

A Tribute to Charles Stein

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In 1956, Charles Stein published an article that was to forever change the statistical approach to high-dimensional estimation. His stunning discovery that the usual estimator of the normal mean vector could be dominated in dimensions 3 and higher amazed many at the time, and became the catalyst for a vast and rich literature of substantial importance to statistical theory and practice. As a tribute to Charles Stein, this special issue on minimax shrinkage estimation is devoted to developments that ultimately arose from Stein's investigations into improving on the UMVUE of a multivariate normal mean vector. Of course, much of the early literature on the subject was due to Stein himself, including a key technical lemma commonly referred to as Stein's Lemma, which leads to an unbiased estimator of the risk of an almost arbitrary estimator of the mean vector.

The following ten papers assembled in this volume represent some of the many areas into which shrinkage has expanded (a one-dimensional pun, no doubt). Clearly, the shrinkage literature has branched out substantially since 1956, the many contributors and the breadth of theory and practice being now far too large to cover with any degree of completeness in a review issue such as this one. But what these papers do show is the lasting impact of Stein (1956), and the ongoing vitality of the huge area that he catalyzed.

- Berger, Jefferys and Müller (Bayesian nonparametric shrinkage applied to Cepheid Star oscillations) model Cepheid star oscillations via Bayesian analysis of a wavelet expansion. They illustrate how two types of shrinkage occur, the shrinkage of parameters toward some prespecified subsets, and the setting of some parameters to zero, which in this setting induces smoothness of the function estimate.
- Brandwein and Strawderman (Stein estimation for spherically symmetric distributions: Recent developments) update an earlier *Statistical Science* review

paper (Brandwein and Strawderman, 1990). Going further, this paper emphasizes the distributional robustness properties of a class of Stein estimators when a residual vector is available to estimate scale.

- Brown and Zhao (A geometrical explanation of Stein shrinkage) flesh out Stein's initial geometric heuristic for the inadmissibility of the usual estimator. By exploiting the spherical symmetry of the problem and reducing it conceptually to a two-dimensional framework, the geometry is clarified to provide justification for the fact that dimension 3 is the critical threshold for inadmissibility.
- Cai (Minimax and adaptive inference in nonparametric function estimation) takes us into the realm of nonparametric function estimation where minimax shrinkage estimation has turned out to play a major role. Cai describes how such function estimation problems are equivalent to problems of estimating infinite-dimensional multivariate normal means constrained to lie within compact subsets, and how three different but connected problems lead to similar minimax results but strikingly different adaptivity results.
- Casella and Hwang (Shrinkage confidence procedures) review the rich developments in confidence set estimation that have paralleled developments in shrinkage point estimation. Possible improvements over the classical equivariant region include not only recentering the usual confidence sets at shrinkage estimators, but also shrinking the volume of the set, and/or changing the shape of the set.
- Fourdrinier and Wells (On loss estimation) discuss the issue of postdictive assessment of statistical procedures through the study of loss estimation. One key connection in this literature which links it with much of classical shrinkage literature is the use of differential identities and inequalities related to the standard Stein identities, but here the expressions are typically of higher order. You will especially love the bi-Laplacian!
- George, Liang and Xu (From minimax shrinkage estimation to minimax shrinkage prediction) describe the parallels between the developments of minimax mean estimation under quadratic risk and minimax predictive density estimation under Kullback–

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Leibler risk. The strong connections reveal a remarkable similarity in the development of risk calculations between the two problems that allows, at least in some cases, an almost immediate proof of domination of a Bayesian shrinkage predictive density estimator over the usual best equivariant one.

- Ghosh and Datta (Small area shrinkage estimation) study the practice of “borrowing strength” as it relates to small area estimation, where survey data gathered for decision making at, say, the state level must be also used to make decisions at the county or town level. As the amount of survey data at the town level is often quite small, estimates based only on local data tend to be overly variable. The use of shrinkage techniques described by the authors is a practical tool of great utility in this setting.
- Morris and Lysy (Shrinkage estimation in multi-level normal models) lay out the evolution of the hierarchical Bayes approach to shrinkage estimation which was catalyzed by Stein’s early work. Moving through the equal variance to the unequal variance case, through multilevel model formulations, they

describe and illustrate the rich variety of shrinkage estimators and tools that have evolved for their evaluation.

- Perlman and Chaudhuri (Reversing the Stein effect) provide further insight into one of the most shocking aspects of Stein’s initial discoveries, that one could obtain risk improvement over equivariant estimators near any preselected target, thereby showing that prior information should not be ignored. They offer a cautionary tale about what can happen when attempting to relax the requirement that the shrinkage target selection be independent of the data.

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