Wonderful Examples, but Let's not Close Our Eyes

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Abstract. The papers in this collection are superb illustrations of the power of modern Bayesian methods. They give examples of problems which are well suited to being tackled using such methods, but one must not lose sight of the merits of having multiple different strategies and tools in one's inferential armoury.

Key words and phrases: Frequentist, likelihood inference, Neyman–Pearson hypothesis testing, schools of inference.

Space prohibits me from making specific comments on each of these informative and thought-provoking papers—they each merit an extended discussion in their own right. Instead, I will make some general comments about the collection.

The papers provide marvelous examples of the power of modern statistics and, in particular, of the power of modern Bayesian methods. The adjective "modern" here is intended mainly to indicate that it is the power of the computer which has made practical solutions such as those illustrated in these papers. But I have to ask, is the emphasis on "Bayesian" necessary? That is, do we need further demonstrations aimed at promoting the merits of Bayesian methods? Surely the case is proven: Bayesian methods are very well suited to tackling many problems, leading to solutions which would be hard to arrive at by alternative methods.

The examples in this special issue were selected first by the authors, who decided what to write about, and, then, second by the editors, in deciding the extent to which the articles conformed to their desiderata of being Bayesian success stories: that they "present actual data processing stories where a non-Bayesian solution would have failed or produced suboptimal results." In a way I think this is unfortunate. I am certainly convinced of the power of Bayesian inference for tackling many problems, but the generality and power of the method is not really demonstrated by a collection specifically selected on the grounds that this approach works and others fail. To take just one example, choosing problems which would be difficult to attack using the Neyman-Pearson hypothesis testing strategy would not be a convincing demonstration of a weakness of that approach if those problems lay outside the class that approach was designed to attack. One of the basic premises of science is that you must not select the data points which support your theory, discarding those which do not. In fact, on the contrary, one should test one's theory by challenging it with tough problems or new observations. (This contrasts with political party rallies, where the candidates speak to a cheering audience of those who already support them.) So the fact that the articles in this collection provide wonderful stories illustrating the power of modern Bayesian methods is rather tarnished by the one-sidedness of the story. If I wasn't already convinced of the power of the Bayesian paradigm, I might be tempted to wonder if there was too much protestation going on.

Or perhaps, if one is going to have a collection of papers demonstrating the power of one particular inferential school, then, in the journalistic spirit of balanced reporting, we should invite a series of similar articles which "present actual data processing stories where a nonfrequentist/nonlikelihood/non-[fill in your favorite school of inference] solution would have failed or produced suboptimal results." Or even examples of the power of each of the *other* 46655 different varieties of Bayesian approach (Good, 1971).

The editors emphasized that they were not looking for "argumentative rehashes of the Bayesian versus frequentist debate." I can only commend them on that.

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On the other hand, times move on, ideas develop, and understanding deepens, so while "argumentative rehashes" might not be desirable, reexamination from a more sophisticated perspective might be. The editors went on to say "we the editors are convinced of the generic appeal of 'doing it Bayes' way,' while non-Bayesians are convinced of the opposite." I think this is a slightly unfortunate phrasing. I would (admittedly, perhaps naively) like to think that any modern statistician would look at each problem on its merits, and decide what "way" was best suited to tackle that problem. I am always a little uncomfortable when I hear about "the one true way" of looking at things.

An interesting question, perhaps in part sociological, is why different scientific communities tend to favor different schools of inference. Astronomers favor Bayesian methods, particle physicists and psychologists seem to favor frequentist methods. Is there something about these different domains which makes them more amenable to attack by different approaches?

In general, when building statistical models, we must not forget that the aim is to understand something about the real world. Or predict, choose an action, make a decision, summarize evidence, and so on, but always about the real world, not an abstract mathematical world: our models are not the reality-a point well made by George Box in his oft-cited remark that "all models are wrong, but some are useful" (Box, 1979). So, likewise, if different models suit different purposes, why should we expect one approach to inference to be universally applicable? The internal mathematical coherence of Bayesian methods is very attractive, but it must not be allowed to take priority over the ultimate aim, which is to say something about the reality we are studying. As Albert Einstein put it: "as far as the propositions of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality." (Einstein, 1921). As an aside, there is also the question of what exactly is meant by "Bayesian." Cox and Donnelly [(2011), page 144] remark that "the word Bayesian, however, became ever more widely used, sometimes representing a regression to the older usage of 'flat' prior distributions supposedly representing initial ignorance, sometimes meaning models in which the parameters of interest are regarded as random variables and occasionally meaning little more than that the laws of probability are somewhere invoked."

Turning to the papers themselves, the Bayesian approach to statistics, with its interpretation of parameters as random variables, has the merit of formulating everything in a consistent manner. Instead of trying

to fit together objects of various different kinds, one merely has a single common type of brick to use, which certainly makes life easier. In particular, this means that very elaborate models can be handled with relative ease. As is elegantly demonstrated in the papers, although the model formulation requires deep and careful thought, at some level the Bayesian procedure is attractively straightforward.

On the basis of these papers, one can certainly see the sorts of problems which lend themselves to attack by Bayesian methods and which are difficult to approach in other ways. Common characteristics seem to be complex models, fragmentary and indirect evidence, the task being evidence synthesis or explicitly to develop a probability distribution, and so on. Each of these are tough problems to cope with, and one should be reassured that statisticians now have the tools to tackle them.

But reassurance should not drift into complacency. When presented with fragmentary evidence, for example, one should proceed with caution. In such circumstances, the opportunity for undetected selection bias is considerable. Assumptions about the missing data mechanism may be untestable, perhaps even unnoticed. Data can be missing only in the context of a larger model, and one might not have any idea about what model might be suitable. Having an inferential strategy which can cope with such problems should not tempt one to ignore the fact that they are there, along with the consequent qualifications and reservations about the conclusions drawn.

Likewise, the power of Bayesian methods to handle complex models is very exciting. Many problems statisticians are asked to tackle are complex, and a complex model is necessary. So the fact that we statisticians now have a paradigm which will allow us to tackle increasingly complex models is certainly to be applauded. But I do have this nagging feeling that sometimes a more approximate solution might be more suitable. On the one hand, very elaborate models have many ways to be misspecified, and, on the other, statisticians rarely work on practical problems in isolation, but typically in conjunction with domain experts in the area being explored. The statistician brings statistical expertise, but at the end of it all the answer must be comprehensible to the other scientists: one aspect of a model being "useful," to use Box's word, is that it should be comprehensible. And there are other related aspects. Timeliness, a corollary of simplicity, is one. I am reminded of a comment made by David Lawrence of Citicorp: "In one business, we waited more than 20 months for a professor of statistics to come up with the 'Cadillac' of scoring systems, while all the business needed was a 'Chevrolet' that would work." (Lawrence, 1984, page 55).

You will see that I am trying to argue the case of balance. Despite that, and however you look at it, the editors are to be congratulated on collating a superb collection of papers illustrating the power of modern statistics to handle complex problems. Moreover, within the remit of what they set out to do—to demonstrate the power of modern Bayesian methods—they certainly succeeded. I shall definitely draw the attention of my students to this excellent collection of articles.

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