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Research Article

Exact Solutions of the Symmetric Regularized Long Wave Equation and the Klein-Gordon-Zakharov Equations

Isaiah Elvis Mhlanga and Chaudry Masood Khalique

International Institute for Symmetry Analysis and Mathematical Modelling, Department of Mathematical Sciences, North-West University, Mafikeng Campus, Private Bag X 2046, Mmabatho 2735, South Africa

Correspondence should be addressed to Chaudry Masood Khalique; masood.khalique@nwu.ac.za

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We study two nonlinear partial differential equations, namely, the symmetric regularized long wave equation and the Klein-Gordon-Zakharov equations. The Lie symmetry approach along with the simplest equation and exp-function methods are used to obtain solutions of the symmetric regularized long wave equation, while the travelling wave hypothesis approach along with the simplest equation method is utilized to obtain new exact solutions of the Klein-Gordon-Zakharov equations.

1. Introduction

The investigation of exact travelling wave solutions of nonlinear partial differential equations (NLPDEs) is important for the understanding of most nonlinear physical phenomena that appear in many areas of scientific fields such as plasma physics, solid state physics, fluid dynamics, optical fibers, mathematical biology, and chemical kinetics [1, 2]. A number of methods have been developed for finding travelling wave solutions to NLPDEs. These include the homogeneous balance method [3], the ansatz method [4, 5], variable separation approach [6], inverse scattering transform method [2], Bäcklund transformation [7], Darboux transformation [8], Hirota bilinear method [9], the (G'/G)-expansion method [10], the reduction mKdV equation method [11], the trifunction method [12, 13], the projective Riccati equation method [14], the sine-cosine method [15, 16], the Jacobi elliptic function expansion method [17, 18], the F-expansion method [19], the exp-function expansion method [20], dynamical system method [21–23], and Lie symmetry method [24–28].

In this paper we study two nonlinear partial differential equations, namely, the symmetric regularized long wave equation and the Klein-Gordon-Zakharov equations. The Lie symmetry approach along with the simplest equation and exp-function methods are used to obtain solutions of the symmetric regularized long wave equation, while the travelling wave hypothesis approach along with the simplest equation method is utilized to obtain new exact solutions of the Klein-Gordon-Zakharov equations.

2. The Symmetric Regularized Long Wave Equation

We first consider that the symmetric regularized long wave equation (SRLW) as given by

$$u_{tt} - u_{xx} + \frac{1}{2}\left(u^2\right)_{xt} - u_{xxtt} = 0 \tag{1}$$

is a nonlinear evolution equation which arises in several physical applications, for example in sound waves in a plasma [29]. Exact travelling wave solutions of this equation were obtained using the (G'/G)-expansion method [29]. In the present work, Lie symmetry method along with the simplest equation method and the exp-function method are used to construct exact solutions for this equation. First the Lie point symmetries of the SRLW equation (1) are found using the Lie algorithm [25]. These Lie point symmetries are then used to transform (1) into an ordinary differential equation. The simplest equation method [30] and the exp-function method [20] are then used to construct exact solutions of the ordinary differential equation, which leads to the exact solutions of the SRLW equation.

2.1. Lie Point Symmetries of (1) *and Symmetry Reduction.* The symmetry group of the SRLW equation (1) is generated by the vector field

$$X = \xi^1(t,x,u)\frac{\partial}{\partial t} + \xi^2(t,x,u)\frac{\partial}{\partial x} + \eta(t,x,u)\frac{\partial}{\partial u}. \quad (2)$$

Applying the fourth prolongation of X to (1) and solving the resultant overdetermined system of linear partial differential equations, we obtain the following two translation symmetries:

$$X_1 = \frac{\partial}{\partial t}, \quad X_2 = \frac{\partial}{\partial x}. \quad (3)$$

Now taking the linear combination of these translation symmetries $X_1 = \partial/\partial t$ and $X_2 = \partial/\partial x$, namely, the symmetry $X = X_1 + \nu X_2$, where ν is a constant, leads to the two invariants

$$z = x - \nu t, \qquad u = F(z). \quad (4)$$

Treating F as the new dependent variable and z as the new independent variable and then substituting the value of u into the SRLW equation (1) transform (1) into a fourth-order nonlinear ordinary differential equation:

$$\left(\nu^2 - 1\right)F''(z) - \nu F(z)F''(z) - \nu\left(F'(z)\right)^2 + \nu^2 F(z)'''' = 0. \quad (5)$$

2.2. Exact Solutions of (1) *Using Simplest Equation Method.* Now the simplest equation method [30, 31] is used to solve (5), and henceforth one obtains the exact solutions of the SRLW equation (1). The Bernoulli and Riccati equations will be used as the simplest equations. The Bernoulli and Riccati equations are well-known equations whose solutions can be expressed in terms of elementary functions [28].

The Bernoulli equation which we use here is given by

$$H'(z) = cH(z) + dH^2(z), \quad (6)$$

where c and d are constants. Its solution is given by

$$H(z) = c\left\{\frac{\cosh[c(z+C)] + \sinh[c(z+C)]}{1 - d\cosh[c(z+C)] - d\sinh[c(z+C)]}\right\}, \quad (7)$$

where C is a constant of integration [28].

For the Riccati equation

$$G'(z) = cG^2(z) + dG(z) + e, \quad (8)$$

where c, d, and e are constants, the solutions to be used are

$$\begin{aligned} H(z) &= -\frac{d}{2c} - \frac{\theta}{2c}\tanh\left[\frac{1}{2}\theta(z+C)\right], \\ H(z) &= -\frac{d}{2c} - \frac{\theta}{2c}\tanh\left(\frac{1}{2}\theta z\right) \\ &\quad + \frac{\operatorname{sech}(\theta z/2)}{C\cosh(\theta z/2) - (2c/\theta)\sinh(\theta z/2)}, \end{aligned} \quad (9)$$

with $\theta^2 = d^2 - 4ce > 0$ and C being a constant of integration [28].

2.2.1. Solutions of (1) *Using Bernoulli as the Simplest Equation.* The solutions of the ODE (5) are considered to be in the form

$$F(z) = \sum_{i=0}^{M} \mathscr{A}_i (H(z))^i, \quad (10)$$

where $H(z)$ satisfies the Bernoulli or Riccati equations, M is a positive integer that can be determined by balancing the highest order derivative term with the highest order nonlinear term [31], and $\mathscr{A}_i$, $(i = 0, 1, \ldots, M)$ are parameters to be determined.

The balancing procedure yields $M = 2$, so the solutions of (5) are of the form

$$F(z) = \mathscr{A}_0 + \mathscr{A}_1 H + \mathscr{A}_2 H^2. \quad (11)$$

Substituting (11) into (5), making use of the Bernoulli equation (6), and then equating all coefficients of the function H^i to zero, we obtain the following algebraic system of equations in terms of $\mathscr{A}_0$, $\mathscr{A}_1$, and $\mathscr{A}_2$:

$$\begin{aligned} &-10\nu\mathscr{A}_2^2 d^2 - 120\nu^2\mathscr{A}_2 d^4 = 0, \\ &-24\nu^2\mathscr{A}_1 d^4 - 12\nu\mathscr{A}_1 d^2\mathscr{A}_2 \\ &\quad - 336\nu^2\mathscr{A}_2 d^3 c - 18\nu\mathscr{A}_2^2 dc = 0, \\ &-\nu\mathscr{A}_0\mathscr{A}_1 c^2 - \mathscr{A}_1 c^2 - \nu^2\mathscr{A}_1 c^4 + \nu^2\mathscr{A}_1 c^2 = 0, \\ &-8\nu\mathscr{A}_2^2 c^2 - 6\mathscr{A}_2 d^2 + 6\nu^2\mathscr{A}_2 d^2 - 21\nu\mathscr{A}_1 d\mathscr{A}_2 c - 3\nu\mathscr{A}_1^2 d^2 \\ &\quad - 60\nu^2\mathscr{A}_1 d^3 c - 6\nu\mathscr{A}_0\mathscr{A}_2 d^2 - 330\nu^2\mathscr{A}_2 d^2 c^2 = 0, \\ &-3\mathscr{A}_1 dc + 3\nu^2\mathscr{A}_1 dc - 4\nu\mathscr{A}_0\mathscr{A}_2 c^2 - 4\mathscr{A}_2 c^2 + 4\nu^2\mathscr{A}_2 c^2 \\ &\quad - 3\nu\mathscr{A}_0\mathscr{A}_1 dc - 2\nu\mathscr{A}_1^2 c^2 \\ &\quad - 16\nu^2\mathscr{A}_2 c^4 - 15\nu^2\mathscr{A}_1 dc^3 = 0, \\ &-2\mathscr{A}_1 d^2 - 50\nu^2\mathscr{A}_1 d^2 c^2 + 10\nu^2\mathscr{A}_2 dc - 5\nu\mathscr{A}_1^2 dc \\ &\quad - 130\nu^2\mathscr{A}_2 dc^3 + 2\nu^2\mathscr{A}_1 d^2 - 9\nu\mathscr{A}_1 c^2\mathscr{A}_2 \\ &\quad - 2\nu\mathscr{A}_0\mathscr{A}_1 d^2 - 10\nu\mathscr{A}_0\mathscr{A}_2 dc - 10\mathscr{A}_2 dc = 0. \end{aligned} \quad (12)$$

Solving this system, with the aid of Maple, we obtain the following values for the constants:

$$\begin{aligned} \mathscr{A}_0 &= \frac{-c^2\nu^2+\nu^2-1}{\nu}, \\ \mathscr{A}_1 &= -12cd\nu, \\ \mathscr{A}_2 &= -12\nu d^2. \end{aligned} \tag{13}$$

As a result a solution of the symmetric regularized long wave equation (1) using the Bernoulli equation as the simplest equation is

$$\begin{aligned} u(t,x) &= \mathscr{A}_0 + \mathscr{A}_1 c\left\{\frac{\cosh[c(z+C)]+\sinh[c(z+C)]}{1-d\cosh[c(z+C)]-d\sinh[c(z+C)]}\right\} \\ &\quad + \mathscr{A}_2 c^2\left\{\frac{\cosh[c(z+C)]+\sinh[c(z+C)]}{1-d\cosh[c(z+C)]-d\sinh[c(z+C)]}\right\}^2, \end{aligned} \tag{14}$$

where $z=x-\nu t$ and C is a constant of integration.

2.2.2. Solutions of (1) *Using Ricatti as the Simplest Equation.* The balancing procedure yields $M=2$, so the solutions of (5) are of the form

$$F(z)=\mathscr{A}_0+\mathscr{A}_1 G(z)+\mathscr{A}_2 G^2(z). \tag{15}$$

Substituting (15) into (5), making use of the Ricatti equation (8), and then equating all coefficients of the function G^i to zero, we obtain an algebraic system of equations in terms of $\mathscr{A}_0$, $\mathscr{A}_1$, and $\mathscr{A}_2$. Solving the resultant algebraic equations, we obtain the following set of values:

$$\begin{aligned} \mathscr{A}_0 &= -\frac{8ce\nu^2+d^2\nu^2-\nu^2+1}{\nu}, \\ \mathscr{A}_1 &= -12cd\nu, \\ \mathscr{A}_2 &= -12c^2\nu. \end{aligned} \tag{16}$$

It follows that the solutions for the symmetric regularized long wave equation (1) using the Ricatti equation as the simplest equation are

$$\begin{aligned} u(t,x) &= \mathscr{A}_0+\mathscr{A}_1\left\{-\frac{d}{2c}-\frac{\theta}{2c}\tanh\left[\frac{1}{2}\theta(z+C)\right]\right\} \\ &\quad + \mathscr{A}_2\left\{-\frac{d}{2c}-\frac{\theta}{2c}\tanh\left[\frac{1}{2}\theta(z+C)\right]\right\}^2, \end{aligned}$$

$$\begin{aligned} u(t,x) &= \mathscr{A}_0+\mathscr{A}_1\left\{-\frac{d}{2c}-\frac{\theta}{2c}\tanh\left(\frac{1}{2}\theta z\right)\right. \\ &\qquad \left.+\frac{\operatorname{sech}(\theta z/2)}{C\cosh(\theta z/2)-(2c/\theta)\sinh(\theta z/2)}\right\} \\ &\quad +\mathscr{A}_2\left\{-\frac{d}{2c}-\frac{\theta}{2c}\tanh\left(\frac{1}{2}\theta z\right)\right. \\ &\qquad \left.+\frac{\operatorname{sech}(\theta z/2)}{C\cosh(\theta z/2)-(2c/\theta)\sinh(\theta z/2)}\right\}^2, \end{aligned} \tag{17}$$

where $z=x-\nu t$ with $\theta^2=d^2-4ce>0$ and C is a constant of integration.

2.3. Solution of (1) *Using the Exp-Function Method.* In this section we use the exp-function method [20] to solve the symmetric regularized long wave equation (1). We consider solutions of (5) in the form

$$F(z)=\frac{\sum_{n=-b}^{c} a_n e^{nz}}{\sum_{m=-p}^{q} b_m e^{mz}}, \tag{18}$$

where b, c, p, and q are positive integers to be determined and a_n and b_m are arbitrary constants [20]. The balancing procedure of the exp-function method produces $p=b$ and $q=c$. For simplicity, we set $p=b=1$ and $q=c=1$ so that (18) is reduced to

$$F(z)=\frac{a_1e^z+a_0+a_{-1}e^{-z}}{b_1e^z+b_0+b_{-1}e^{-z}}. \tag{19}$$

Substituting (19) into (5) and solving the resultant ODE, with the help of Maple, one possible set of values of the constants is

$$\begin{aligned} a_{-1} &= -\frac{b_{-1}}{\nu}, \\ a_0 &= \frac{b_0\left(6\nu^2-1\right)}{\nu}, \\ a_1 &= \frac{-b_0^2}{4\nu b_{-1}}, \\ b_1 &= \frac{b_0^2}{4b_{-1}}. \end{aligned} \tag{20}$$

As a result we obtain the solution

$$u(t,x)=\frac{a_1e^{x-\nu t}+a_0+a_{-1}e^{-(x-\nu t)}}{b_1e^{x-\nu t}+b_0+b_{-1}e^{-(x-\nu t)}}. \tag{21}$$

3. The Klein-Gordon-Zakharov Equations

The Klein-Gordon-Zakharov (KGZ) equations [32]

$$u_{tt}-u_{xx}+u+uv+|u|^2u=0, \tag{22a}$$

$$v_{tt}-v_{xx}-\left(|u|^2\right)_{xx}=0, \tag{22b}$$

are a coupled system of nonlinear partial differential equations of two functions $u(x,t)$ and $v(x,t)$. This model describes the interaction of the Langmuir wave and the ion acoustic wave in plasma. The function $u(x,t)$ denotes the fast time scale component of electric field raised by electrons and the function $v(x,t)$ denotes the deviation of ion density from its equilibrium. Here $u(x,t)$ is a complex function and $v(x,t)$ is a real function. Note that if we remove the term $|u|^2u$, then this system reduces to the classical Klein-Gordon-Zakharov system [33]

$$u_{tt} - u_{xx} + u + uv = 0, \tag{23a}$$

$$v_{tt} - v_{xx} - \left(|u|^2\right)_{xx} = 0. \tag{23b}$$

A number of studies have been conducted for this system ((23a) and (23b)) in different time space [34–38]. However, for the KGZ equations (22a) and (22b), Chen [39] considered orbital stability of solitary waves, while Shi et al. [33] employed the sine-cosine method and the extended tanh method to construct exact solutions of the KGZ equations (22a) and (22b).

In this paper, we employ an entirely different approach, namely, the travelling wave variable approach along with the simplest equation method to obtain exact solutions of the KGZ equations (22a) and (22b).

3.1. Solution of (22a) *and* (22b) *Using the Travelling Wave Variable Approach.* The travelling wave variable approach converts the system of nonlinear partial differential equations into a system of nonlinear ordinary differential equations, which we then solve to obtain exact solutions of the system.

In order to solve the KGZ equations (22a) and (22b), we first transform it into a system of nonlinear ordinary differential equations which can then be solved in order to obtain its exact solutions.

We make the wave variable transformation

$$\begin{aligned} u &= e^{i\phi}u(z), \qquad v = v(z), \\ \phi &= px + rt, \qquad z = kx + dt, \end{aligned} \tag{24}$$

where p, r, k, and d are real constants and $d \neq k$. Using this transformation, (22a) and (22b) transform into

$$\begin{aligned} &\left(p^2 - r^2 + 1\right)u + i\left(2rd - 2pk\right)u' \\ &\quad + \left(d^2 - k^2\right)u'' + uv + u^3 = 0, \end{aligned} \tag{25a}$$

$$\left(d^2 - k^2\right)v'' - \left(u^2\right)'' = 0. \tag{25b}$$

Integrating (25b) twice and taking the constants of integration to be zero, we obtain

$$v = \frac{u^2}{d^2 - k^2}. \tag{26}$$

Now substituting (26) into (25a), we get

$$u'' = \left(\frac{r^2 - p^2 - 1}{d^2 - k^2}\right)u + \left(\frac{d^2 - k^2 + 1}{\left(d^2 - k^2\right)^2}\right)u^3, \tag{27}$$

which can be written in the form

$$u'' = Pu + Qu^3, \tag{28}$$

where

$$P = \frac{r^2 - p^2 - 1}{d^2 - k^2}, \qquad Q = \frac{d^2 - k^2 + 1}{\left(d^2 - k^2\right)^2}. \tag{29}$$

Solving (28), with the aid of Mathematica, we obtain the solution

$$u(z) = \pm\frac{1}{H}i\,\mathrm{sn}\,(F \mid \omega), \tag{30}$$

where $\mathrm{sn}(F \mid \omega)$ is a Jacobian elliptic function of the sine amplitude [40],

$$\begin{aligned} F &= \frac{\sqrt{\left(\sqrt{P^2 - 2Qc_1} - P\right)\left(z + c_2\right)^2}}{\sqrt{2}}, \\ H &= \sqrt{\frac{Q}{\sqrt{P^2 - 2Qc_1} + P}}, \\ \omega &= \frac{-Qc_1 + P\left(\sqrt{P^2 - 2Qc_1} + P\right)}{Qc_1} \end{aligned} \tag{31}$$

is the modulus of the elliptic function with $0 < \omega < 1$. Here c_1 and c_2 are constants of integration. Reverting back to our original variables, we can now write the solution of our Klein-Gordon-Zakharov equations as

$$u(x,t) = \pm\frac{1}{H}i\,\mathrm{sn}\,(F \mid \omega), \tag{32}$$

where

$$F = \frac{\sqrt{\left(\sqrt{P^2 - 2Qc_1} - P\right)\left(kx + dt + c_2\right)^2}}{\sqrt{2}}, \tag{33}$$

and ω and H are as above.

Now $v(x,t)$ can be obtained from (26).

It should be noted that the solution (32) is valid for $0 < \omega < 1$, as ω approaches zero, the solution becomes the normal sine function, $\sin z$, and as ω approaches 1, the solution tends to the tanh function, $\tanh z$.

The profile of the solution (32) is given in Figure 1.

3.2. Solutions of (22a) *and* (22b) *Using the Simplest Equation Method.* We consider the solutions of (27) in the form

$$u(z) = \sum_{i=0}^{M} A_i (G(z))^i, \tag{34}$$

where $G(z)$ satisfies the Bernoulli or the Riccati equation.

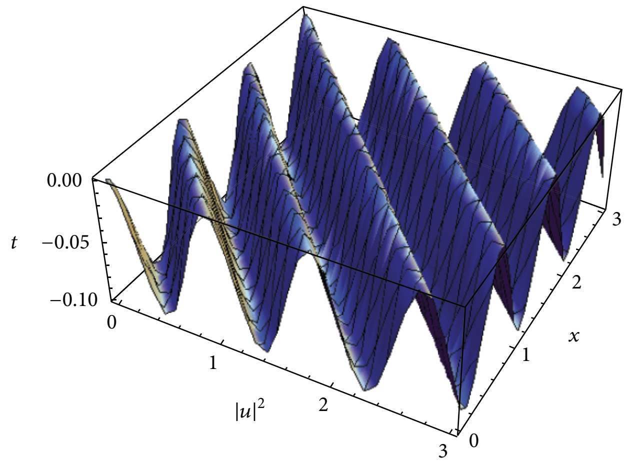

FIGURE 1: Profile of solution (32).

3.2.1. Solutions of (22a) *and* (22b) *Using Bernoulli as the Simplest Equation.* We consider the Bernoulli equation

$$G'(z) = aG(z) + bG^2(z), \tag{35}$$

where a and b are constants.

The balancing procedure yields $M = 1$, so the solution of (27) is of the form

$$u(z) = A_0 + A_1 H(z). \tag{36}$$

Substituting (36) into (27), making use of the Bernoulli equation (35), and then equating all coefficients of the function H^i to zero, we obtain the following algebraic system of equations:

$$\begin{aligned}
&2A_1k^4b^2 - k^2A_1^3 + d^2A_1^3 - 4A_1d^2k^2b^2 \\
&\quad + A_1^3 + 2A_1d^4b^2 = 0, \\
&3A_1k^4ab + 3d^2A_0A_1^2 - 3k^2A_0A_1^2 - 6A_1d^2k^2ab \\
&\quad + 3A_0A_1^2 + 3A_1d^4ab = 0, \\
&p^2A_0d^2 - p^2A_0k^2 - r^2A_0d^2 + r^2A_0k^2 - A_0k^2 \\
&\quad + d^2A_0^3 - k^2A_0^3 + A_0d^2 + A_0^3 = 0, \\
&A_1d^2 + 3d^2A_0^2A_1 - p^2A_1k^2 + r^2A_1k^2 + p^2A_1d^2 \\
&\quad + A_1d^4a^2 - 3k^2A_0^2A_1 + A_1k^4a^2 + 3A_0^2A_1 \\
&\quad - 2A_1d^2k^2a^2 - r^2A_1d^2 - A_1k^2 = 0.
\end{aligned} \tag{37}$$

Solving this system, with the aid of Maple, we obtain the following values for the constants:

$$\begin{aligned}
A_0 &= \frac{a\left(d^2 - k^2\right)}{\sqrt{2\left(k^2 - d^2 - 1\right)}}, \\
A_1 &= \frac{\sqrt{2}b\left(d^2 - k^2\right)}{k^2 - d^2 - 1}, \\
p &= \sqrt{\frac{r^2d^2 - r^2k^2 + k^2 - d^2A_0^2 + k^2A_0^2 - d^2 - A_0^2}{d^2 - k^2}}.
\end{aligned} \tag{38}$$

As a result, a solution of the Klein-Gordon-Zakharov equations (22a) and (22b), using the Bernoulli equation as the simplest equation, is

$$\begin{aligned}
u(x,t) &= e^{i(px+rt)}\Bigg[\Big(\Big(\sqrt{2}ab\left(d^2 - k^2\right)\left(\cosh\left(a\left(kx + dt + c\right)\right)\right. \\
&\qquad \left. + \sinh\left(a\left(kx + dt + c\right)\right)\right)\Big) \\
&\qquad \times \Big(\sqrt{k^2 - d^2 - 1}\left(1 - b\cosh\left(a\left(kx + dt + c\right)\right)\right. \\
&\qquad \left. - b\sinh\left(a\left(kx + dt + c\right)\right)\right)\Big)^{-1}\Big) \\
&\qquad + \frac{a\left(d^2 - k^2\right)}{\sqrt{2\left(k^2 - d^2 - 1\right)}}\Bigg],
\end{aligned} \tag{39}$$

where c is a constant of integration.

3.2.2. Solutions of (22a) *and* (22b) *Using Riccati as the Simplest Equation.* We use the Riccati equation given by

$$G'(z) = aG^2(z) + bG(z) + c, \tag{40}$$

where a, b, and c are constants. The balancing procedure yields $M = 1$, so the solution of (27) is of the form

$$u(z) = A_0 + A_1G(z). \tag{41}$$

Similar calculations yield the following set of values:

$$\begin{aligned}
A_0 &= \frac{b\left(d^2 - k^2\right)}{\sqrt{2\left(k^2 - d^2 - 1\right)}}, \\
A_1 &= -\frac{\sqrt{2}a\left(d^2 - k^2\right)\sqrt{k^2 - d^2 - 1}}{d^2 - k^2 + 1}, \\
c &= -\frac{\sqrt{2\left(k^2 - d^2 - 1\right)}\left(d^2b^2 - k^2b^2 - 2 - 2p^2 + 2r^2\right)}{4A_1\left(d^2 - k^2 + 1\right)}.
\end{aligned} \tag{42}$$

As a result the two solutions of (22a) and (22b) are

$$u(x,t) = e^{i\phi}\left[A_0 + A_1\left\{-\frac{b}{2a} - \frac{\theta}{2a}\tanh\left[\frac{1}{2}\theta(z+C)\right]\right\}\right],$$

$$\begin{aligned} u(x,t) &= e^{i\phi}\Bigg[A_0 \\ &\quad + A_1\Bigg\{-\frac{b}{2a} - \frac{\theta}{2a}\tanh\left(\frac{1}{2}\theta z\right) \\ &\quad + \frac{\operatorname{sech}(\theta z/2)}{C\cosh(\theta z/2) - (2a/\theta)\sinh(\theta z/2)}\Bigg\}\Bigg], \end{aligned} \tag{43}$$

where $\phi = px + rt$ and $z = kx + dt$. θ is given by $\sqrt{b^2 - 4ac}$, C is a constant of integration, and A_0 and A_1 are as obtained above.

It should be noted that by substituting the above value of $u(x,t)$ into (26), one can now obtain the solution for the variable $v(x,t)$.

4. Conclusion

In this paper we studied two nonlinear partial differential equations. Firstly, Lie symmetry approach along with the simplest equation and the Exp-function method were used to obtain travelling wave solutions of the symmetric regularized long wave equation. Secondly, the travelling wave hypothesis approach along with the simplest equation method is utilized to obtain new exact solutions of the Klein-Gordon-Zakharov equations.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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