function defined in the interval (0, 1) and derives the estimate in this class with the least asymptotic variance. Blom (4B) treats the order statistics as sample quantiles and obtains linear unbiased estimates with optimum asymptotic properties. Blom's article therefore duplicates the work of Ogawa (Chapter 5), though the reader will find it difficult to establish the identity of results owing to differences in the formidable array of notations used. Gumbel writes a useful summary of his book, *Statistics of Extremes* in

Chapter 6. His equations (6.2.7a) and consequently his definitions (6.2.7b) are however wrong in respect of the characteristic smallest value. A considerable amount of work has been done in recent years on what is known as "quick and dirty methods" of analysis. Analogues of the conventional Normal Theory procedures, with range based estimates of the standard deviation, are well known members of this class. David (Chapter 7) writes an excellent review on this subject and provides a useful range of accompanying tables. Walsh (Chapter 8) describes the distribution free tolerance limits and confidence intervals on the population percentiles based on order statistics, though no reference is made to Nair's early work on confidence intervals for the median (*Sankhyā* **4** (1940)). Order statistics also enter directly or indirectly in various other nonparametric procedures. These however the editors have found expeditious to exclude from their purview. Part I is concluded with a review paper by David on Multiple Decision and Multiple Comparison procedures.

The classification of articles in Part II is based on the type of parent population considered. Chapter 10, for example, includes all papers relating to the Normal Distribution. Some were already included in Part I. Ruben (10A) is concerned with the evaluation of contents of hyperspherical simplices which are used in the computation of moments of the extreme observations and of the range in normal samples. A short table is appended. Teichroew (10B) gives a table of expected values of order statistics and of their cross products, obtained by numerical integration performed on the SWAC, and also the Sarhan-Greenberg tables of variance and covariances based on these calculations. Sarhan and Greenberg tabulate the coefficients in the best linear systematic estimates of the normal mean and standard deviation in uncensored as well as singly and doubly censored samples (10C). Alternative simple unbiased linear estimates, obtained when the actual variance covariance matrix of the order statistics is replaced by the unit matrix, are also considered and shown to be surprisingly good. Ogawa (10E) considers the problem of optimal spacing of the k sample quantiles to be included in linear estimates and provides the necessary tables for small values of k. Dixon (10G) writes a comprehensive summary of recent results dealing with "rogue" observations and "contaminated" samples, and gives a useful range of tables connected with the various test criteria that have been devised. Chapter 11 is similarly concerned with the exponential distribution and includes two papers by Sarhan and Greenberg giving the variances and covariances of order statistics and the best linear systematic estimates of the two parameters (11A and B), one paper by Epstein (11C) describing estimates arising in life testing procedures, and two papers by Ogawa (11D and E) on the use of quantiles in linear estimation, in testing, and in confidence interval estimation.

Chapter 12 is the residue class of this collection, and, of the five papers included, Gumbel's paper (12C) dealing at length on an application of extreme value theory to the estimation of endurance limit in fatigue tests deserves special mention.

The editors are certainly to be complimented for having conceived the idea,

and executed with a large measure of success, the bringing out a separate treatise on order statistics. The volume, however, is not free from the defects of a multi-author book. The contributors write with varying degrees of rigour and apparently without any well defined standard and objective. (To cite just one example, we fail to appreciate the need for reporting a theorem due to Hoeffding as done in Chapter 2, Section 2.1.) The reader is consequently tossed back and forth from a highly mathematical and sophisticated writing to a down to earth, practical "know-how" sort of discussion as he proceeds through the volume.

No doubt there will be situations where order statistics have to be used because of deficiency in data which do not permit a conventional approach, or when a distribution free approach is desired. Or we may allow its use in routine data analysis where really quick (and dirty) methods are profitable in the long run. It may be noted that not all methods based on order statistics are "quick" and then we should not be exposed to the criticism of using "dirty" methods. Further, some methods seem to depend heavily on the specification of the distribution function (such as normal, exponential, etc.) in the parent population from which observations are drawn and any recommendation of their use in practical work should be preceded by an investigation of "robustness" of these methods. There is no word like robustness in the well detailed index to the book!