

## Research Article

# An Analysis of the Quality of Repeated Plate Load Tests Using the Harmony Search Algorithm

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We report a characteristic function to determine whether repetitive static plate load test (RSPLT) moduli are within an acceptable quality range, based on a comparison between the initial loading values and reloading moduli. The results of RSPLTs depend on the experience and expertise of the engineer carrying out the test, as well as the loading device, hydraulic jack assembly, and bearing plates. To identify outlier data points, well-tested data were used to develop a characteristic function model using a harmony search algorithm error minimization technique. This measure was applied to determine the reliability of RSPLT data.

## 1. Introduction

Compacting multilayered soil structures is of particular importance. A repetitive static plate load test (RSPLT) can be carried out to obtain moduli of the soil based on the loading and deformation and evaluate whether sufficient compaction has occurred. The moduli of soil can be practically evaluated using a conventional *in situ* test method; however, the deformation-modulus curves obtained from RSPLT evaluations are typically nonlinear, resulting in secant, tangent, unloading, reloading, and cyclic moduli [1–3]. Because of variations in the experience of the engineer who may be carrying out the test, as well as variations in the RSPLT experimental equipment, the results of these tests may lead to spurious differences from measurement to measurement.

For these reasons, we have developed a quality evaluation process for the RSPLT process, with a focus on railway applications. To determine the quality of RSPLT measurements, we used well-tested data from an expert in RSPLT evaluations following the compaction of soil or granular material. Using these RSPLT data, we developed a characteristic function defined by a series of log-sigmoidal functions. The parameters of these functions were determined by minimizing

a predefined error term expressing the difference between measurement of the RSPLT data and the predicted values from the characteristic function, where a harmony search (HS) algorithm was used to minimize the error. The primary goal of this work was to assess whether an RSPLT has been carried out properly.

## 2. RSPLT

The RSPLT apparatus consists of a circular load plate made of steel with a radius of 150 mm, which is placed on the compacted soil to be tested. A thin layer of fine sand may be placed between the two to provide a flat surface, which ensures proper load transmission across the entire plate. Incremental loading steps are applied to the plate and the vertical displacements are recorded. Subsequently, the loading is incrementally reduced until the applied load is zero, while the displacement continues to be monitored. This procedure is then repeated up to a peak load of 0.5 MN/m<sup>2</sup>. An example two-step set of load-deformation curves is shown in Figure 1; these load and deformation data typically exhibit hysteresis.

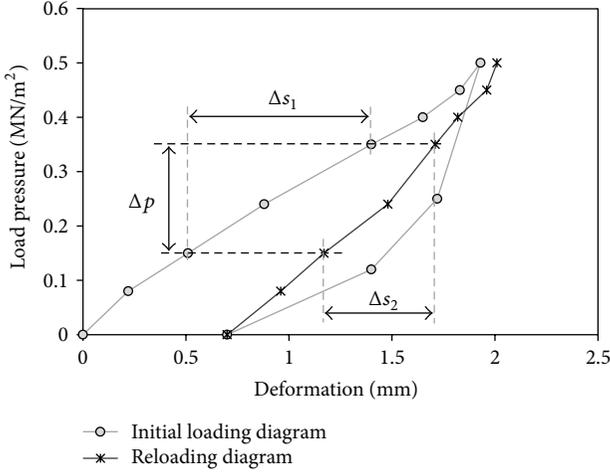


FIGURE 1: Example load-deformation curves for soil.

The moduli can be obtained from the initial loading and reloading cases using the following equation:

$$E = (1 - \mu^2) \frac{\pi D p}{4 \delta}, \quad (1)$$

where  $\mu$  is Poisson's ratio,  $D$  is the diameter of the plate,  $p$  is the applied pressure on the bearing plate, and  $\delta$  is the average settlement of the plate. As shown in Figure 1, two deformation increments,  $\Delta s_1$  and  $\Delta s_2$ , can be defined, which correspond to the first and second loadings, where  $\Delta p$  is the difference in pressure due to these incremental loadings. Therefore, two moduli are obtained: an initial modulus,  $E_{ini}$ , and a reloading modulus,  $E_{re}$ ; that is,

$$E_{ini} = (1 - \mu^2) \frac{\pi D \Delta p}{4 \Delta s_1}, \quad (2a)$$

$$E_{re} = (1 - \mu^2) \frac{\pi D \Delta p}{4 \Delta s_2}. \quad (2b)$$

The stiffness ratio  $E_{re}/E_{ini}$  is used to evaluate the elasticity and plasticity of the soil. A higher stiffness ratio corresponds to more plastic deformation during the initial loading test and is related to a soft initial modulus. Further soil compaction is required if the stiffness ratio is too large. A low stiffness ratio corresponds to more elastic deformation, since similar initial and reloading moduli were obtained; the soil compaction may be considered adequate if the stiffness ratio is below a given threshold. In this study, we focus on developing a characteristic function to evaluate whether or not an RSPLT has been satisfied.

### 3. Harmony Search Algorithm

The HS algorithm is a metaheuristic optimization tool that is inspired by the musical improvisation process and has been widely used to solve optimization problems consisting of discrete and continuous objective functions. Unlike conventional derivative-based optimization algorithms, HS is

based on a random stochastic search with no gradient data. For this reason, the HS algorithm can be easily implemented using a harmony memory consideration rate (HMCR), pitch adjustment rate (PAR), and harmony memory (HM) [4–9].

The HS algorithm may be formulated as an optimization problem with  $N$  variables as follows:

$$\begin{aligned} &\text{Minimize} && f(\mathbf{x}), \\ &\text{subject to} && l_i \leq x_i \leq u_i, \quad i = 1, \dots, N, \end{aligned} \quad (3)$$

where  $f(\mathbf{x})$  is the objective function,  $\mathbf{x} = (x_1, x_2, \dots, x_N)$  is a possible solution vector,  $l_i$  and  $u_i$  are the lower and upper bounds of the search domain, respectively, and  $N$  is the number of decision variables. To implement an HS algorithm, the optimization procedure is as follows.

*Step 1* (initialize the HS parameters). The HS algorithm parameters, which consist of the harmony memory size (HMS), HMCR, PAR, and maximum number of improvisations (MNI), are initialized.

*Step 2* (generate the harmony memory (HM)). The HM is randomly initialized from a uniform distribution in the range between  $l_i$  and  $u_i$ ; that is,

$$\text{HM} = \begin{bmatrix} x_1^1 & x_2^1 & \dots & x_N^1 \\ x_1^2 & x_2^2 & \dots & x_N^2 \\ \vdots & \vdots & \dots & \vdots \\ x_1^{\text{HMS}} & x_2^{\text{HMS}} & \dots & x_N^{\text{HMS}} \end{bmatrix}, \quad (4)$$

where the superscripts and subscripts of the elements represent each random trial row vector and the decision variables used to evaluate the objective function given in (3), respectively.

*Step 3* (improvise new harmonies). The new harmony is denoted by  $\mathbf{x}^{\text{new}} = (x_1^{\text{new}}, x_2^{\text{new}}, \dots, x_N^{\text{new}})$  and is implemented using the HMCR, PAR, and stochastic variables. The HMCR is the probability of selecting an element of the HM matrix; therefore, it is in the range 0-1, and  $(1 - \text{HMCR})$  is the probability of randomly selecting a value within the feasible domain. For example,  $\text{HMCR} = 0.85$  means that the HS algorithm chooses values of the solution vector from the HM with an 85% probability and from stochastic processes with a 15% probability. In addition, the PAR parameter, which is analogous to the pitch adjustment of each instrument during tuning of an ensemble, is used to improve the solutions and escape local optima. If the stochastic variable is less than PAR,  $x_i^{\text{new}}$  is replaced by  $x_i^{\text{new}} \pm \text{bw} \times \text{rand}()$ , where  $\text{bw}$  is the bandwidth and  $\text{rand}()$  is a randomly generated number in the range 0-1. Thus, pitch adjustment occurs only once a value has been chosen by the HM matrix, and  $(1 - \text{PAR})$  is the probability of doing nothing. If  $\text{PAR} = 0.5$ , the algorithm chooses a neighboring value with a probability of 50%.

*Step 4* (update the harmony memory). If the newly generated harmony vector,  $\mathbf{x}^{\text{new}}$ , represents an improvement over the least satisfactory harmony vector in the HM in terms of the objective function, then the least satisfactory vector is replaced by  $\mathbf{x}^{\text{new}}$ .

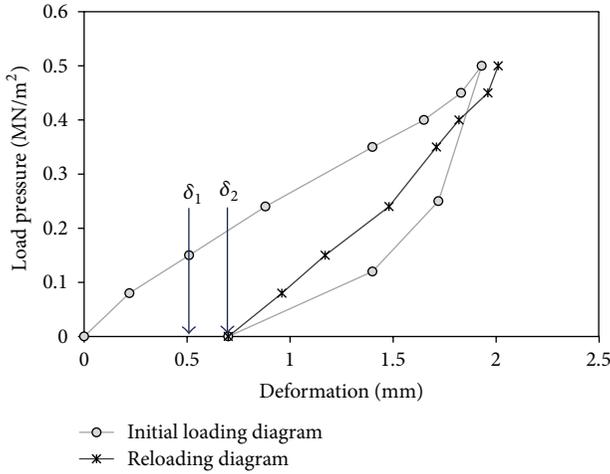


FIGURE 2: Determination of  $\delta_1$  and  $\delta_2$ .

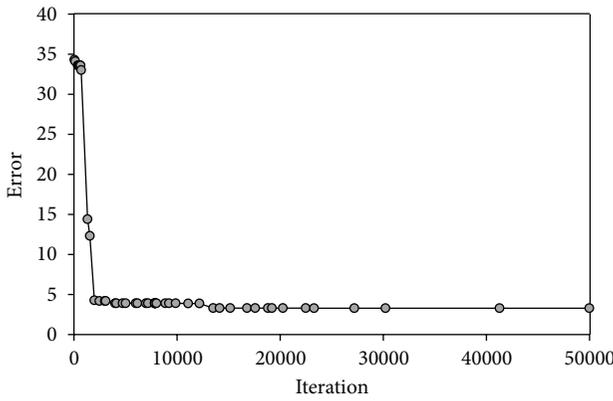


FIGURE 3: Error in each iteration of the HS algorithm.

Step 5 (check the termination criterion). If the convergence criteria are satisfied, the HS algorithm is terminated. Otherwise, go to Step 3.

#### 4. Characteristic Function Model Describing the RSPLT Quality

In order to develop a characteristic function model to determine the quality of the RSPLT, well-tested data obtained from an expert operator of RSPLTs were used. In addition, the RSPLT data were separated into training data and testing data to develop and evaluate the characteristic function model, respectively. Using the training RSPLT data, the reloading modulus, which describes the relationship of the initial modulus and characteristic deformations, can be defined by a series of log-sigmoidal functions as follows:

$$E_{re} = x_1 E_{ini} + \sum_{i=1}^2 \frac{1}{\{1 + x_{i+1} \exp(x_{i+2} \delta_i)\}}, \quad (5)$$

where  $x_1, x_2, \dots, x_5$  are the parameters, and  $\delta_1$  and  $\delta_2$  are the deformations shown in Figure 2. These deformations were

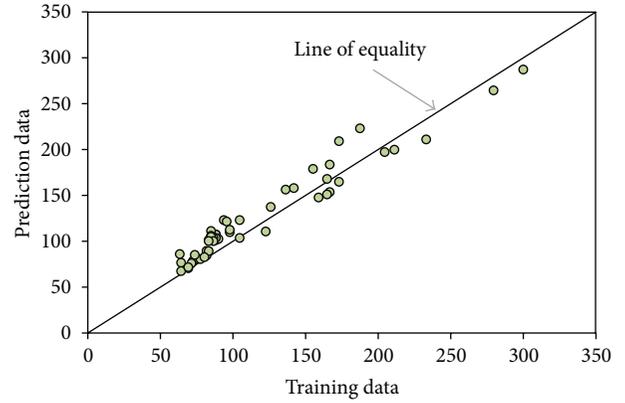


FIGURE 4: Comparison between training data and predicted data.

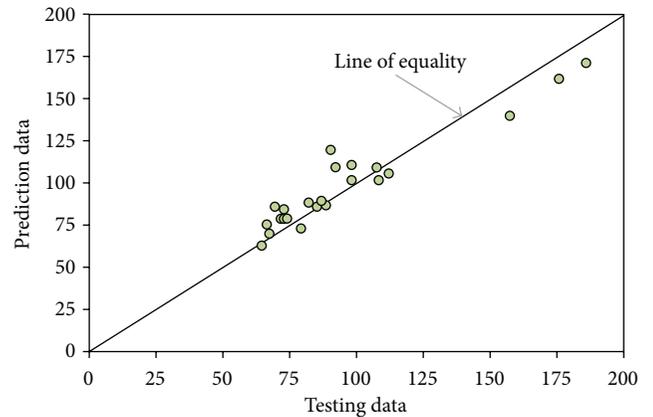


FIGURE 5: Comparison between test data and predicted data.

determined at a load of  $0.15 \text{ MN/m}^2$  and at the end of the initial loading, respectively. The log-sigmoidal function has been widely used in applications including artificial neural networks [10]. The reloading modulus can be defined using (5). If some anomalous RSPLT data were used to calculate  $E_{re}$ , this can then be detected as an outlier.

The parameters of (5) were determined by minimizing a predefined error function expressing the difference between the training data and the predicted results of the characteristic function; that is,

$$\sum_{j=1}^M \left| \text{TD}_j - \left[ x_1 E_{ini} + \sum_{i=1}^2 \frac{1}{\{1 + x_{i+1} \exp(x_{i+2} \delta_i)\}} \right] \right|, \quad (6)$$

where  $|\cdot|$  denotes the absolute value, TD is the training data, and  $M$  is the total number data points in the training data. Two datasets consisting of 53 and 23 data were used for training data and testing data, respectively. The HS algorithm was used to minimize the error of (6).

The HS algorithm parameters were  $\text{HMS} = 10$ ,  $\text{HMCR} = 0.95$ ,  $\text{PAR} = 0.35$ , and  $\text{MNI} = 50\,000$ . Figure 3 shows the procedure used to minimize the HS error. The parameters of the characteristic function model were  $x_1 = 1.96515$ ,  $x_2 = -2$ ,  $x_3 = 0.04542$ ,  $x_4 = 0.001$ , and  $x_5 = 4.631$ .

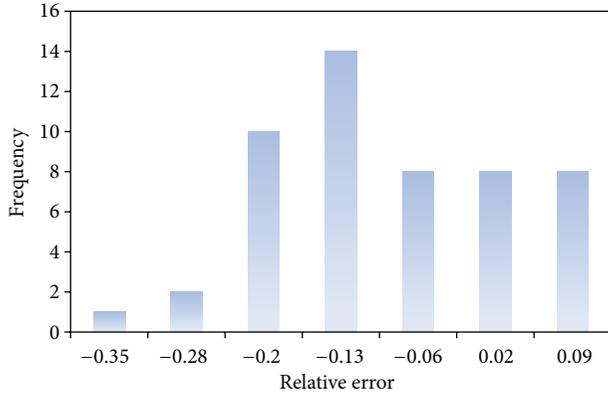


FIGURE 6: Histogram showing the frequency of the relative errors.

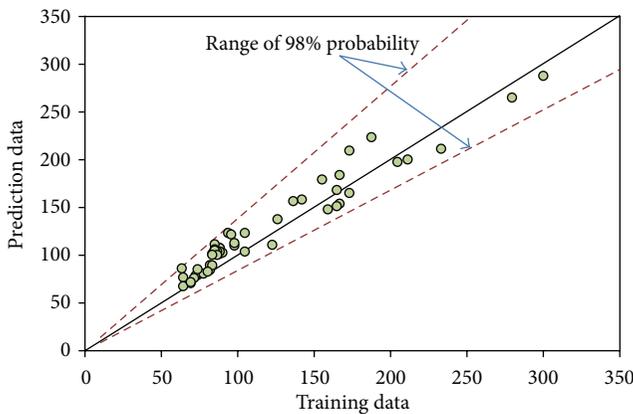


FIGURE 7: Range of 98% probability, based on the characteristic function model.

The training dataset, which was not used for the error minimization, was used to evaluate the characteristic function model. As shown in Figures 4 and 5, the predicted data of the training and testing sets were in good agreement. Based on the characteristic function model, outliers could be determined by introducing a relative error function; that is,

$$RE_j = \frac{TD_j - \left[ x_1 E_{ini} + \sum_{i=1}^2 \left( \frac{1}{1 + x_{i+1} \exp(x_{i+2} \delta_i)} \right) \right]_j}{TD_j}, \quad (7)$$

where  $RE_j$  is the  $j$ th relative error in a sequence of the training data.

The average and standard deviation of the relative error were calculated in order to determine a 98% probability of the characteristic function model, where a uniform distribution of the relative error was assumed. The mean and the standard deviation of the relative error were  $-0.11019$  and  $0.11644$ , respectively. A histogram showing the relative errors is shown in Figure 6. Using the mean and standard deviation, the range of 98% probability can be provided as shown in Figure 7. The outliers (i.e., those points in the remaining 2% probability window) can be identified and used to check whether

the RSPLT setup or data acquisition process is satisfactory. In this way, the quality of the RSPLT may be evaluated.

## 5. Conclusions

The objective of this study was to develop a characteristic function describing the initial data from the first RSPLT hysteresis curves using the HS error minimization technique. Reloading moduli data from the RSPLT were compared with predicted moduli obtained from the characteristic function to assess whether the predicted value was an outlier, that is, whether it was in the 98% probability range. If a reloading modulus was out of this range, it was assumed to be an outlier. In this way, RSPLT data can be monitored, including load transmission across the entire plate, testing surface, hydraulic jack assembly, and bearing plate.

## Conflict of Interests

The authors declare that they have no conflict of interests regarding the publication of this paper.

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