Research Article

Mathematical Model of Schistosomiasis under Flood in Anhui Province

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Based on the real observation data in Tongcheng city, this paper established a mathematical model of schistosomiasis transmission under flood in Anhui province. The delay of schistosomiasis outbreak under flood was considered. Analysis of this model shows that the disease free equilibrium is locally asymptotically stable if the basic reproduction number is less than one. The stability of the unique endemic equilibrium may be changed under some conditions even if the basic reproduction number is larger than one. The impact of flood on the stability of the endemic equilibrium is studied and the results imply that flood can destabilize the system and periodic solutions can arise by Hopf bifurcation. Finally, numerical simulations are performed to support these mathematical results and the results are in accord with the observation data from Tongcheng Schistosomiasis Control Station.

1. Introduction

As we know, schistosomiasis is a serious water-borne disease. It is not easy to control because of many reasons such as flood. Many reports have shown that flood leads to a serious outbreak of schistosomiasis [1–3]. During the flood period there are a lot of people that come into contact with contaminated water, which may lead to the fact that a lot of people are infected by schistosome [1–3]. In China, Anhui province often encounters floods; in particular in 1998 the flood was one of the most serious flood [1]. Based on observation data from Tongcheng Schistosomiasis Control Station in Anhui province (Figure 1), we can see that the number of patients and the area of snails increase by a big margin after 1998 in Tongcheng city in Anhui province. Although people know the phenomenon that schistosomiasis will be serious after flood, people do not know the reason and there are only some live reports. Hence, it is necessary to investigate theoretically the effect of flood on the schistosomiasis transmission.

After flood the infected human by cercaria will have an incubation period to become an infectious human. In fact, it is about five weeks from the time of cercaria penetration through skins of human to the time when eggs are discharged [4]. Adult schistosomes in human are capable of producing eggs for a number of years [5]. This leads to breakout of schistosomiasis in many places after flood. For example, the catastrophic flood in 1998 brought a serious impact on the prevalence of schistosomiasis in Anhui province from 1998 to 2000 [1]. Furthermore, the data from Tongcheng Schistosomiasis Control Station (Figure 1) and the report of Ge et al. [1] both show that schistosomiasis is more serious in three years after flood than in the flood year. This phenomenon is called the delayed effect of flood [1]. In this paper we want to investigate how flood affects the dynamical behavior of schistosomiasis.

Many schistosomiasis models have involved many aspects such as drug-resistant, age-structure, incubation period of snail, and chemotherapy [6–10]. Their results imply that many factors affect the transmission of schistosomiasis. However,
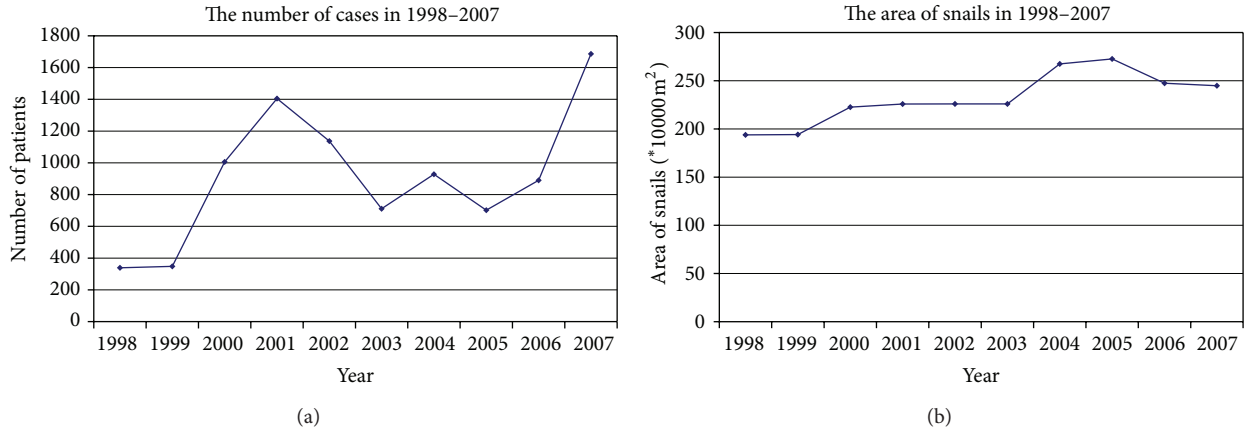
(a)

(b)

FIGURE 1: The observation data in Tongcheng city from Tongcheng Schistosomiasis Control Station in 1998–2007.

there are few mathematical models considering the effect of flood in previous papers.

To study the delayed effect of flood, we use a time delay to reflect the incubation period in the infected human. We modified the model in [11]. Distribute human into susceptible $x_s(t)$ and infectious $x_i(t)$ and snails into susceptible $y_s(t)$, preshedding $y_e(t)$, and infectious $y_i(t)$. The model in [11] reads

$$\frac{dx_s}{dt} = A_x - \mu_x x_s - \beta_x x_s y_i,$$

$$\frac{dx_i}{dt} = \beta_x x_s y_i - (\mu_x + \alpha_x) x_i,$$

$$\frac{dy_s}{dt} = A_y - \mu_y y_s - \beta_y x_i y_s, \tag{1}$$

$$\frac{dy_e}{dt} = \beta_y x_i y_s - (\mu_y + \theta) y_e,$$

$$\frac{dy_i}{dt} = \theta y_e - (\mu_y + \alpha_y) y_i,$$

where A_x is the recruitment rate of human, μ_x is the death rate of human, α_x is the disease-induced death rate of human, β_x is the transmission rate from infectious snails to susceptible human, A_y is the recruitment rate of snail host, μ_y is the death rate of snail host, α_y is the disease-induced death rate of snail host, β_y is the transmission rate from infectious human to susceptible snails, and θ is the translate rate from infected and preshedding snails to shedding snails. In the model, we have studied the stability of equilibria and preferable control strategies.

The goal of this paper is to study the impact of flood on the basic reproduction number and the dynamics of the schistosomiasis transmission. This paper is organized as follows. In Section 2 we establish a schistosomiasis model with a time delay and define the basic reproduction number R_0. The stability of the disease free equilibrium is obtained in Section 3. We devote Section 4 to the Hopf bifurcation analysis. Section 5 examines mathematical results by numerical simulations.

2. The Delayed Model

By incorporating a time delay in human, we have the following model:

$$\frac{dx_s}{dt} = A_x - \mu_x x_s(t) - \beta_x x_s(t-\tau) y_i(t-\tau) e^{-\mu_x \tau},$$

$$\frac{dx_i}{dt} = \beta_x x_s(t-\tau) y_i(t-\tau) e^{-\mu_x \tau} - (\mu_x + \alpha_x) x_i(t),$$

$$\frac{dy_s}{dt} = A_y - \mu_y y_s(t) - \beta_y x_i(t) y_s(t), \tag{2}$$

$$\frac{dy_e}{dt} = \beta_y x_i(t) y_s(t) - (\mu_y + \theta) y_e(t),$$

$$\frac{dy_i}{dt} = \theta y_e(t) - (\mu_y + \alpha_y) y_i(t),$$

where τ is the incubation period in the infected human, that is, the time from cercaria penetration through skins to the time when eggs are discharged.

Define the basic reproduction number according to biological meanings:

$$R_0 = \frac{A_x A_y \theta \beta_x e^{-\mu_x \tau} \beta_y}{\mu_x \mu_y (\mu_x + \alpha_x)(\mu_y + \alpha_y)(\mu_y + \theta)}. \tag{3}$$

These quantities have a clear biological interpretation. Consider the case when an infectious snail is introduced into a purely susceptible people population with size A_x/μ_x. The size of susceptible people who become infectious people per unit time is $\beta_x(A_x/\mu_x)$. $1/(\mu_x + \alpha_x)$ is the mean infective period of the infectious people and $e^{-\mu_x \tau}$ represents the survived rate of people during his infection. On the other hand, infectious people can infect $\beta_y(A_y/\mu_y)$ susceptible snails which should get through the latent time where the rate of transmission is θ and then infective period is $1/(\mu_y + \alpha_y)(\mu_y + \theta)$. Thus, R_0 gives the total number of secondary infectious snails produced by a typical infected snail during its entire period of infectiousness in a completely susceptible population. The following section shows that the

basic reproduction number R_0 provides a threshold condition for parasite extinction.

Theorem 1. *There exist at most two equilibria:*

(i) *if $R_0 \leq 1$, system (2) has a disease free equilibrium $E_0 = (A_x/\mu_x, 0, A_y/\mu_y, 0, 0)$;*

(ii) *if $R_0 > 1$, system (2) has two equilibria, the disease free equilibrium E_0 and the unique endemic equilibrium $E = (x_s^*, x_i^*, y_s^*, y_e^*, y_i^*)$, where*

$$x_s^* = \frac{A_x \left(A_x \beta_y + \mu_y (\mu_x + \alpha_x)\right)}{\mu_x \left(A_x \beta_y + \mu_y R_0 (\mu_x + \alpha_x)\right)},$$

$$x_i^* = \frac{A_x \mu_y (R_0 - 1)}{A_x \beta_y + \mu_y R_0 (\mu_x + \alpha_x)},$$

$$y_s^* = \frac{A_y \left(A_x \beta_y + \mu_y R_0 (\mu_x + \alpha_x)\right)}{\mu_y R_0 \left(A_x \beta_y + \mu_y (\mu_x + \alpha_x)\right)}, \tag{4}$$

$$y_e^* = \frac{\mu_x \mu_y (\mu_x + \alpha_x)(\mu_y + \alpha_y)(R_0 - 1)}{\theta \beta_x e^{-\mu_x \tau} \left(A_x \beta_y + \mu_y (\mu_x + \alpha_x)\right)},$$

$$y_i^* = \frac{\mu_x \mu_y (\mu_x + \alpha_x)(R_0 - 1)}{\beta_x e^{-\mu_x \tau} \left(A_x \beta_y + \mu_y (\mu_x + \alpha_x)\right)}.$$

Next we will discuss the stabilities of E_0 and E in system (2).

3. Stability Analysis of E_0

In this section, we will analyze the stability of the disease free equilibrium E_0 of the delayed model (2) in the two cases: $R_0 < 1$ and $R_0 > 1$.

Theorem 2. *The disease free equilibrium E_0 of the system (2) is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$.*

Proof. Denote $b = \mu_x + \alpha_x, c = \mu_y + \alpha_y, d = \mu_y + \theta$. By linearizing the system (2) around E_0 we can obtain the characteristic roots that are $-\mu_x, -\mu_y$ and roots of the following equation:

$$\lambda^3 + (b + c + d)\lambda^2 + (bc + bd + cd)\lambda$$
$$+ bcd - \frac{A_x A_y \theta \beta_x e^{-\mu_x \tau} \beta_y}{\mu_x \mu_y} e^{-\lambda \tau} = 0. \tag{5}$$

Denote the left-hand side of (5) as $F(\lambda, \tau)$. It is easy to see that

$$F(0, \tau) = bcd - \frac{A_x A_y \theta \beta_x e^{-\mu_x \tau} \beta_y}{\mu_x \mu_y} e^{-\lambda \tau}$$

$$= bcd (1 - R_0),$$

$$F_\lambda'(\lambda, \tau) = 3\lambda^2 + 2(b + c + d)\lambda + (bc + bd + cd)$$

$$+ \tau \frac{A_x A_y \theta \beta_x e^{-\mu_x \tau} \beta_y}{\mu_x \mu_y} e^{-\lambda \tau}. \tag{6}$$

(i) If $R_0 > 1$, $F(0, \tau) < 0$, $F_\lambda'(\lambda, \tau) > 0$ for $\lambda \geq 0$ and $\tau > 0$. Thus, (5) has a positive real solution for $\tau > 0$ and the disease free equilibrium E_0 is unstable.

(ii) If $R_0 < 1$, $F(0, \tau) > 0$. Since $F_\lambda'(\lambda, \tau) > 0$ for $\lambda \geq 0$ and $\tau > 0$, (5) does not have nonnegative real roots for $\tau > 0$. Hence, if (5) has roots with nonnegative real parts they must be complex roots. Moreover these complex roots should be obtained from a pair of complex conjugate roots crossing the imaginary axis. Thus, (5) must have a pair of purely imaginary roots $\lambda = \pm \omega i$ for some $\tau > 0$. Without loss of generality we assume that $\omega > 0$. Then ω must be a positive solution of the following equation:

$$-\omega^3 i - (b + c + d)\omega^2 + (bc + bd + cd)\omega i + bcd$$

$$- \frac{A_x A_y \theta \beta_x e^{-\mu_x \tau} \beta_y}{\mu_x \mu_y} (\cos(\omega\tau) - i \sin(\omega\tau)) = 0, \tag{7}$$

which is equivalent to

$$-\omega^3 + (bc + bd + cd)\omega$$

$$+ \frac{A_x A_y \theta \beta_x e^{-\mu_x \tau} \beta_y}{\mu_x \mu_y} \sin(\omega\tau) = 0,$$

$$-(b + c + d)\omega^2 + bcd$$

$$- \frac{A_x A_y \theta \beta_x e^{-\mu_x \tau} \beta_y}{\mu_x \mu_y} \cos(\omega\tau) = 0. \tag{8}$$

Let $A_x A_y \theta \beta_x e^{-\mu_x \tau} \beta_y / \mu_x \mu_y = e$. Hence,

$$\omega^6 + \left(b^2 + c^2 + d^2\right)\omega^4 + \left(b^2 c^2 + b^2 d^2 + c^2 d^2\right)\omega^2$$
$$+ \left(b^2 c^2 d^2 - e^2\right) = 0. \tag{9}$$

Assuming $z = \omega^2$, we can obtain

$$z^3 + \alpha z^2 + \beta z + \gamma = 0, \tag{10}$$

where $\alpha = b^2 + c^2 + d^2 > 0, \beta = b^2 c^2 + b^2 d^2 + c^2 d^2 > 0, \gamma = b^2 c^2 d^2 - e^2 > 0$ as $R_0 < 1$.

From [12, Lemma 3.31], if $\alpha \geq 0$, $\beta > 0$, $\gamma \geq 0$, then (10) has no positive real roots. This implies that (7) does not have positive solution ω since $R_0 < 1$. Therefore, (5) does not have purely imaginary roots. Consequently, the real parts of all eigenvalues of E_0 are negative for all positive τ. This indicates that the disease free equilibrium E_0 is locally asymptotically stable if $R_0 < 1$. □

4. Hopf Bifurcation Analysis

In this section, we turn to the study of the stability of the endemic equilibrium E when $R_0 > 1$. Notice that $R_0 > 1$ is equivalent to

$$\tau < \tau^* = \frac{1}{\mu_x} \ln \frac{A_x A_y \theta \beta_x \beta_y}{\mu_x \mu_y (\mu_x + \alpha_x)(\mu_y + \alpha_y)(\mu_y + \theta)}. \quad (11)$$

The characteristic equation of E is

$$\lambda^5 + a_1 \lambda^4 + a_2 \lambda^3 + a_3 \lambda^2 + a_4 \lambda + a_5$$
$$= e^{-\lambda \tau} \left(b_1 \lambda^4 + b_2 \lambda^3 + b_3 \lambda^2 + b_4 \lambda + b_5 \right), \quad (12)$$

where

$$a_1 = \mu_x + \mu_y + b + c + d > 0,$$

$$a_2 = bc + bd + b\mu_x + b\mu_y + cd + c\mu_x + c\mu_y$$
$$+ d\mu_x + d\mu_y + \mu_x \mu_y > 0,$$

$$a_3 = bcd + bc\mu_x + bc\mu_y + cd\mu_x$$
$$+ cd\mu_y + d\mu_x \mu_y,$$

$$a_4 = bcd\mu_x + bcd\mu_y + cd\mu_x \mu_y,$$

$$a_5 = bcd\mu_x \mu_y,$$

$$b_1 = -\beta_x y_i^* e^{-\mu_x \tau} < 0,$$

$$b_2 = -\beta_x y_i^* e^{-\mu_x \tau} \left(b + d + c + \mu_y \right) < 0,$$

$$b_3 = -\beta_x y_i^* e^{-\mu_x \tau} \left(b\mu_y + bd + d\mu_y + bc + cd + c\mu_y \right) + bcd,$$

$$b_4 = -\beta_x y_i^* e^{-\mu_x \tau} \left(bd\mu_y + bc\mu_y + cd\mu_y + bcd \right)$$
$$+ bcd \left(\mu_x + \mu_y \right),$$

$$b_5 = -\beta_x y_i^* e^{-\mu_x \tau} bcd\mu_y + bcd\mu_x \mu_y. \quad (13)$$

In the following, it can be shown that (12) does not have nonnegative real roots for $\tau > 0$. Let

$$\widetilde{a}_3 = a_3 - bcde^{-\lambda \tau},$$

$$\widetilde{a}_4 = a_4 - bcd \left(\mu_x + \mu_y \right) e^{-\lambda \tau},$$

$$\widetilde{a}_5 = a_5 - bcd\mu_x \mu_y e^{-\lambda \tau},$$

$$\widetilde{b}_3 = -\beta_x y_i^* e^{-\mu_x \tau} \left(b\mu_y + bd + d\mu_y + bc + cd + c\mu_y \right) < 0,$$

$$\widetilde{b}_4 = -\beta_x y_i^* e^{-\mu_x \tau} \left(bd\mu_y + bc\mu_y + cd\mu_y + bcd \right) < 0,$$

$$\widetilde{b}_5 = -\beta_x y_i^* e^{-\mu_x \tau} bcd\mu_y < 0. \quad (14)$$

Note that $\widetilde{a}_3 > 0, \widetilde{a}_4 > 0, \widetilde{a}_5 > 0$ for all $\lambda \geq 0$ and $\tau > 0$. We rewrite (12) in the following form:

$$\lambda^5 + a_1 \lambda^4 + a_2 \lambda^3 + \widetilde{a}_3 \lambda^2 + \widetilde{a}_4 \lambda + \widetilde{a}_5$$
$$= e^{-\lambda \tau} \left(b_1 \lambda^4 + b_2 \lambda^3 + \widetilde{b}_3 \lambda^2 + \widetilde{b}_4 \lambda + \widetilde{b}_5 \right). \quad (15)$$

It is easy to see that the left-hand side in (15) is positive while the right-hand side is negative for all $\lambda \geq 0$. Then (12) does not have nonnegative real solutions. Now we consider whether or not (12) has purely imaginary solutions.

Suppose $\lambda = \omega i$, $\omega > 0$ for some $\tau > 0$, is a root of (12). Then we have

$$\omega^5 i + a_1 \omega^4 - a_2 \omega^3 i - a_3 \omega^2 + a_4 \omega i + a_5$$
$$= [\cos(\omega \tau) - i \sin(\omega \tau)] \quad (16)$$
$$\times \left(b_1 \omega^4 - b_2 \omega^3 i - b_3 \omega^2 + b_4 \omega i + b_5 \right).$$

Therefore

$$\omega^5 - a_2 \omega^3 + a_4 \omega = \cos(\omega \tau) \left(-b_2 \omega^3 + b_4 \omega \right)$$
$$- \sin(\omega \tau) \left(b_1 \omega^4 - b_3 \omega^2 + b_5 \right),$$

$$a_1 \omega^4 - a_3 \omega^2 + a_5 = \cos(\omega \tau) \left(b_1 \omega^4 - b_3 \omega^2 + b_5 \right)$$
$$+ \sin(\omega \tau) \left(-b_2 \omega^3 + b_4 \omega \right). \quad (17)$$

From (17), we obtain

$$\left(a_1 \omega^4 - a_3 \omega^2 + a_5 \right)^2 + \left(\omega^5 - a_2 \omega^3 + a_4 \omega \right)^2$$
$$= \left(b_1 \omega^4 - b_3 \omega^2 + b_5 \right)^2 + \left(-b_2 \omega^3 + b_4 \omega \right)^2; \quad (18)$$

that is,

$$\omega^{10} + \left(a_1^2 - 2a_2 - b_1^2 \right) \omega^8 + \left(a_2^2 + 2a_4 - 2a_1 a_3 \right.$$
$$\left. - b_2^2 + 2b_1 b_3 \right) \omega^6$$
$$+ \left(a_3^2 - 2a_2 a_4 + 2a_1 a_5 + 2b_2 b_4 - b_3^2 - 2b_1 b_5 \right) \omega^4$$
$$+ \left(a_4^2 - 2a_3 a_5 - b_4^2 + 2b_3 b_5 \right) \omega^2 + \left(a_5^2 - b_5^2 \right) = 0. \quad (19)$$

Let $z = \omega^2$ again; we obtain

$$z^5 + c_1 z^4 + c_2 z^3 + c_3 z^2 + c_4 z + c_5 = 0, \quad (20)$$

where

$$c_1 = a_1^2 - 2a_2 - b_1^2,$$

$$c_2 = a_2^2 + 2a_4 - 2a_1 a_3 - b_2^2 + 2b_1 b_3,$$

$$c_3 = a_3^2 - 2a_2 a_4 + 2a_1 a_5 + 2b_2 b_4 - b_3^2 - 2b_1 b_5, \quad (21)$$

$$c_4 = a_4^2 - 2a_3 a_5 - b_4^2 + 2b_3 b_5,$$

$$c_5 = a_5^2 - b_5^2.$$

Because (20) is very complex, the roots cannot easily be found. However, we know there are positive roots in some conditions. For example, if $c_5 < 0$, then (20) has at least a positive root, say z_0, and (19) has at least a positive root $\omega_0 = \sqrt{z_0}$. Consequently, the endemic equilibrium E may lose stability and lead to oscillations because the time delay $\tau > 0$. In this case, we will do bifurcation analysis by τ as bifurcation parameter in the following.

Let $\lambda(\tau) = \xi(\tau) + i\omega(\tau)$ be a root of (12) such that $\xi(\tau_0) = 0$, $\omega(\tau_0) = \omega_0$ ($\omega_0 > 0$) for some initial value of the bifurcation parameter τ_0. From (17) we can obtain

$$
\begin{aligned}
\tau_j &= \frac{1}{\omega_0}\arccos\Bigg(\big((a_1 b_1 - b_2)\,\omega_0^8 + (b_4 + a_2 b_2 - a_1 b_3 - a_3 b_1)\,\omega_0^6 \\
&\qquad + (-a_2 b_4 - a_4 b_2 + a_1 b_5 + a_3 b_3 + a_5 b_1)\,\omega_0^4 \big) \\
&\qquad \times \Big(\big(b_1\omega_0^4 - b_3\omega_0^2 + b_5 \big)^2 + \big(-b_2\omega_0^3 + b_4\omega_0 \big)^2 \Big)^{-1} \\
&\qquad + \frac{(a_4 b_4 - a_3 b_5 - a_5 b_3)\,\omega_0^2 + a_5 b_5}{\big(b_1\omega_0^4 - b_3\omega_0^2 + b_5 \big)^2 + \big(-b_2\omega_0^3 + b_4\omega_0 \big)^2} \Bigg) \\
&\quad + \frac{2j\pi}{\omega_0}, \quad j = 0, 1, 2, \dots .
\end{aligned}
$$
(22)

Now we can show the transversal condition $(d\,\mathrm{Re}\,\lambda(\tau)/d\tau)|_{\tau=\tau_0} \ne 0$.

Differentiating (12) with respect to τ yields

$$
\begin{aligned}
&\big(5\lambda^4 + 4a_1\lambda^3 + 3a_2\lambda^2 + 2a_3\lambda + a_4 + \tau e^{-\lambda\tau} \\
&\quad \times \big(b_1\lambda^4 + b_2\lambda^3 + b_3\lambda^2 + b_4\lambda + b_5 \big) \\
&\quad - e^{-\lambda\tau}\big(4b_1\lambda^3 + 3b_2\lambda^2 + 2b_3 + b_4 \big) \big)\frac{d\lambda}{d\tau} \\
&= -\lambda e^{-\lambda\tau}\big(b_1\lambda^4 + b_2\lambda^3 + b_3\lambda^2 + b_4\lambda + b_5 \big).
\end{aligned}
$$
(23)

Using (12), we obtain

$$
\begin{aligned}
\left(\frac{d\lambda}{d\tau}\right)^{-1} &= \big(5\lambda^4 + 4a_1\lambda^3 + 3a_2\lambda^2 + 2a_3\lambda + a_4 \\
&\quad + \tau e^{-\lambda\tau}\big(b_1\lambda^4 + b_2\lambda^3 + b_3\lambda^2 + b_4\lambda + b_5 \big) \\
&\quad - e^{-\lambda\tau}\big(4b_1\lambda^3 + 3b_2\lambda^2 + 2b_3 + b_4 \big) \big) \\
&\quad \times \big(-\lambda e^{-\lambda\tau}\big(b_1\lambda^4 + b_2\lambda^3 + b_3\lambda^2 + b_4\lambda + b_5 \big) \big)^{-1} \\
&= \frac{5\lambda^4 + 4a_1\lambda^3 + 3a_2\lambda^2 + 2a_3\lambda + a_4}{-\lambda\big(\lambda^5 + a_1\lambda^4 + a_2\lambda^3 + a_3\lambda^2 + a_4\lambda + a_5 \big)} \\
&\quad + \frac{4b_1\lambda^3 + 3b_2\lambda^2 + 2b_3 + b_4}{\lambda\big(b_1\lambda^4 + b_2\lambda^3 + b_3\lambda^2 + b_4\lambda + b_5 \big)} - \frac{\tau}{\lambda}.
\end{aligned}
$$
(24)

Then,

$$
\begin{aligned}
&\mathrm{sign}\left\{\frac{d\,\mathrm{Re}\,\lambda}{d\tau}\right\}_{\lambda=i\omega_0} \\
&= \mathrm{sign}\left\{\mathrm{Re}\left(\frac{d\lambda}{d\tau}\right)^{-1}\right\}_{\lambda=i\omega_0} \\
&= \mathrm{sign}\left\{\mathrm{Re}\left[\frac{5\lambda^4 + 4a_1\lambda^3 + 3a_2\lambda^2 + 2a_3\lambda + a_4}{-\lambda\big(\lambda^5 + a_1\lambda^4 + a_2\lambda^3 + a_3\lambda^2 + a_4\lambda + a_5 \big)}\right]_{\lambda=i\omega_0} \right. \\
&\qquad \left. + \mathrm{Re}\left[\frac{4b_1\lambda^3 + 3b_2\lambda^2 + 2b_3 + b_4}{\lambda\big(b_1\lambda^4 + b_2\lambda^3 + b_3\lambda^2 + b_4\lambda + b_5 \big)}\right]_{\lambda=i\omega_0}\right\} \\
&= \mathrm{sign}\left\{\mathrm{Re}\left[\big((-2a_3\omega_0 + 4a_1\omega_0^3) \right.\right. \\
&\qquad + (-3a_2\omega_0^2 + 5\omega_0^4 + a_4)\,i\big) \\
&\qquad \times \big(\omega_0\big((a_1\omega_0^4 - a_3\omega_0^2 + a_5) \\
&\qquad\qquad + (\omega_0^5 - a_2\omega_0^3 + a_4\omega_0)\,i \big)\big)^{-1}\Big] \\
&\qquad \left.+ \mathrm{Re}\left[\frac{(2b_3\omega_0 - 4b_1\omega_0^3) + (3b_2\omega_0^2 - b_4)\,i}{\omega_0\big((b_1\omega_0^4 + b_5 - b_3\omega_0^2) + (b_4\omega_0 - b_2\omega_0^3)\,i \big)}\right]\right\} \\
&= \mathrm{sign}\left\{\big(5\omega_0^8 + (4a_1^2 - 8a_2 - 4b_1^2)\,\omega_0^6 \right. \\
&\qquad + (3a_2^2 - 6a_1 a_3 + 6a_4 + 6b_1 b_3 - 3b_2^2)\,\omega_0^4 \\
&\qquad + (2a_3^2 - 4a_2 a_4 + 4a_1 a_5 + 4b_2 b_4 - 2b_3^2 - 4b_1 b_5)\,\omega_0^2 \\
&\qquad + (a_4^2 - 2a_3 a_5 + 2b_3 b_5 - b_4^2)\big) \\
&\qquad \times \Big(\big(a_1\omega_0^4 - a_3\omega_0^2 + a_5 \big)^2 \\
&\qquad \left.+ \big(\omega_0^5 - a_2\omega_0^3 + a_4\omega_0 \big)^2 \Big)^{-1}\right\} .
\end{aligned}
$$
(25)

If we denote $z_0 = \omega_0^2$, we get

$$
\begin{aligned}
&\mathrm{sign}\left\{\frac{d\,\mathrm{Re}\,\lambda}{d\tau}\right\}_{\lambda=i\omega_0} \\
&= \mathrm{sign}\left\{\frac{5z_0^4 + 4c_1 z_0^3 + 3c_2 z_0^2 + 2c_3 z_0 + c_4}{\big(a_1\omega_0^4 - a_3\omega_0^2 + a_5 \big)^2 + \big(\omega_0^5 - a_2\omega_0^3 + a_4\omega_0 \big)^2}\right\} .
\end{aligned}
$$
(26)

Denote $f(z) = z^5 + c_1 z^4 + c_2 z^3 + c_3 z^2 + c_4 z + c_5$. Suppose ω_0 is the largest positive simple root of (19); from [12, Lemma 3.32 and Theorem 3.32], we have

$$
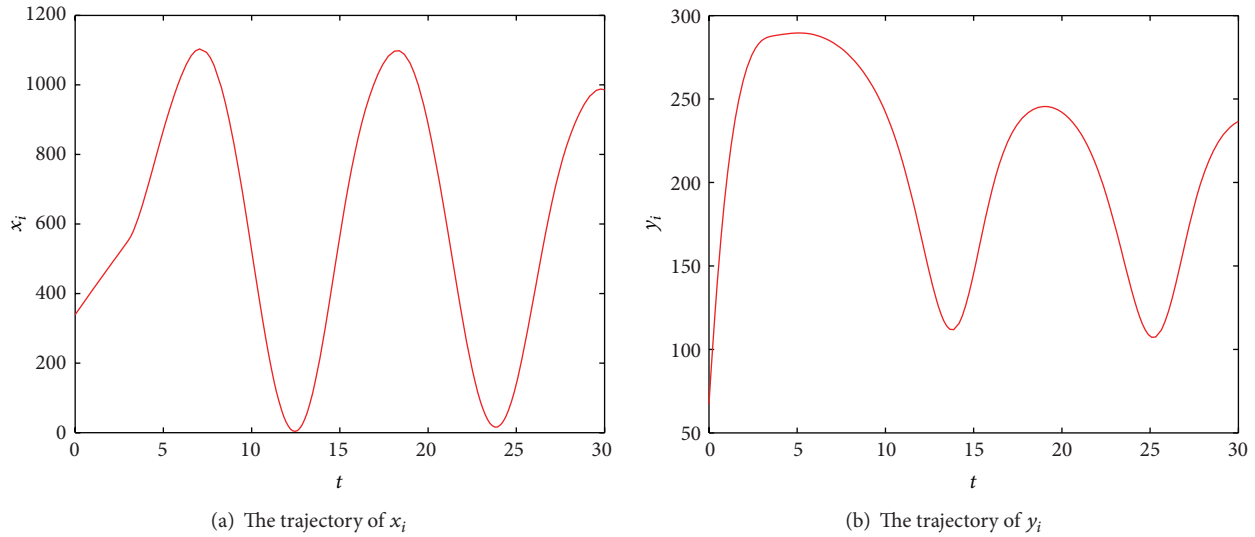\left.\frac{df(z)}{dz}\right|_{z=z_0} = 5z_0^4 + 4c_1 z_0^3 + 3c_2 z_0^2 + 2c_3 z_0 + c_4 > 0. \quad (27)
$$

(a) The trajectory of x_i

(b) The trajectory of y_i

FIGURE 2: The trajectories of x_i and y_i occur oscillations when $\tau_0 = 3$.

Thus,

$$\text{sign} \left\{ \frac{d \operatorname{Re} \lambda}{d\tau} \right\}_{\lambda = i\omega_0}$$

$$= \text{sign} \left\{ \left(\frac{df(z)}{dz} \Big|_{z=z_0} \right) \right.$$

$$\times \left(\left(a_1 \omega_0^4 - a_3 \omega_0^2 + a_5 \right)^2 \right.$$

$$\left. \left. + \left(\omega_0^5 - a_2 \omega_0^3 + a_4 \omega_0 \right)^2 \right)^{-1} \right\} = +1.$$

(28)

Summarizing the above analysis, we have the following results.

Theorem 3. If $R_0 > 1$, $c_5 < 0$ and ω_0 is the largest positive simple root of (19), a Hopf bifurcation occurs around the endemic equilibrium E of the delayed model (2).

5. Numerical Simulations

Based on the observation data from the investigation of Tongcheng Schistosomiasis Control Station in Anhui province, we estimated transmission rates in our model. Also according to the previous papers [7–9, 11, 13], we choose the parameter values in Table 1. Thus, $R_0 > 1$, $\tau^* = 327$, and $c_5 < 0$ when $\tau = 0.1$.

Note that the bifurcation parameter $\tau_0 = 3$ at this time. We performed some simulations and obtained Figure 2. From Figure 2 we can see that Hopf bifurcation can occur when $\tau_0 = 3$. This implies that schistosomiasis will break out in about three years after flood. It is also in accord with the investigation of Tongcheng Schistosomiasis Control Station. This phenomenon is also in accord with the report of the whole Anhui province [1]. From our theoretical results and

TABLE 1: Values of parameters.

Parameters	Values (per capita per year)	References
A_x	6	[8, 9]
μ_x	0.014	[8, 9, 11]
α_x	10^{-5}	[8, 9]
β_x	0.003	Estimated
A_y	100	[8, 9]
μ_y	0.3	[8, 9, 11]
α_y	0.01	[9]
β_y	0.001	Estimated
θ	9.125	[13]

the reports we can see that schistosomiasis will break out in about the third year after a flood. Hence, we can get the result that the delayed effect of flood may be caused by the incubation period of schistosome in the infected human.

6. Discussion

In this paper, based on the observation data in Tongcheng Schistosomiasis Control Station in Anhui province we have modified our previous model by including a time delay that describes the incubation period of schistosome within infected human. We define the basic reproduction number R_0 according to biological meanings and give the existence of the disease free equilibrium and the endemic equilibrium. We find that, if $R_0 < 1$, then the disease free equilibrium is locally asymptotically stable. However, the stability of the unique endemic equilibrium may be changed under some condition even if the basic reproduction number is larger than one. The results imply that the time delay can destabilize the system and periodic solutions can arise by Hopf bifurcation.

Numerical simulations imply that schistosomiasis will break out in about three years after flood. Furthermore the

observation data show that schistosomiasis will be the most serious in about the third year after flood. From Figure 1, we can see that the number of patients and snails did not greatly change in 1998 and 1999. However, in 2001 the number of patients became about 5 times that of 1998, and the area of snails became about two times that of 1998. In our simulations, there is a little difference. Our results are higher than the observation data. We think the reason may be that after flood the government dispatched a large number of manpower and material resources to control the spread of the disease. In summary, our theoretical results are in accord with the investigation of Tongcheng Schistosomiasis Control Station and the report of Anhui Province Institute of Schistosomiasis for the whole Anhui province. Hence, we can obtain the result that after flood the delayed effect of flood may be caused by the incubation period of schistosome in the definitive host. Furthermore, the period of outbreak is about three years after flood in Anhui province.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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