BOOK REVIEW

Correspondence concerning reviews should be addressed to the Book Review Editor, Professor James F. Hannan, Department of Statistics, Michigan State University, East Lansing, Michigan 48823.

FELLER, WILLIAM, *An Introduction to Probability Theory and Its Applications.* Volume I. John Wiley and Sons, New York, 1966. xvii + 626 pp. $12.00.

Review by FRANK SPITZER

Cornell University

This is the long awaited continuation of the author's famous 'volume one'. It begins with three chapters concerning probability densities on one and higher dimensional Euclidean space. They fully recapture the spirit of volume one; indeed this material would well supplement a beginner's course from volume one, which deals only with discrete probability spaces. And just as in volume one the author delights in giving many deceptively simple results which tease the probabilistic intuition or which would require sophisticated proof if viewed outside their natural probabilistic context. The atmosphere of the text turns more austere in Chapters 4 and 5 which introduce measure on Euclidean space in fair detail, and on an abstract probability space in elegant outline (the usual extension theorems, as well as the results of Fubini and Radon-Nikodym are stated without proof). Chapter 6 develops basic notions from the theory of processes with independent increments, including the first deep results concerning the classification of random walk and the solutions of the associated Poisson (renewal) equation. Indeed this chapter lays the foundation for what appear to be the two major goals of this volume. The study of the Poisson equation is continued in Chapter 11 where it culminates in the Feller-Orey renewal theorem for random walk on the line. Together with the fluctuation theory of random walk in Chapters 12 and 18, this treatment gives a highly polished introduction to random walk. However, in view of the conflicting demands of current research on one hand and the many attractive applications to concrete probability models on the other it is not clear that a treatment designed for the non-specialist provides the best training for the serious student of probability. The latter might benefit more from systematic use of the notion of stopping times and the underlying ideas from potential theory. Nevertheless, the methods chosen by the author leave nothing to be desired in the way of motivation and clarity, although they do demand a high degree of technical ability.

The second major theory developed in this volume is much more classical and, with the appearance of this volume, perhaps a nearly closed field. It begins with Chapters 7 and 8 which present an attractive elementary treatment of operators on the space of bounded continuous functions on the line. Particular emphasis is given to convolution operators, which leads to Trotter's proof of the central limit theorem. Then Chapter 9 applies the same methods of functional analysis to exhibit the generator of the most general infinitely divisible law (semi-group), and from this result are derived the classical limit theorems for triangular arrays. This approach gives practically the complete theory (however the unique factorization of infinitely divisible laws yields only to the Fourier analytical methods developed later on) and represents a major pedagogical innovation. Nevertheless, after Chapter 15 which introduces characteristic functions, the entire subject of infinitely divisible and stable laws is redeveloped in Chapter 17, in the classical manner, but with many improvements along the way. This harmonic analysis treatment is self contained except for its dependence on the author's elegant version of the theory of functions of regular variation which was developed in Chapter 8.

That leaves only five chapters which are somewhat less essential in the framework of the reviewer's (highly subjective) evaluation of this book as a treatise primarily on convolution of probability measures or addition of random variables. Chapter 13 has its logical place in that it develops Laplace transform theory, Tauberian theorems (H. König's beautiful proof) and the Hille Yosida theorem, for all of which the ground was beautifully prepared in Chapters 7 and 8. Chapter 16 presents technical but useful refinements of the central limit theorem, and Chapter 19 is a very brief introduction to stationary processes and stochastic integrals. Finally, we come to Chapters 10 and 14, more or less one unit conceptually, although the second of these follows and uses heavily the theory of Laplace transforms. Here the reader is introduced to a variety of topics in Markov processes. The treatment is partly heuristic, as it must be for diffusion for instance, in the absence of careful sample function analysis. (In this connection the reviewer found the excellent index helpful in bridging gaps and finding scattered statements and references, the latter unfortunately often incomplete.) Only one class of Markov processes with continuous time is treated in detail, namely those with countable state space, with particular emphasis on birth and death processes, completely superseding the treatment in volume one. It is to be hoped that the formidable difficulties in presenting a more complete account of Markov processes to an audience of nonspecialists will one day be surmounted in as admirable a manner as were the difficulties in developing the material for this volume.