

Discussion of “Impact of Frequentist and Bayesian Methods on Survey Sampling Practice: A Selective Appraisal” by J. N. K. Rao

Glen Meeden

It has been pointed out that when apologists for competing systems like capitalism and socialism or the frequentist and Bayesian approaches to survey sampling argue about the relative merits of their systems, they often compare their ideal to the other’s reality. Since the ideal is always quite different than reality it is easy for each of them to score points. I wish to thank Rao for avoiding this trap and giving a fair reading to both sides in his survey. Beyond that, I particularly liked the sections on the early development of frequentist methods.

How should prior information about the population be used in survey sampling? It can inform how the sample is selected and is used when making inferences after the data have been collected. Formally, at each of the two stages, the frequentist and Bayesian approaches are quite different but practically, I believe, they are often more alike than is commonly supposed.

THE FREQUENTIST APPROACH

In theory, for non-model-based frequentists, the sampling design is the most important place to use prior information. In Section 2 Rao described some of the early fundamental advances in survey design based on the frequentist approach. He explained why in stratified sampling and in stratified two-stage cluster sampling, where one cluster within each stratum is drawn, self-weighting of the units is a very desirable property. Such examples led to the notion of assigning a weight, which is the reciprocal of its inclusion probability, to each unit in the sample. A unit’s weight is the number of units in the population that it represents. A theoretical justification for this notion that is often given is that

under the sampling design the resulting estimator is unbiased. Rao argues, however, that large sample consistency of an estimator is a more important property than unbiasedness. Although it is hard to find sensible estimators which are badly biased, I agree with him that unbiasedness in and of itself is not an important property. Whatever justification there is for the notion of a weight, it should not be based on unbiasedness.

What I have sometimes found puzzling about weights is that after the sample has been selected they are often adjusted. Information that may not have been used at the design stage is used to make the sampled units and their weights more accurately reflect what is known about the population of interest. Calibration and the model-assisted approach are two common methods for achieving this end. An estimator based on the adjusted weights will no longer be design-unbiased, but there is theory to show that it can be design-consistent. Practice, however, can be more complicated especially when there are missing observations. But more importantly, the whole reweighting technology seems to me to mix up an unconditional argument (selecting the sampling design) and a conditional argument (using population information to get a good estimate after the sample has been observed). I am not suggesting that such adjustments should not be done, only that there can be more art than science in finding a good set of weights.

I believe that frequentists would be better served in their analysis if they more explicitly recognized these two different stages in the inferential process. In the first stage, one uses the information that is relevant to select the sampling design. In the second stage, after the sample has been selected, one should ignore the design but use *all* the information when constructing an estimator. In effect that is what one does when the sampling weights are adjusted. However, in the second stage, the design weights need not be used explicitly as long as all the information is being taken into account.

Glen Meeden is Professor, School of Statistics, University of Minnesota, 224 Church St. SE, Minneapolis, Minnesota 55455-0493, USA (e-mail: glen@stat.umn.edu).

If all the information has been used wisely, then the resulting estimate should work well whenever this particular sample has been observed no matter how it was selected. Again, I am not suggesting that much of current frequentist practice is badly flawed. But I believe design-based practitioners should realize that when trying to decide how to select a good sample they are arguing unconditionally and when adjusting the weights at the estimation stage they are arguing conditionally and in the latter stage they should pay less explicit attention to the design.

THE BAYESIAN APPROACH

The usual Bayesian approach requires the specification of a prior distribution over the possible population values. Basu argued (Basu, 1988), correctly I believe, that for a Bayesian after the sample has been observed, how it was selected should not enter into the analysis. This assumes of course that any information used in selecting the design was also available to the Bayesian when choosing the prior distribution. This does not mean that a Bayesian is indifferent to how a sample is selected. In theory a Bayesian should use the prior distribution to select an optimal, purposeful sample. However, as a practical matter this almost never happens. The reason is that the typical kinds of prior information available in the finite population setting seldom lend themselves to summarization in a prior distribution. The most common situation where a Bayesian can find an optimal sample is when the prior distribution is exchangeable and any sample is just as good or bad as any other. Therefore it seems unlikely to me that the Bayesian approach will be useful when deciding how to select a sample. Meeden and Noorbaloochi (2010) argue, however, that in some situations the sampling design can be thought of as part of the prior distribution.

After the sample is observed, inferences for a Bayesian are in theory straightforward. Given a sample, one uses the posterior distribution to simulate many complete copies of the population. For each simulated complete copy one calculates the population quantity of interest, say a mean or a quantile. The average of these simulated values is their point estimate and the 0.025 quantile and the 0.975 quantile of the values forms an approximate 0.95 credible interval. As Rao noted, Bayes methods have proven useful in small area estimation where certain model assumptions lead to fairly simple hierarchical priors. The more general lack of the

utilization of Bayesian methods, despite these attractive features, results from two factors. Specifying sensible prior distributions can be very difficult, and even with the recent advances in Markov chain Monte Carlo methods simulating complete copies of the population from a posterior distribution can also be difficult.

Consider situations where, given a sample, the statistician believes the unsampled or unseen units are like the sampled or seen units. This happens, for example, under simple random sampling. In such cases the Polya posterior (PP) yields a nonparametric objective pseudo/Bayesian justification for many of the standard methods. The PP does not arise from a single prior distribution but is actually a family of posteriors that arise from a sequence of prior distributions. Ghosh and Meeden (1997) give the underlying stepwise Bayes theory for this approach which proves the admissibility of many of the standard estimators.

The stepwise Bayes theory allows for a more flexible Bayesian-like approach. Rather than selecting a single prior before the sample is chosen, one selects a posterior, after the sample has been observed, which uses the sample and all available information about the population to relate the unseen to the seen. One needs to verify that all the posteriors fit together in a stepwise Bayes manner to guarantee the admissibility of the resulting procedure for any design. Even so, this can be much easier to do than specifying a single prior.

Lazar, Meeden and Nelson (2008) showed how population information about auxiliary variables can be incorporated into the PP after a sample has been observed. Examples include knowing the population mean or median of auxiliary variables and more generally only knowing that they fall in some known intervals. No model assumptions are made about how the auxiliary variables are related to the variable of interest. For this constrained version of the PP, the R (RDC Team, 2005) package *polyapost* is available to generate simulated copies of the complete population.

FINAL REMARKS

I believe that once a sample has been selected the key issue is how the unseen are related to the seen. I believe that this is in line with much frequentist practice although this is obscured by the prominent and unnecessary role played by the design weights after the sample has been selected. For a Bayesian, with a prior distribution, this happens automatically through the posterior distribution, but has been of limited value in practice. I believe that the stepwise Bayesian approach should

make it easier to select useful posteriors which make use of all the prior information present. But as Rao pointed out, this approach needs to be extended to more complicated sampling designs.

In most of sample survey, given a design, any procedure, be it frequentist or Bayesian, should be evaluated by how it behaves under repeated sampling from the design. For point estimators either their average mean squared error loss or average absolute error loss is the quantity of interest. Rather than focusing on getting an estimate of variance for the estimator to measure its precision, one should focus on using the estimator to find approximate 95% confidence intervals for the parameter of interest.

REFERENCES

- BASU, D. (1988). *Statistical Information and Likelihood: A Collection of Critical Essays. Lecture Notes in Statist.* **45** (J. K. Ghosh, ed.). Springer, New York. [MR0953081](#)
- GHOSH, M. and MEEDEN, G. (1997). *Bayesian Methods for Finite Population Sampling. Monogr. Statist. Appl. Probab.* **79**. Chapman & Hall, London. [MR1469494](#)
- LAZAR, R., MEEDEN, G. and NELSON, D. (2008). A noninformative Bayesian approach to finite population sampling using auxiliary variables. *Survey Methodol.* **34** 51–64.
- MEEDEN, G. and NOORBALOOCHI, S. (2010). Ordered designs and Bayesian inference in survey sampling. *Sankhya A* **72** 119–135. [MR2658167](#)
- RDC TEAM (2005). *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing, Vienna. Available at www.R-project.org.