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Research Article

Hopf Bifurcation in a Delayed SEIQRS Model for the Transmission of Malicious Objects in Computer Network

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A delayed SEIQRS model for the transmission of malicious objects in computer network is considered in this paper. Local stability of the positive equilibrium of the model and existence of local Hopf bifurcation are investigated by regarding the time delay due to the temporary immunity period after which a recovered computer may be infected again. Further, the properties of the Hopf bifurcation are studied by using the normal form method and center manifold theorem. Numerical simulations are also presented to support the theoretical results.

1. Introduction

The action of malicious objects throughout a network can be studied by using epidemic models due to the high similarity between malicious objects and biological viruses [1–7]. In [2], Thommes and Coates proposed a modified version of the SEI model to predict virus propagation in a network. In [3], Mishra and Pandey proposed a SEIRS epidemic model for the transmission of worms with vertical transmission. Recently, antivirus counter measures such as virus immunization and quarantine strategy have been introduced into some epidemic models in order to study the prevalence of virus. In [7], Mishra and Jha proposed the following SEIQRS model for the transmission of malicious objects in computer network:

$$\begin{aligned}
\frac{dS(t)}{dt} &= A - \beta S(t) I(t) - dS(t) + \eta R(t),\\
\frac{dE(t)}{dt} &= \beta S(t) I(t) - (d+\mu) E(t),\\
\frac{dI(t)}{dt} &= \mu E(t) - (d+\alpha+\gamma+\delta) I(t),\\
\frac{dQ(t)}{dt} &= \delta I(t) - (d+\alpha+\varepsilon) Q(t),\\
\frac{dR(t)}{dt} &= \gamma I(t) + \varepsilon Q(t) - (d+\eta) R(t),
\end{aligned} \tag{1}$$

where $S(t)$, $E(t)$, $I(t)$, $Q(t)$, and $R(t)$ denote the sizes of nodes at time t in the states susceptible, exposed, infectious, quarantined, and recovered, respectively. The parameters A, d, and β are positive constants. A is the recruitment rate of susceptible nodes to the computer network. d is the crashing rate of nodes due to the reason other than the attack of malicious objects. α is the crashing rate of nodes due to the attack of malicious objects. β is the transmission rate. μ, γ, δ, ε, and η are the state transition rates. Mishra and Jha [7] investigated the global stability of the unique endemic equilibrium for the system (1).

It is well known that time delays can cause a stable equilibrium to become unstable and make a system bifurcate periodic solutions and dynamical systems with delay have been studied by many scholars [8–15]. In [9], Feng et al. investigated the Hopf bifurcation of a delayed viral infection model in computer networks by using theories of stability and bifurcation. In [11], Dong et al. proposed a computer virus model with time delay based on an SEIR model and studied the dynamical behaviors such as local stability and local Hopf bifurcation by regarding the time delay as a bifurcating parameter. Motivated by the work above and considering the temporary immunity period after which a recovered computer may be infected again, we introduce the time delay due to the temporary immunity period into system (1) and get the following system with time delay:

$$\begin{aligned}\frac{dS(t)}{dt} &= A - \beta S(t)I(t) - dS(t) + \eta R(t-\tau),\\ \frac{dE(t)}{dt} &= \beta S(t)I(t) - (d+\mu)E(t),\\ \frac{dI(t)}{dt} &= \mu E(t) - (d+\alpha+\gamma+\delta)I(t),\\ \frac{dQ(t)}{dt} &= \delta I(t) - (d+\alpha+\varepsilon)Q(t),\\ \frac{dR(t)}{dt} &= \gamma I(t) + \varepsilon Q(t) - dR(t) - \eta R(t-\tau),\end{aligned} \tag{2}$$

where $\tau \ge 0$ is the time delay due to the temporary immunity period.

The main purpose is to investigate the effects of the delay on system (2) and this paper is organized as follows. Sufficient conditions for the local stability and existence of local Hopf bifurcation are obtained by regarding the delay due to the temporary immunity period after which a recovered computer may be infected again as a bifurcating parameter in Section 2. Direction of the Hopf bifurcation and stability of the bifurcating periodic solutions are determined by the normal form method and center manifold theorem in Section 3. In Section 4, we give a numerical example to support the theoretical results in the paper.

2. Stability and Existence of Local Hopf Bifurcation

According to the analysis in [7], we can get that if the condition (H_1): $R_{cq} = \beta(A/d)/(\mu+\alpha+\delta+\gamma+d) > 1$ holds, then system (2) has a unique positive equilibrium $P^*(S^*, E^*, I^*, Q^*, R^*)$, where

$$\begin{aligned}S^* &= \frac{A/d}{R_{cq}}, \qquad E^* = \frac{d(R_{cq}-1)}{\beta},\\ I^* &= \frac{R_{cq}-1}{\beta}\left(\frac{\mu d}{d+\alpha}\right),\\ Q^* &= \frac{R_{cq}-1}{\beta}\left(\frac{\delta\mu}{\varepsilon+d+\alpha}\right),\\ R^* &= \frac{R_{cq}-1}{\beta}\left(\gamma + \frac{\varepsilon\delta\eta}{\eta+\varepsilon+d+\alpha}\right).\end{aligned} \tag{3}$$

Let $\overline{S}(t) = S(t) - S^*$, $\overline{E}(t) = E(t) - E^*$, $\overline{I}(t) = I(t) - I^*$, $\overline{Q}(t) = Q(t) - Q^*$, and $\overline{R}(t) = R(t) - R^*$. Dropping the bars for the sake of simplicity, system (2) can be rewritten as the following system:

$$\begin{aligned}\frac{dS(t)}{dt} &= a_1 S(t) + a_2 I(t) + b_1 R(t-\tau) - \beta S(t)I(t),\\ \frac{dE(t)}{dt} &= a_3 S(t) + a_4 E(t) + a_5 I(t) + \beta S(t)I(t),\\ \frac{dI(t)}{dt} &= a_6 E(t) + a_7 I(t),\\ \frac{dQ(t)}{dt} &= a_8 I(t) + a_9 Q(t),\\ \frac{dR(t)}{dt} &= a_{10} I(t) + a_{11} Q(t) + a_{12} R(t) + b_2 R(t-\tau),\end{aligned} \tag{4}$$

where

$$\begin{gathered}a_1 = -(\beta I^* + d), \qquad a_2 = -\beta S^*,\\ a_3 = \beta I^*, \qquad a_4 = -(d+\mu),\\ a_5 = \beta S^*, \qquad a_6 = \mu,\\ a_7 = -(d+\alpha+\gamma+\delta), \qquad a_8 = \delta,\\ a_9 = -(d+\alpha+\varepsilon), \qquad a_{10} = \gamma,\\ a_{11} = \varepsilon, \qquad a_{12} = -d, \qquad b_1 = \eta, \qquad b_2 = -\eta.\end{gathered} \tag{5}$$

Then, we can get the linearized system of system (4) as follows:

$$\begin{aligned}\frac{dS(t)}{dt} &= a_1 S(t) + a_2 I(t) + b_1 R(t-\tau),\\ \frac{dE(t)}{dt} &= a_3 S(t) + a_4 E(t) + a_5 I(t),\\ \frac{dI(t)}{dt} &= a_6 E(t) + a_7 I(t),\\ \frac{dQ(t)}{dt} &= a_8 I(t) + a_9 Q(t),\\ \frac{dR(t)}{dt} &= a_{10} I(t) + a_{11} Q(t) + a_{12} R(t) + b_2 R(t-\tau).\end{aligned} \tag{6}$$

Thus, the characteristic equation of system (6) at the positive equilibrium P^* is

$$\begin{aligned}&\lambda^5 + m_4\lambda^4 + m_3\lambda^3 + m_2\lambda^2 + m_1\lambda + m_0\\ &\quad + \left(n_4\lambda^4 + n_3\lambda^3 + n_2\lambda^2 + n_1\lambda + n_0\right)e^{-\lambda\tau} = 0,\end{aligned} \tag{7}$$

where

$$m_4 = -\left(a_1 + a_4 + a_7 + a_9 + a_{12}\right),$$
$$m_3 = a_1a_4 + a_1a_7 + a_1a_9 + a_1a_{12} + a_4a_7 + a_4a_9 + a_4a_{12} + a_7a_9 + a_7a_{12} + a_9a_{12} - a_5a_6,$$
$$m_2 = a_1a_5a_6 + a_5a_6a_9 + a_5a_6a_{12} - a_2a_3a_6 - a_1a_4a_7 - a_1a_4a_9 - a_1a_4a_{12} - a_1a_7a_9 - a_1a_7a_{12} - a_4a_7a_{12} - a_4a_9a_{12} - a_7a_9a_{12},$$
$$m_1 = a_1a_4a_7a_9 + a_1a_4a_7a_{12} + a_1a_4a_9a_{12} + a_1a_7a_9a_{12} + a_4a_7a_9a_{12} + a_2a_3a_6a_9 + a_2a_3a_6a_{12} - a_1a_5a_6a_9 - a_1a_5a_6a_{12} + a_5a_6a_9a_{12},$$
$$m_0 = a_1a_5a_6a_9a_{12} - a_1a_4a_7a_9a_{12} - a_2a_3a_6a_9a_{12},$$
$$n_4 = -b_2, \qquad n_3 = \left(a_1 + a_4 + a_7 + a_9\right)b_2,$$
$$n_2 = \left(a_5a_6 - a_1a_4 - a_1a_7 - a_1a_9 - a_4a_7 - a_4a_9 - a_7a_9\right)b_2,$$
$$n_1 = \left(a_1a_4a_7 + a_1a_4a_9 + a_1a_7a_9 + a_4a_7a_9 + a_2a_3a_6 - a_1a_5a_6 - a_5a_6a_9\right)b_2 + a_3a_6a_{10}b_1,$$
$$n_0 = a_1a_5a_6a_9b_2 - a_1a_4a_7a_9b_2 - a_2a_3a_6a_9b_2 - a_3a_6a_8a_{11}b_1 - a_3a_6a_9a_{10}b_1. \tag{8}$$

For the existence of local Hopf bifurcation of system (2), we give the following result.

Theorem 1. *For system* (2)*, if conditions* (H_2)*–*(H_4) *hold, then the positive equilibrium* $P^*(S^*, E^*, I^*, Q^*, R^*)$ *of system* (2) *is asymptotically stable for* $\tau \in [0, \tau_0)$ *and system* (2) *undergoes a Hopf bifurcation at the positive equilibrium* $P^*(S^*, E^*, I^*, Q^*, R^*)$ *when* $\tau = \tau_0$*, where the conditions* (H_2)*–*(H_4) *and the expression of* τ_0 *are defined in the following analysis.*

Proof. For $\tau = 0$, (7) becomes

$$\lambda^5 + d_4\lambda^4 + d_3\lambda^3 + d_2\lambda^2 + d_1\lambda + d_0 = 0, \tag{9}$$

where

$$d_4 = m_4 + n_4, \qquad d_3 = m_3 + n_3,$$
$$d_2 = m_2 + n_2, \qquad d_1 = m_1 + n_1, \tag{10}$$
$$d_0 = m_0 + n_0.$$

Obviously, $D_1 = d_4 > 0$. Therefore, if condition (H_2): (11) holds, then the positive equilibrium P^* is locally asymptotically stable without delay. Consider

$$D_2 = \det\begin{pmatrix} d_4 & 1 \\ d_2 & d_3 \end{pmatrix} > 0,$$
$$D_3 = \det\begin{pmatrix} d_4 & 1 & 0 \\ d_2 & d_3 & d_4 \\ 0 & d_1 & d_2 \end{pmatrix} > 0,$$
$$D_4 = \det\begin{pmatrix} d_4 & 1 & 0 & 0 \\ d_2 & d_3 & d_4 & 1 \\ d_0 & d_1 & d_2 & d_3 \\ 0 & 0 & d_0 & d_1 \end{pmatrix} > 0, \tag{11}$$
$$D_5 = \det\begin{pmatrix} d_4 & 1 & 0 & 0 & 0 \\ d_2 & d_3 & d_4 & 1 & 0 \\ d_0 & d_1 & d_2 & d_3 & d_4 \\ 0 & 0 & d_0 & d_1 & d_2 \\ 0 & 0 & 0 & 0 & d_0 \end{pmatrix} > 0.$$

For $\tau > 0$, let $\lambda = i\omega$ $(\omega > 0)$ be a root of (7). Then, we can get

$$\left(n_1\omega - n_3\omega^3\right)\sin\tau\omega + \left(n_4\omega^4 - n_2\omega^2 + n_0\right)\cos\tau\omega = m_2\omega^2 - m_4\omega^4 - m_0, \tag{12}$$
$$\left(n_1\omega - n_3\omega^3\right)\cos\tau\omega - \left(n_4\omega^4 - n_2\omega^2 + n_0\right)\sin\tau\omega = -\omega^5 + m_3\omega^3 - m_1\omega, \tag{13}$$

from which we obtain

$$\omega^{10} + c_4\omega^8 + c_3\omega^6 + c_2\omega^4 + c_1\omega^2 + c_0 = 0, \tag{14}$$

with

$$c_4 = m_4^2 - 2m_3 - n_4^2,$$
$$c_3 = m_3^2 + 2m_1 - 2m_2m_4 - n_3^2 + 2n_2n_4,$$
$$c_2 = m_2^2 + 2m_0m_4 - 2m_1m_3 - n_2^2 - 2n_0n_4 + 2n_1n_3, \tag{15}$$
$$c_1 = m_1^2 + 2m_0m_2 - n_1^2 + 2n_0n_2,$$
$$c_0 = m_0^2 - n_0^2.$$

Let $\omega^2 = v$, then (14) becomes

$$v^5 + c_4v^4 + c_3v^3 + c_2v^2 + c_1v + c_0 = 0. \tag{16}$$

In order to give the main results in this paper, we make the following assumption.

(H_3) Equation (16) has at least one positive real root.

Suppose that condition (H_3) holds. Without loss of generality, we suppose that (16) has five positive roots, which are denoted as $v_1, v_2, \ldots, v_5$, respectively. Then, (14) has five positive roots $\omega_k = \sqrt{v_k}, k = 1, 2, \ldots, 5$. For every fixed ω_k, the corresponding critical value of time delay is

$$\tau_k^{(j)} = \frac{1}{\omega_k}\arccos\frac{g(\omega_k)}{h(\omega_k)} + \frac{2j\pi}{\omega_k}, \quad k = 1, 2, \ldots, 5, \quad j = 0, 1, 2, \ldots, \tag{17}$$

where

$$\begin{aligned} g(\omega_k) = {} & (n_3 - m_4 n_4)\,\omega_k^8 + (m_2 n_4 + m_4 n_2 - m_3 n_3 - n_1)\,\omega_k^6 \\ & + (m_1 n_3 + m_3 n_1 - m_0 n_4 - m_2 n_2 - m_4 n_0)\,\omega_k^4 \\ & + (m_0 n_2 + m_2 n_0 - m_1 n_1)\,\omega_k^2 - m_0 n_0, \\ h(\omega_k) = {} & n_4^2 \omega_k^8 + (n_3^2 - 2 n_2 n_4)\,\omega_k^6 \\ & + (n_2^2 + 2 n_0 n_4 - 2 n_1 n_3)\,\omega_k^4 \\ & + (n_1^2 - 2 n_0 n_2)\,\omega_k^2 + n_0^2. \end{aligned} \tag{18}$$

Let

$$\tau_0 = \min\left\{\tau_k^{(0)}\right\}, \quad k \in \{1, 2, \ldots, 5\}, \quad \omega_0 = \omega_k\big|_{\tau=\tau_0}. \tag{19}$$

Taking the derivative of λ with respect to τ in (7), we obtain

$$\begin{aligned} \left[\frac{d\lambda}{d\tau}\right]^{-1} = {} & -\frac{5\lambda^4 + 4m_4\lambda^3 + 3m_3\lambda^2 + 2m_2\lambda + m_1}{\lambda(\lambda^5 + m_4\lambda^4 + m_3\lambda^3 + m_2\lambda^2 + m_1\lambda + m_0)} \\ & + \frac{4n_4\lambda^3 + 3n_3\lambda^2 + 2n_2\lambda + n_1}{\lambda(n_4\lambda^4 + n_3\lambda^3 + n_2\lambda^2 + n_1\lambda + n_0)} - \frac{\tau}{\lambda}. \end{aligned} \tag{20}$$

Then, we have

$$\operatorname{Re}\left[\frac{d\lambda}{d\tau}\right]^{-1}_{\tau=\tau_0} = \frac{f'(v_*)}{(m_2\omega_0^2 - m_4\omega_0^4 - m_0)^2}, \tag{21}$$

where $f(v) = v^5 + c_4 v^4 + c_3 v^3 + c_2 v^2 + c_1 v + c_0$ and $v_* = \omega_0^2$. Obviously, if condition (H_4) $f'(v_*) \neq 0$ holds, then $\operatorname{Re}[d\lambda/d\tau]^{-1}_{\tau=\tau_0} \neq 0$. That is, if condition (H_4) holds, then the transversality condition is satisfied. The proof of Theorem 1 is completed. □

3. Properties of the Hopf Bifurcation

In the previous section, we have obtained the conditions under which system (2) undergoes Hopf bifurcation at the positive equilibrium $P^*(S^*, E^*, I^*, Q^*, R^*)$ when the delay τ crosses though the critical value τ_0. In this section, we give the formula that determines the direction of Hopf bifurcation and stability of the bifurcating periodic solutions of system (2).

Define

$$\begin{aligned} \mu_2 &= -\frac{\operatorname{Re}\{C_1(0)\}}{\operatorname{Re}\{\lambda'(\tau_0)\}}, \\ \beta_2 &= 2\operatorname{Re}\{C_1(0)\}, \\ T_2 &= -\frac{\operatorname{Im}\{C_1(0)\} + \mu_2 \operatorname{Im}\{\lambda'(\tau_0)\}}{\tau_0\omega_0}, \end{aligned} \tag{22}$$

where $C_1(0)$ is defined in the following analysis. Further, we give the following result with respect to the direction of Hopf bifurcation and stability of the bifurcating periodic solutions of system (2).

Theorem 2. *For system* (2)*, if $\mu_2 > 0$ ($\mu_2 < 0$), the Hopf bifurcation is supercritical (subcritical). If $\beta_2 < 0$ ($\beta_2 > 0$), the bifurcating periodic solutions are stable (unstable). If $T_2 > 0$ ($T_2 < 0$), the period of the bifurcating periodic solutions increases (decreases).*

Proof. Let $\tau = \tau + \mu$, $\mu \in R$, so that $\mu = 0$ is the Hopf bifurcation value of system (2). Rescaling the time delay by $t \to (t/\tau)$. $u_1(t) = S(t) - S^*$, $u_2(t) = E(t) - E^*$, $u_3(t) = I(t) - I^*$, $u_4(t) = Q(t) - Q^*$, and $u_5(t) = R(t) - R^*$, then system (2) can be transformed into the following form:

$$\dot{u}(t) = L_\mu u_t + F(\mu, u_t), \tag{23}$$

where $u_t = (u_1(t), u_2(t), u_3(t), u_4(t), u_5(t))^T \in C = C([-1, 0], R^5)$ and $L_\mu : C \to R^5$, $F : R \times C \to R^5$ are given, respectively, by

$$\begin{aligned} L_\mu\phi = {} & (\tau_0 + \mu)\begin{pmatrix} a_1 & 0 & a_2 & 0 & 0 \\ a_3 & a_4 & a_5 & 0 & 0 \\ 0 & a_6 & a_7 & 0 & 0 \\ 0 & 0 & a_8 & a_9 & 0 \\ 0 & 0 & a_{10} & a_{11} & a_{12} \end{pmatrix}\phi(0) \\ & + (\tau_0 + \mu)\begin{pmatrix} 0 & 0 & 0 & 0 & b_1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & b_2 \end{pmatrix}\phi(-1), \end{aligned} \tag{24}$$

$$F(\mu, \phi) = (\tau_0 + \mu)\begin{pmatrix} -\beta\phi_1(0)\phi_3(0) \\ \beta\phi_1(0)\phi_3(0) \\ 0 \\ 0 \\ 0 \end{pmatrix}. \tag{25}$$

By the Riesz representation theorem, there exists a 5×5 matrix function $\eta(\theta, \mu) : [-1, 0] \to R^5$ whose elements are of bounded variation such that

$$L_\mu\phi = \int_{-1}^{0} d\eta(\theta, \mu)\,\phi(\theta), \quad \phi \in C. \tag{26}$$

In fact, we choose

$$\eta(\theta,\mu) = (\tau_0+\mu)\begin{pmatrix} a_1 & 0 & a_2 & 0 & 0\\ a_3 & a_4 & a_5 & 0 & 0\\ 0 & a_6 & a_7 & 0 & 0\\ 0 & 0 & a_8 & a_9 & 0\\ 0 & 0 & a_{10} & a_{11} & a_{12}\end{pmatrix}\delta(\theta) + (\tau_0+\mu)\begin{pmatrix} 0 & 0 & 0 & 0 & b_1\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & b_2\end{pmatrix}\delta(\theta+1), \tag{27}$$

where δ is the Dirac delta function.

For $\phi \in C([-1,0],R^5)$, we define

$$A(\mu)\phi = \begin{cases} \dfrac{d\phi(\theta)}{d\theta}, & -1\le\theta<0,\\ \displaystyle\int_{-1}^{0} d\eta(\theta,\mu)\phi(\theta), & \theta=0,\end{cases}$$
$$R(\mu)\phi = \begin{cases} 0, & -1\le\theta<0,\\ F(\mu,\phi), & \theta=0.\end{cases} \tag{28}$$

Then system (23) can be transformed into the following operator equation:

$$\dot u(t) = A(\mu)u_t + R(\mu)u_t, \tag{29}$$

where $u_t = u(t+\theta)$ for $\theta\in[-1,0]$.

The adjoint operator A^* of A is defined by

$$A^*(\varphi) = \begin{cases} -\dfrac{d\varphi(s)}{ds}, & 0<s\le 1,\\ \displaystyle\int_{-1}^{0} d\eta^T(s,0)\varphi(-s), & s=0,\end{cases} \tag{30}$$

associated with a bilinear form

$$\langle\varphi(s),\phi(\theta)\rangle = \overline{\varphi}(0)\phi(0) - \int_{\theta=-1}^{0}\int_{\xi=0}^{\theta}\overline{\varphi}(\xi-\theta)\,d\eta(\theta)\,\phi(\xi)\,d\xi, \tag{31}$$

where $\eta(\theta)=\eta(\theta,0)$.

Let $q(\theta) = (1,q_2,q_3,q_4,q_5)^T e^{i\omega_0\tau_0\theta}$ be the eigenvector of A corresponding to $i\omega_0\tau_0$ and $q^*(s) = D(1,q_2^*,q_3^*,q_4^*,q_5^*)e^{i\omega_0\tau_0 s}$ the eigenvector of A^* corresponding to $-i\omega_0\tau_0$. From the definition of $A(0)$ and $A^*(0)$ and by a simple computation, we obtain

$$\begin{gathered} q_2 = \frac{a_3+a_5q_3}{i\omega_0-a_4},\\ q_3 = \frac{a_3a_6}{(i\omega_0-a_4)(i\omega_0-a_7)-a_5a_6},\\ q_4 = \frac{a_3a_8}{i\omega_0-a_9},\qquad q_5 = \frac{i\omega_0-a_1a_3-a_2q_3}{b_4e^{-i\tau_0\omega_0}},\\ q_2^* = -\frac{i\omega_0+a_1}{a_3},\qquad q_3^* = \frac{a_4(i\omega_0+a_1)}{a_3(i\omega_0+a_6)},\\ q_4^* = -\frac{a_{11}q_5^*}{i\omega_0+a_9},\\ q_5^* = -\frac{b_1e^{i\tau_0\omega_0}}{i\omega_0+a_{12}+b_2e^{i\tau_0\omega_0}}. \end{gathered} \tag{32}$$

From (31), we have

$$\begin{aligned} \langle q^*(s),q(\theta)\rangle = \overline{D}\big[&1+q_2\overline{q}_2^*+q_3\overline{q}_3^*+q_4\overline{q}_4^*\\ &+q_5\overline{q}_5^*+q_5\tau_0e^{-i\tau_0\omega_0}(b_1+b_2\overline{q}_5^*)\big]. \end{aligned} \tag{33}$$

Let

$$\begin{aligned} \overline{D} = \big[&1+q_2\overline{q}_2^*+q_3\overline{q}_3^*+q_4\overline{q}_4^*+q_5\overline{q}_5^*\\ &+q_5\tau_0e^{-i\tau_0\omega_0}(b_1+b_2\overline{q}_5^*)\big]^{-1}. \end{aligned} \tag{34}$$

Then, $\langle q^*,q\rangle = 1$, $\langle q^*,\overline{q}\rangle = 0$.

Next, we can obtain the coefficients determining the properties of the Hopf bifurcation by the algorithms introduced in [16] as follows:

$$\begin{aligned} g_{20} &= 2\beta\tau_0\overline{D}(\overline{q}_2^*-1)q^{(1)}(0)q^{(3)}(0),\\ g_{11} &= \beta\tau_0\overline{D}(\overline{q}_2^*-1)\left(\overline{q}^{(1)}(0)q^{(3)}(0)+q^{(1)}(0)\overline{q}^{(3)}(0)\right),\\ g_{02} &= 2\beta\tau_0\overline{D}(\overline{q}_2^*-1)\overline{q}^{(1)}(0)\overline{q}^{(3)}(0),\\ g_{21} &= 2\beta\tau_0\overline{D}(\overline{q}_2^*-1)\\ &\quad\times\Big(W_{11}^{(1)}(0)q^{(3)}(0)+\frac{1}{2}W_{20}^{(1)}(0)\overline{q}^{(3)}(0)\\ &\quad+W_{11}^{(3)}(0)q^{(1)}(0)+\frac{1}{2}W_{20}^{(3)}(0)\overline{q}^{(1)}(0)\Big), \end{aligned} \tag{35}$$

with

$$\begin{aligned} W_{20}(\theta) &= \frac{ig_{20}q(0)}{\tau_0\omega_0}e^{i\tau_0\omega_0\theta} + \frac{i\overline{g}_{02}\overline{q}(0)}{3\tau_0\omega_0}e^{-i\tau_0\omega_0\theta}+E_1e^{2i\tau_0\omega_0\theta}, \end{aligned} \tag{36}$$

$$W_{11}(\theta) = -\frac{ig_{11}q(0)}{\tau_0\omega_0}e^{i\tau_0\omega_0\theta}+\frac{i\overline{g}_{11}\overline{q}(0)}{\tau_0\omega_0}e^{-i\tau_0\omega_0\theta}+E_2,$$

where E_1 and E_2 can be determined by the following equations, respectively:

$$E_1 = 2\begin{pmatrix} 2i\omega_0 - a_1 & 0 & -a_2 & 0 & -b_1 e^{-2i\omega_0\tau_0} \\ -a_3 & 2i\omega_0 - a_4 & -a_5 & 0 & 0 \\ 0 & -a_6 & 2i\omega_0 - a_7 & 0 & 0 \\ 0 & 0 & -a_8 & 2i\omega_0 - a_9 & 0 \\ 0 & 0 & -a_{10} & -a_{11} & 2i\omega_0 - a_{12} - b_2 e^{-2i\omega_0\tau_0} \end{pmatrix}^{-1} \begin{pmatrix} E_1^{(1)} \\ E_1^{(2)} \\ 0 \\ 0 \\ 0 \end{pmatrix},$$
$$E_2 = -\begin{pmatrix} a_1 & 0 & a_2 & 0 & b_1 \\ a_3 & a_4 & a_5 & 0 & 0 \\ 0 & a_6 & a_7 & 0 & 0 \\ 0 & 0 & a_8 & a_9 & 0 \\ 0 & 0 & a_{10} & a_{11} & a_{12} + b_2 \end{pmatrix}^{-1} \begin{pmatrix} E_2^{(1)} \\ E_2^{(2)} \\ 0 \\ 0 \\ 0 \end{pmatrix}, \tag{37}$$

with

$$\begin{aligned} E_1^{(1)} &= -\beta q^{(1)}(0)\, q^{(3)}(0), \\ E_1^{(2)} &= \beta q^{(1)}(0)\, q^{(3)}(0), \\ E_2^{(1)} &= -\beta\left(\overline{q}^{(1)}(0)\, q^{(3)}(0) + q^{(1)}(0)\,\overline{q}^{(3)}(0)\right), \\ E_2^{(2)} &= \beta\left(\overline{q}^{(1)}(0)\, q^{(3)}(0) + q^{(1)}(0)\,\overline{q}^{(3)}(0)\right). \end{aligned} \tag{38}$$

Then, we can get the expression of $C_1(0)$ as follows:

$$C_1(0) = \frac{i}{2\tau_0\omega_0}\left(g_{11}g_{20} - 2\left|g_{11}\right|^2 - \frac{\left|g_{02}\right|^2}{3}\right) + \frac{g_{21}}{2}. \tag{39}$$

Further we have

$$\begin{aligned} \mu_2 &= -\frac{\operatorname{Re}\{C_1(0)\}}{\operatorname{Re}\{\lambda'(\tau_0)\}}, \\ \beta_2 &= 2\operatorname{Re}\{C_1(0)\}, \\ T_2 &= -\frac{\operatorname{Im}\{C_1(0)\} + \mu_2 \operatorname{Im}\{\lambda'(\tau_0)\}}{\tau_0\omega_0}, \end{aligned} \tag{40}$$

where the sign μ_2 determines the direction of the Hopf bifurcation, the sign β_2 determines the stability of the bifurcating periodic solutions, and the sign of T_2 determines the period of the bifurcating periodic solutions. The proof of Theorem 2 is completed. □

4. Numerical Simulation

In this section, we use a numerical example to support the theoretical analysis above in this paper. We take the following particular case of system (2) in which $A = 0.3$, $\beta = 0.75$, $d = 0.1$, $\eta = 0.2$, $\mu = 0.3$, $\alpha = 0.2$, $\gamma = 0.18$, $\delta = 0.38$, and $\varepsilon = 0.3$. Consider

$$\begin{aligned} \frac{dS(t)}{dt} &= 0.3 - 0.75S(t)I(t) - 0.1S(t) + 0.2R(t-\tau), \\ \frac{dE(t)}{dt} &= 0.75S(t)I(t) - 0.4E(t), \\ \frac{dI(t)}{dt} &= 0.3E(t) - 0.86I(t), \\ \frac{dQ(t)}{dt} &= 0.38I(t) - 0.6Q(t), \\ \frac{dR(t)}{dt} &= 0.18I(t) + 0.3Q(t) - 0.1R(t) - 0.2R(t-\tau). \end{aligned} \tag{41}$$

It is easy to verify that $R_{cq} = 1.9397 > 1$. Thus, condition (H_1) holds. The positive equilibrium $P^*(1.5466, 0.1253, 0.1253, 0.2381, 0.2612)$ can be obtained by solving the equations in system (41). By some complex computations, we obtain that (16) has one positive root $v_* = 0.2217$, and further we have $\omega_0 = 0.4709$, $\tau_0 = 8.9829$. First, we choose $\tau = 8.0025 < \tau_0$; the corresponding phase plots are shown in Figures 1 and 2; it is easy to see from Figures 1 and 2 that system (41) is asymptotically stable. Then, we choose $\tau = 10.025 > \tau_0$. The corresponding phase plots are shown in Figures 3 and 4. It is easy to see that system (41) undergoes Hopf bifurcation. In addition, we have $\lambda'(\tau_0) = 0.0161 - 0.0088i$, $C_1(0) = -3.2609 + 1.2090i$. Thus, we have $\mu_2 = 202.5404 > 0$, $\beta_2 = -6.5218 < 0$, and $T_2 = 0.1355 > 0$. From Theorem 2, we can conclude that the Hopf bifurcation is supercritical and the bifurcating periodic solutions are stable, and the period of the periodic solutions increases.

5. Conclusions

This paper is concerned with a delayed SEIQRS model for the transmission of malicious objects in computer network. The theoretical analysis for the delayed model is given and the main results are given in terms of local stability and local Hopf bifurcation. By regarding the delay due to the temporary

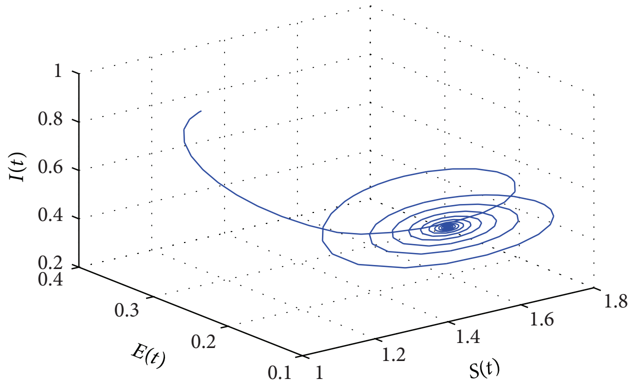

Figure 1: The phase plot of the states S, E, and I for $\tau = 8.0025 < 8.9829 = \tau_0$.

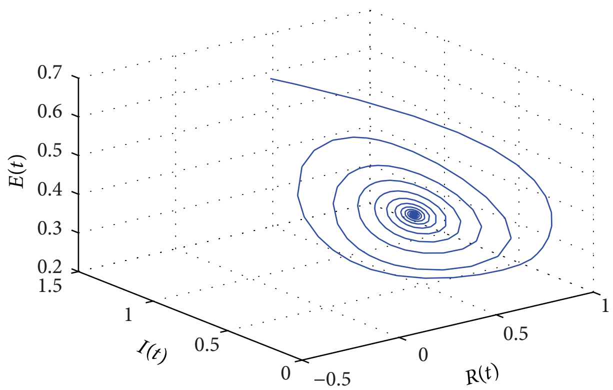

Figure 2: The phase plot of the states E, I, and R for $\tau = 8.0025 < 8.9829 = \tau_0$.

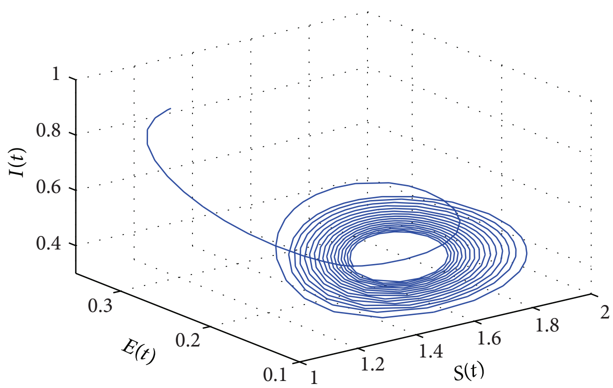

Figure 3: The phase plot of the states S, E, and I for $\tau = 10.025 > 8.9829 = \tau_0$.

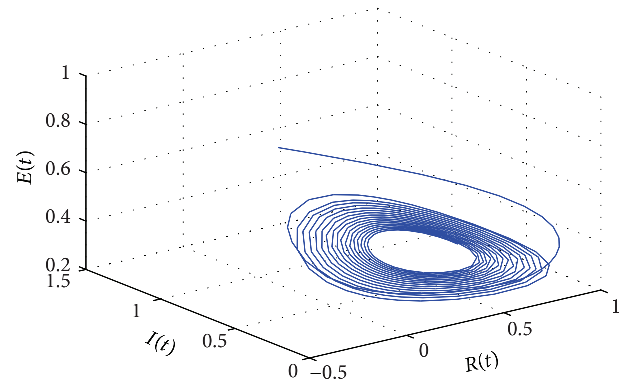

Figure 4: The phase plot of the states E, I, and R for $\tau = 10.025 > 8.9829 = \tau_0$.

immunity period after which a recovered computer may be infected again, we have proven that when the delay passes through the critical value, the model undergoes a Hopf bifurcation. The occurrence of Hopf bifurcation means that the state of virus prevalence changes from a positive equilibrium to a limit cycle, which is not welcomed in networks. Hence, we should control the phenomenon by combining some bifurcation control strategies and other relative features of virus prevalence. Further, the properties of the Hopf bifurcation are studied by using the normal form method and center manifold theorem. Finally, some numerical simulations are presented to clarify our theoretical results.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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