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Research Article

Adaptive Impulsive Observer for Outer Synchronization of Delayed Complex Dynamical Networks with Output Coupling

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The synchronization problem of two delayed complex dynamical networks with output coupling is investigated by using impulsive hybrid control schemes, where only scalar signals need to be transmitted from the drive network to the response one. Based on the Lyapunov stability theorem and the impulsive hybrid control method, some sufficient conditions guaranteeing synchronization of such complex networks are established for both the cases of coupling delay and node delay are considered, respectively. Finally, two illustrative examples with numerical simulations are given to show the feasibility and efficiency of theoretical results.

1. Introduction

A complex network is composed of a number of coupled nodes, where each node is a dynamical system and can only access the local neighboring information. In our daily life, many nature and artificial systems can be described by the complex dynamical networks, such as the World Wide Web, various wireless communication networks, metabolic networks, biological neural works, epidemic network, traffic network, and many others [1–3]. In the past decade, the synchronization problem for complex networks has attracted much attention from various disciplines [4–11]. Many kinds of synchronization have been proposed, such as complete synchronization, lag synchronization, cluster synchronization, and generalized synchronization. For a network which cannot achieve synchronization by itself, one can design some appropriate controllers to force the network to synchronize onto a homogenous trajectory, such as adaptive control [12–14], pinning control [15–18], intermittent control [19, 20], and impulsive control [21–27].

Generally, network synchronization can be classified into inner synchronization and outer synchronization. Briefly, the synchronization inside a network is called inner synchronization, that is, the coherent behavior of all the nodes within a network, while outer synchronization aims at the study of dynamics between coupled networks [28]. Li et al. [29] theoretically and numerically demonstrated the possibility of outer synchronization between two networks having the same topological structures. Subsequently, through the adaptive control or impulsive control, synchronization between two networks is also studied in [30–34], which could deal with more complicated cases, such as different node dynamics, nonidentical topological structures, or time-varying delays. However, these schemes demand all the states of the drive network to be sent to the response network. This is impractical and not economical for real network applications, such as communication networks, where too many links or a too wide bandwidth in communication channels among uses is very unlikely. To resolve this problem, observer-based synchronization schemes have been used where the receiver uses an observer to estimate the states of the drive. This will decrease the number of coupling signals between drive and response. Recently, the state observer approach has been applied to chaos synchronization [35–37] and synchronization of complex networks [38, 39]. Based on the state observer approach, Jiang et al. [38] formulated a complex network model and derived some criteria to investigate the local synchronization in the network. In [39], a new scheme for outer synchronization has been proposed based on the pinning-state-observer approach, where only part of nodes are controlled in response networks and only scalar signals need to be transmitted from the drive network to the response one. However, they did not take into account the time delay. Ignoring them may lead to design flaws and incorrect analysis conclusions. Hence, time delays in couplings or in dynamical nodes have received considerable attention. On the other hand, in most of the impulsive synchronization methods [21–27], response system needs to have access to all of the drive system states. This means that all the drive system states should be transmitted to the response system which will decrease the security and capacity of the communication channel and the complexity of the communication system will be increased. To the best of our knowledge, synchronization between two delayed complex dynamical networks with output coupling has received very little research attention despite its significance in practice; therefore, the main purpose of this paper is to shorten such a gap. The main novelty of this paper can be summarized as follows. First, the adaptive impulsive observer problem for outer synchronization of delayed complex dynamical networks with output coupling is considered in this paper. Second, only by using the output of the drive network at discrete instant times, the response network would be able to estimate states of the drive network.

Motivated by the above discussions, in this paper, we are going to design an outer synchronization scheme where receiver only needs the output of the drive network. Applying the effective, robust, and low-cost impulsive hybrid control method, we study outer synchronization between the drive network and the response network with delay. The cases of coupling delay and node delay are considered, respectively. Sufficient conditions of synchronization of two coupled networks are derived based on the comparison theorem of an impulsive differential system. The proposed scheme herein will be very useful for practical engineering applications, such as network monitoring and network communications.

The rest of this paper is organized as follows. In Section 2, model description and some necessary preliminaries are given. The impulsive state observer is proposed, and outer synchronization criteria are derived in Section 3. In Section 4, numerical results are given to validate the theoretical analysis. Finally, some concluding remarks are stated in Section 5.

2. Model Description and Preliminaries

This section provides some mathematical preliminaries to derive the main results of this paper.

2.1. Notations. The notations in this paper are quite standard. R^n and $R^{n\times n}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n \times n$ real matrices. The superscript T denotes matrix or vector transposition. I_n is the $n \times n$ identity matrix. $\lambda_{\max}(A)$ means the maximum eigenvalue of matrix A. The Euclidean norm in R^n is defined as $\|\cdot\|$, for vector $x \in R^n$, $\|x\| = x^T x$, for matrix $A \in R^{n\times n}$, $\|A\| = \sqrt{\lambda_{\max}(A^T A)}$. $\otimes$ is the Kronecker product of two matrices. The matrices, if their dimensions are not explicitly stated, are assumed to have appropriate dimensions.

2.2. Comparison Theorem. The following comparison theorem is important to study the impulsive control of delayed complex dynamical networks with output coupling.

Lemma 1 (see [40]). *Let $0 \le \tau(t), \tau_1(t), \tau_2(t), \dots, \tau_m(t) \le \tilde{\tau}$, $\tilde{\tau} = \max(\tau, \tau_1, \tau_2, \dots, \tau_m)$, $F(t, u, \overline{u}_1, \dots, \overline{u}_m) : R^+ \times \overbrace{R \times \cdots \times R}^{m+1} \to R$ be nondecreasing in $\overline{u}_i$ for each fixed $(t, u, \overline{u}_1, \dots, \overline{u}_{i-1}, \overline{u}_{i+1}, \overline{u}_m)$, $i = 1, 2, \dots, m$, and $I_k(u) : R \to R$ be nondecreasing in u. Suppose that $u(t)$ and $v(t)$ satisfy*

$$\begin{gathered} D^+ u(t) \le F(t, u(t), u_1(t - \tau_1(t)), \dots, u_m(t - \tau_m(t))), \\ t \ge 0, \\ u(t_k) \le I_k u(t_k^-), \quad k \in N, \\ D^+ v(t) > F(t, v(t), v_1(t - \tau_1(t)), \dots, v_m(t - \tau_m(t))), \\ t \ge 0, \\ v(t_k) \ge I_k v(t_k^-), \quad k \in N, \end{gathered} \tag{1}$$

where the right and upper Dini derivative $D^+ u(t)$ is defined as $D^+ u(t) = \overline{\lim}_{h\to 0^+}(u(t+h) - u(t))/h$, where $h \to 0^+$ means that h approaches zero from the right-hand side. Then $u(t) \le v(t)$ for $-\tilde{\tau} \le t \le 0$ implies that $u(t) \le v(t)$ for $t \ge 0$.

2.3. Model Description. The drive network and the response network with output coupling delay can generally be described as follows:

$$\begin{gathered} \dot{x}_i(t) = f(t, x_i(t)) + \varepsilon \sum_{j=1}^{N} c_{ij} \Gamma y_j(t - \tau(t)), \\ y_j(t) = H x_j(t), \end{gathered} \tag{2}$$

$$\begin{gathered} \dot{\hat{x}}_i(t) = f(t, \hat{x}_i(t)) + \varepsilon \sum_{j=1}^{N} c_{ij} \Gamma \hat{y}_j(t - \tau(t)) + u_i(t), \\ \hat{y}_j(t) = H \hat{x}_j(t), \end{gathered} \tag{3}$$

where $u_i(t)$ is control input, $1 \le i \le N$, $x_i(t) = (x_{i1}(t), x_{i2}(t), \dots, x_{in}(t))^T \in R^n$ is the state vector of the ith node, $y_i(t) \in R^p$ is the output variable of the ith node, $1 \le p \le n$, $\hat{x}_i(t) \in R^n$ is the estimated state vector, $\hat{y}_i(t) \in R^p$ is the estimated output vector, $f : R^+ \times R^n \to R^n$ is a smooth nonlinear function, $H \in R^{p\times n}$ is the output matrix of each node, $\Gamma \in R^{n\times p}$ is the inner coupling matrix, and the time-varying delay $\tau(t)$ is bounded by a known constant; that is $0 \le \tau(t) \le \tau$, and $C = (c_{ij})_{N\times N}$ is the delayed outer coupling configuration matrix with zero-sum rows, in which $c_{ij} \ne 0$ if there is a link from node i to node j $(i \ne j)$, and $c_{ij} = 0$ otherwise.

On the other hand, two output coupled networks with dynamical nodes delay are described by

$$\dot{x}_i(t) = f(t, x_i(t), x_i(t-\tau(t))) + \varepsilon \sum_{j=1}^{N} c_{ij}\Gamma y_j(t), \quad (4)$$
$$y_i(t) = Hx_i(t),$$

$$\dot{\hat{x}}_i(t) = f(t, \hat{x}_i(t), \hat{x}_i(t-\tau(t))) + \varepsilon \sum_{j=1}^{N} c_{ij}\Gamma \hat{y}_j(t) + u_i(t),$$
$$\hat{y}_i(t) = H\hat{x}_i(t). \quad (5)$$

Assumption 2. Assuming that there is a positive-definite diagonal matrix $P = \operatorname{diag}(p_1, p_2, \ldots, p_n)$, such that f satisfies the following inequality:

$$\begin{aligned}(x-y)^T P(f(t,x,x(t-\tau)) - f(t,y,y(t-\tau))) &\\ \leq K(x-y)^T(x-y) &\\ + L(x(t-\tau) - y(t-\tau))^T (x(t-\tau) - y(t-\tau)),&\end{aligned} \quad (6)$$

for $K > 0$, $L > 0$, all $x, y \in R^n$ and $t > 0$.

Remark 3. Assumption 2 gives some requirements for the dynamics of isolated nodes in network. If the function describing each node satisfies uniform Lipschitz condition with respect to the time, that is, $\|f(t,x,x(t-\tau)) - f(t,y,y(t-\tau))\| \leq K_1\|x-y\| + L_1\|x(t-\tau) - y(t-\tau)\|$, one can choose $K_1 = K + \varepsilon L/2$, $L_1 = L/(2\varepsilon)$, and $P = I_n$ to satisfy Assumption 2, where ε is a positive constant. It is easy to verify that many chaotic systems with delays or without delays satisfy Assumption 2, for example, Chua's oscillator, Rössler system, Lorenz system, Chen system, and Lü system as well as the delayed Lorenz system, delayed Hopfield neural networks, and delayed cellular neural networks.

Definition 4. Two networks are said to achieve globally exponential synchronization if $\|\hat{x}_i(t) - x_i(t)\| \leq M_0 e^{-\theta t}$, $M_0 > 0$, $\theta > 0$, and $t \geq 0$.

3. Adaptive Impulsive Criteria for Network with Output Coupling

In this section, we discuss outer synchronization of the drive-response dynamical network with output coupling via the impulsive hybrid controller under two cases: with node delay and with coupling delay. In order to achieve synchronization of two networks, the impulsive hybrid controller, for the i node, is designed as

$$u_i(t) = u_{1i} + u_{2i}, \quad (7)$$

where $i = 1, 2, \ldots, N$, u_{1i} is the nonlinear feedback controller, the impulsive control $u_{2i} = \sum_{k=1}^{\infty} B_{ik}(\hat{y}_i - y_i)\delta(t - t_k)$, and the impulsive instant sequence $\{t_k\}_{k=1}^{\infty}$ satisfies $0 = t_0 < t_1 < \cdots < t_k < \cdots$ and $\lim_{k\to\infty} t_k = +\infty$. $B_{ik} \in R^{n\times p}$ is the states impulses gain matrix and $\delta(\cdot)$ is the Dirac impulsive function; that is,

$$\delta(t - t_k) = \begin{cases} 1, & t = t_k, \\ 0, & t \neq t_k. \end{cases} \quad (8)$$

3.1. Coupling Delay. Two networks with coupling delays and impulsive control can be equivalently expressed as follows:

$$\begin{aligned}
\dot{x}_i(t) &= f(t, x_i(t)) + \varepsilon \sum_{j=1}^{N} c_{ij}\Gamma y_j(t-\tau(t)),\\
y_j(t) &= Hx_i(t),\\
\dot{\hat{x}}_i(t) &= f(t, \hat{x}_i(t)) + \varepsilon \sum_{j=1}^{N} c_{ij}\Gamma \hat{y}_j(t-\tau(t)) + u_{1i},\\
\hat{y}_j(t) &= H\hat{x}_i(t), \quad t = t_k,\ k = 1, 2, \ldots,\\
\Delta\hat{x}_i &= \hat{x}_i(t_k^+) - \hat{x}_i(t_k^-) = B_{ik}(\hat{y}_i - y_i), \quad t = t_k,
\end{aligned} \quad (9)$$

where $x_i(t_k^+) = \lim_{t\to t_k^+} x_i(t)$ and $x_i(t_k^-) = \lim_{t\to t_k^-} x_i(t)$. Without loss of generality, we assume that $\lim_{t\to t_k^-} x_i(t) = x_i(t_k)$, which means that the solution of (9) is left continuous at time t_k. $B_{ik} \in R^{n\times p}$ is impulsive control gain.

Let $e_i(t) = \hat{x}_i(t) - x_i(t)$; then the synchronization error of two networks can be written as

$$\begin{aligned}
\dot{e}_i(t) &= f(t, \hat{x}_i(t)) - f(t, x_i(t))\\
&\quad + \varepsilon \sum_{j=1}^{N} c_{ij}\Gamma He_j(t-\tau(t)) + u_{1i},\\
e_{yi}(t) &= He_i(t), \quad t \neq t_k,\\
\Delta e_i &= B_{ik}He_i, \quad t = t_k,\ k = 1, 2, \ldots.
\end{aligned} \quad (10)$$

Here, the adaptive controller u_{1i} and updating laws are designed as follows:

$$\begin{aligned}
u_{1i} &= -d_i\Gamma(\hat{y}_i - y_i),\\
\dot{d}_i &= k_i e_i^T(t) e_i(t), \quad k_i > 0.
\end{aligned} \quad (11)$$

Then, we have the following results.

Theorem 5. *Let $0 < \rho = \sup\{t_k - t_{k-1}\} < \infty$. Suppose Assumption 2 holds, the drive network (2) and the response network (3) with impulsive controller (7) will achieve globally exponential synchronization in the following sense:*

$$\|e(t)\| \leq Me^{-(\lambda t/2)} \sup_{-\tau\leq s\leq 0} \|\phi(s)\|, \quad t \geq 0, \quad (12)$$

if

$$\max_k \|I_n + B_{ik}H\|^2 = \rho_k,$$
$$\rho_k \|P\| \lambda_{\max}\left(P^{-1}\right) \le \eta^2, \quad 0 < \eta < 1, \tag{13}$$

$$\frac{\ln \eta}{\rho} + \lambda_{\max}\left(P^{-1}\right)\left[K - \lambda_{\min}\left(D^* \otimes P\Gamma H\right) + \frac{\varepsilon \|(C \otimes P\Gamma H)\|}{\eta}\right] < 0, \tag{14}$$

where $M = (1/\eta)\sqrt{(\lambda_{\max}(P))/(\lambda_{\min}(P))}$, $D^* = d^* I_N$, *and* d^* *is the minimum value of the initial feedback strength* d_{i0}. $\lambda > 0$ *is the solution of* $\lambda - a + be^{\lambda\tau} = 0$ *within which*

$$a = -\frac{2\ln\eta}{\rho} - \lambda_{\max}\left(P^{-1}\right) \times \left[2K - 2\lambda_{\min}\left(D^* \otimes P\Gamma H\right) + \frac{\varepsilon \|(C \otimes P\Gamma H)\|}{\eta}\right], \tag{15}$$
$$b = \lambda_{\max}\left(P^{-1}\right)\frac{\varepsilon \|(C \otimes P\Gamma H)\|}{\eta}.$$

Proof. In order to obtain the criteria of synchronization for the drive-response networks (9), we translate this problem to research the stability of the error system (10) around the zero solution. Consider the following Lyapunov candidate function:

$$V(t) = \sum_{i=1}^{N} e_i^T(t) P e_i(t). \tag{16}$$

When $t = t_k$, $k \in N$, one has

$$\begin{aligned} V\left(t_k^+\right) &= \sum_{i=1}^{N} e_i^T\left(t_k^+\right) P e_i\left(t_k^+\right) \\ &= \sum_{i=1}^{N} e_i^T\left(t_k\right)\left(I_n + B_{ik}H\right)^T P\left(I_n + B_{ik}H\right) e_i\left(t_k\right) \\ &\le \max_k \|I_n + B_{ik}H\|^2 \|P\| \lambda_{\max}\left(P^{-1}\right)\sum_{i=1}^{N} e_i^T\left(t_k\right) P e_i\left(t_k\right) \\ &\le \eta^2 \sum_{i=1}^{N} e_i^T\left(t_k\right) P e_i\left(t_k\right) = \eta^2 V\left(t_k\right), \quad k = 1, 2, \ldots. \end{aligned} \tag{17}$$

For $t \in [t_{k-1}, t_k)$, $k \in N$, differentiating $V(t)$ along the solution of (10), one obtains

$$\begin{aligned} \dot V(t) &= 2\sum_{i=1}^{N} e_i^T(t) P \Bigg[f\left(t, \hat x_i(t)\right) - f\left(t, x_i(t)\right) + \varepsilon \sum_{j=1}^{N} c_{ij}\Gamma H e_j(t - \tau(t)) - d_i \Gamma H e_i(t)\Bigg] \\ &\le 2K\sum_{i=1}^{N} e_i^T(t) e_i(t) - 2d^* \sum_{i=1}^{N} e_i^T(t) P\Gamma H e_i(t) \\ &\quad + 2\varepsilon e^T(t)(C \otimes P\Gamma H) e(t - \tau(t)). \end{aligned} \tag{18}$$

It is clear that

$$\begin{aligned} &2\varepsilon e^T(t)(C \otimes P\Gamma H) e(t - \tau(t)) \\ &\quad\le \frac{\varepsilon}{\eta}\|C \otimes P\Gamma H\| e^T(t) e(t) \\ &\quad\quad + \varepsilon\eta \|C \otimes P\Gamma H\| e^T(t - \tau(t)) e(t - \tau(t)), \end{aligned} \tag{19}$$

where $e(t) = (e_1^T(t), e_2^T(t), \ldots, e_N^T(t))^T$.

Then, we get

$$\begin{aligned} \dot V(t) &\le \left[2K + \frac{\varepsilon}{\eta}\|C \otimes P\Gamma H\|\right]\sum_{i=1}^{N} e_i^T(t) e_i(t) \\ &\quad - 2d^* \sum_{i=1}^{N} e_i^T(t) P\Gamma H e_i(t) \\ &\quad + \eta\varepsilon \|C \otimes P\Gamma H\| \sum_{i=1}^{N} e_i^T(t - \tau(t)) e_i(t - \tau(t)) \\ &\le \lambda_{\max}\left(P^{-1}\right)\left[2K - 2\lambda_{\min}\left(D^* \otimes P\Gamma H\right) + \frac{\varepsilon}{\eta}\|C \otimes P\Gamma H\|\right] V(t) \\ &\quad + \lambda_{\max}\left(P^{-1}\right)\varepsilon\eta \|C \otimes P\Gamma H\| V(t - \tau(t)). \end{aligned} \tag{20}$$

For any $\varepsilon > 0$, let $\upsilon(t)$ be a unique solution of the following impulsive delayed dynamical system:

$$\begin{aligned} \dot\upsilon(t) &= \lambda_{\max}\left(P^{-1}\right)\left[2K - 2\lambda_{\min}\left(D^* \otimes P\Gamma H\right) + \frac{\varepsilon}{\eta}\|C \otimes P\Gamma H\|\right]\upsilon(t) \\ &\quad + \lambda_{\max}\left(P^{-1}\right)\varepsilon\eta\|C \otimes P\Gamma H\|\upsilon(t - \tau(t)) + \varsigma, \quad t \ne t_k, \\ \upsilon\left(t_k\right) &= \eta^2 \upsilon\left(t_k^-\right), \quad k \in N, \\ \upsilon(s) &= \lambda_{\max}(P)\|\phi(s)\|^2, \quad -\tau \le s \le 0, \end{aligned} \tag{21}$$

where $\phi(s) = (\phi_1^T(s), \phi_2^T(s), \ldots, \phi_N^T(s))^T$.

Since $V(s) \le \lambda_{\max}(P)\|\phi(s)\|^2$ for $-\tilde\tau \le s \le 0$, it follows from (20)-(21) and Lemma 1 that

$$0 \le V(t) \le \upsilon(t), \quad \text{for } t \ge 0. \tag{22}$$

By the formula for the variation of parameters, one obtains $v(t)$ from (21) that

$$\begin{aligned} v(t) &= \omega(t,0)\,v(0) \\ &\quad + \int_0^t \omega(t,s)\Big(\lambda_{\max}\left(P^{-1}\right)\varepsilon\eta\,\|C\otimes P\Gamma H\| \\ &\qquad \times v(s-\tau(s)) + \varsigma\Big)\,ds, \end{aligned} \tag{23}$$

where $\omega(t,s)$, $0 \le s \le t$, is Cauchy matrix of the linear system

$$\begin{aligned} \dot{\varsigma}(t) &= \lambda_{\max}\left(P^{-1}\right)\Big[2K - 2\lambda_{\min}\left(D^*\otimes P\Gamma H\right) \\ &\quad + \frac{\varepsilon}{\eta}\|C\otimes P\Gamma H\|\Big]\varsigma(t), \quad t \ne t_k, \\ \varsigma\left(t_k^+\right) &= \eta^2\varsigma\left(t_k^-\right), \quad k \in N. \end{aligned} \tag{24}$$

According to the representation of the Cauchy matrix [41], we get the following estimation of $\omega(t,s)$ since $0 < \eta < 1$ and $t_k - t_{k-1} \le \rho$,

$$\begin{aligned} \omega(t,s) &= e^{\lambda_{\max}(P^{-1})[2K-2\lambda_{\min}(D^*\otimes P\Gamma H)+(\varepsilon/\eta)\|C\otimes P\Gamma H\|](t-s)}\prod_{s<t_k\le t}\eta^2 \\ &\le e^{(-a-(2\ln\eta/\rho))(t-s)}\eta^{2(((t-s)/\rho)-1)} \\ &= \eta^{-2}e^{-a(t-s)}, \quad 0 \le s \le t. \end{aligned} \tag{25}$$

For simplicity, let $\sigma = \eta^{-2}\lambda_{\max}(P)\sup_{-\tau\le s\le 0}\{\|\phi(s)\|^2\}$; from (23) and (25), one has

$$\begin{aligned} v(t) &\le \eta^{-2}e^{-at}v(0) \\ &\quad + \int_0^t e^{-a(t-s)}\eta^{-2}\left[\lambda_{\max}\left(P^{-1}\right)\varepsilon\eta\,\|C\otimes P\Gamma H\| \right.\\ &\qquad \left. \times v(s-\tau(s)) + \varsigma\right]ds \\ &\le \sigma e^{-at} + \int_0^t e^{-a(t-s)}\left[bv(s-\tau(s)) + \frac{\varsigma}{\eta^2}\right]ds. \end{aligned} \tag{26}$$

Define $H(\lambda) = \lambda - a + be^{\lambda\tau}$; from (14), one has $a-b>0$, and also $H(0) < 0$, $H(+\infty) > 0$, and $H'(\lambda) = 1 + b\tau e^{\lambda\tau} > 0$. Therefore, there exists a unique solution $\lambda > 0$ such that $H(\lambda) = 0$.

On the other hand, since ε, λ, $a-b>0$, and $(1/\eta) > 1$, one has

$$\begin{aligned} v(t) \le \eta^{-2}\sup_{-\tau\le s\le 0} v(s) &< \sigma e^{-\lambda t} + \frac{\varsigma}{\eta^2(a-b)}, \\ & -\tau \le t \le 0. \end{aligned} \tag{27}$$

In the following, we will prove that the following inequality holds

$$v(t) < \sigma e^{-\lambda t} + \frac{\varsigma}{\eta^2(a-b)}, \quad t \ge 0. \tag{28}$$

If (26) is not true, that is, it is assumed that there exists a $t^* > 0$ such that

$$v\left(t^*\right) \ge \sigma e^{-\lambda t^*} + \frac{\varsigma}{\eta^2(a-b)}, \tag{29}$$

$$v(t) < \sigma e^{-\lambda t} + \frac{\varsigma}{\eta^2(a-b)}, \quad t < t^*. \tag{30}$$

One has from (24) and (28) that

$$\begin{aligned} v\left(t^*\right) &\le \sigma e^{-at^*} + \int_0^{t^*} e^{-a(t^*-s)}\left[bv(s-\tau(s)) + \frac{\varsigma}{\eta^2}\right]ds \\ &\le e^{-at^*}\left[\sigma + \frac{\varepsilon}{\eta^2(a-b)} + b\sigma e^{\lambda\tau}\int_0^{t^*} e^{-(\lambda-a)s}ds \right.\\ &\qquad \left. + \int_0^{t^*} e^{as}\frac{a\varsigma}{\eta^2(a-b)}ds\right] \\ &= \sigma e^{-\lambda t^*} + \frac{\varsigma}{\eta^2(a-b)}\left[1 - e^{-at^*}\right], \\ &< \sigma e^{-\lambda t^*} + \frac{\varsigma}{\eta^2(a-b)}, \end{aligned} \tag{31}$$

which contradicts with (29), and so (28) holds. Letting $\varsigma \to 0$, we get

$$V(t) \le v(t) \le \sigma e^{-\lambda t}. \tag{32}$$

Moreover,

$$V(t) \ge \lambda_{\min}(P)\,\|e(t)\|^2, \quad t \ge 0. \tag{33}$$

From (32) and (33), we have

$$\|e(t)\| \le \frac{1}{\eta}\sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}}\sup_{-\tau\le s\le 0}\left\{\|\phi(s)\|^2\right\}e^{-(\lambda/2)t}. \tag{34}$$

When $t \to \infty$, the error system (10) is global exponential asymptotically stable, which implies that the drive network and the response network achieve synchronization by using impulsive hybrid controller. This completes the proof of Theorem 5. □

Similarly, we can easily obtain the following result for the case $\eta \ge 1$.

Theorem 6. *Let $\rho = \inf\{t_k - t_{k-1}\} > 0$. Suppose Assumption 2 holds; if there exists a constant $\eta \ge 1$ such that*

$$\max_k\left\|I_n + B_{ik}H\right\|^2 = \rho_k, \qquad \rho_k\,\|P\|\,\lambda_{\max}\left(P^{-1}\right) \le \eta^2, \tag{35}$$

$$\begin{aligned} \frac{\ln\eta}{\rho} &+ \lambda_{\max}\left(P^{-1}\right)\left[K - \lambda_{\min}\left(D^*\otimes P\Gamma H\right)\right. \\ &\qquad \left. + \varepsilon\eta\,\|(C\otimes P\Gamma H)\|\right] < 0, \end{aligned} \tag{36}$$

then the drive network (2) *and the response network* (3) *with impulsive controller* (7) *will achieve globally exponential synchronization in the following sense:*

$$\|e(t)\| \leq Me^{-(\lambda t/2)} \sup_{-\tau \leq s \leq 0} \|\phi(s)\|, \quad t \geq 0, \tag{37}$$

where $M = (1/\eta)\sqrt{(\lambda_{\max}(P))/(\lambda_{\min}(P))}$, $\lambda > 0$ *is the solution of* $\lambda - a + be^{\lambda\tau} = 0$ *within which*

$$\begin{aligned} a &= -\frac{2\ln\eta}{\rho} - \lambda_{\max}\left(P^{-1}\right)[2K - 2\lambda_{\min}\left(D^{*} \otimes P\Gamma H\right) \\ &\quad + \varepsilon\eta \|(C \otimes P\Gamma H)\|], \\ b &= \lambda_{\max}\left(P^{-1}\right)\varepsilon\eta \|(C \otimes P\Gamma H)\|. \end{aligned} \tag{38}$$

Proof. Take the same Lyapunov candidate function $V(t)$ as in Theorem 5. By the proof of Theorem 5, we can get

$$\begin{aligned} \dot{V}(t) &\leq \lambda_{\max}\left(P^{-1}\right)[2K - 2\lambda_{\min}\left(D^{*} \otimes P\Gamma H\right) \\ &\quad + \varepsilon\eta \|C \otimes P\Gamma H\|] V(t) \\ &\quad + \lambda_{\max}\left(P^{-1}\right)\frac{\varepsilon}{\eta}\|C \otimes P\Gamma H\| V(t - \tau(t)). \end{aligned} \tag{39}$$

According to the representation of the Cauchy matrix, we get the following estimation of $\omega(t,s)$, since $\eta > 1$ and $t_k - t_{k-1} \geq \rho$,

$$\begin{aligned} \omega(t,s) &= e^{\lambda_{\max}(P^{-1})[2K - 2\lambda_{\min}(D^{*}\otimes P\Gamma H) + \varepsilon\eta\|C\otimes P\Gamma H\|](t-s)} \prod_{s<t_k\leq t} \eta^{2} \\ &\leq e^{(-a-(2\ln\eta/\rho))(t-s)}\eta^{2(((t-s)/\rho)+1)} \\ &= \eta^{2}e^{-a(t-s)}, \quad 0 \leq s \leq t. \end{aligned} \tag{40}$$

Let $\sigma = \eta^{2}\lambda_{\max}(P)\sup_{-\tau\leq s\leq 0}\{\|\phi(s)\|^{2}\}$, and accordingly

$$\begin{aligned} v(t) &\leq \eta^{2}e^{-at}v(0) \\ &\quad + \int_{0}^{t} e^{-a(t-s)}\eta^{2}\left[\lambda_{\max}\left(P^{-1}\right)\frac{\varepsilon}{\eta}\|C \otimes P\Gamma H\| \right. \\ &\quad \left. \times v(s - \tau(s)) + \varsigma\right] ds \\ &\leq \sigma e^{-at} + \int_{0}^{t} e^{-a(t-s)}\left[bv(s - \tau(s)) + \eta^{2}\varsigma\right] ds. \end{aligned} \tag{41}$$

Let $H(\lambda) = \lambda - a + be^{\lambda\tau}$; from (36), one has $a - b > 0$, and also $H(0) < 0$, $H(+\infty) > 0$, and $H'(\lambda) = 1 + b\tau e^{\lambda\tau} > 0$. Therefore, there exists a unique solution $\lambda > 0$.

Since ε, λ, $a - b > 0$, and $\eta \geq 1$, one has

$$\begin{aligned} v(t) &\leq \eta^{2}\sup_{-\tau\leq s\leq 0} v(s) \\ &< \sigma e^{-\lambda t} + \frac{\varsigma\eta^{2}}{(a-b)}, \quad -\tau \leq t \leq 0. \end{aligned} \tag{42}$$

In the following, we will prove that the following inequality holds

$$v(t) < \sigma e^{-\lambda t} + \frac{\varsigma\eta^{2}}{(a-b)}, \quad t \geq 0. \tag{43}$$

If (28) is not true, that is, it is assumed that there exists a $t^{*} > 0$ such that

$$v(t^{*}) \geq \sigma e^{-\lambda t^{*}} + \frac{\varsigma\eta^{2}}{(a-b)}, \tag{44}$$

$$v(t) < \sigma e^{-\lambda t} + \frac{\varsigma\eta^{2}}{(a-b)}, \quad t < t^{*}. \tag{45}$$

One has from (41) and (45) that

$$\begin{aligned} v(t^{*}) &\leq \sigma e^{-at^{*}} + \int_{0}^{t^{*}} e^{-a(t^{*}-s)}\left[bv(s-\tau(s)) + \eta^{2}\varsigma\right] ds \\ &< e^{-at^{*}}\left(\sigma + \frac{\varsigma\eta^{2}}{(a-b)}\right) \\ &\quad + e^{-at^{*}}\int_{0}^{t^{*}} e^{as}\left[b\left(\sigma e^{-\lambda(s-\tau(s))} + \frac{\varsigma\eta^{2}}{(a-b)}\right) + \varsigma\eta^{2}\right] ds \\ &\leq e^{-at^{*}}\left[\sigma + b\sigma e^{-\lambda\tau}\int_{0}^{t^{*}} e^{(a-\lambda)s}ds + \frac{a\varsigma\eta^{2}}{(a-b)}\int_{0}^{t^{*}} e^{as}ds\right] \\ &= \sigma e^{-\lambda t^{*}} + \frac{\varsigma\eta^{2}}{(a-b)}\left[1 - e^{-at^{*}}\right], \\ &< \sigma e^{-\lambda t^{*}} + \frac{\varsigma\eta^{2}}{(a-b)}, \end{aligned} \tag{46}$$

which contradicts with (44), and so (43) holds. Letting $\varsigma \to 0$, we have

$$V(t) \leq v(t) \leq \sigma e^{-\lambda t}. \tag{47}$$

This completes the proof of Theorem 5. □

For most complex dynamical networks with nodes coupled by state variables [4–34], we can also obtain outer synchronization between two such networks using the proposed scheme as follows:

$$\begin{aligned} \dot{x}_i(t) &= f(t, x_i(t)) + \varepsilon\sum_{j=1}^{N} c_{ij}\Gamma x_j(t - \tau(t)), \\ y_j(t) &= Hx_i(t), \\ \dot{\hat{x}}_i(t) &= f(t, \hat{x}_i(t)) + \varepsilon\sum_{j=1}^{N} c_{ij}\Gamma\hat{x}_j(t - \tau(t)) + u_{1i}, \\ \hat{y}_j(t) &= H\hat{x}_i(t), \quad t = t_k, \ k = 1, 2, \ldots, \\ \Delta\hat{x}_i &= \hat{x}_i(t_k^{+}) - \hat{x}_i(t_k^{-}) = B_{ik}(\hat{y}_i - y_i), \quad t = t_k. \end{aligned} \tag{48}$$

Then, the error system is derived as follows:

$$\dot{e}_i(t) = f(t, \hat{x}_i(t)) - f(t, x_i(t)) + \varepsilon \sum_{j=1}^{N} c_{ij} \Gamma e_j(t - \tau(t)) + u_{1i},$$
$$e_{yi}(t) = He_i(t), \quad t \neq t_k,$$
$$\Delta e_i = B_{ik} He_i, \quad t = t_k, \ k = 1, 2, \ldots. \tag{49}$$

Then, we have the following corollaries.

Corollary 7. *Let* $0 < \rho = \sup\{t_k - t_{k-1}\} < \infty$. *Suppose Assumption 2 holds; the drive-response network* (48) *with impulsive controller* (7) *will achieve globally exponential synchronization if*

$$\max_k \|I_n + B_{ik} H\|^2 = \rho_k,$$
$$\rho_k \|P\| \lambda_{\max}(P^{-1}) \leq \eta^2, \quad 0 < \eta < 1,$$
$$\frac{\ln \eta}{\rho} + \lambda_{\max}(P^{-1}) \left[K - \lambda_{\min}(D^* \otimes P\Gamma) + \frac{\varepsilon \|(C \otimes P\Gamma)\|}{\eta} \right] < 0. \tag{50}$$

Corollary 8. *Let* $\rho = \inf\{t_k - t_{k-1}\} > 0$. *Suppose Assumption 2 holds, the drive-response network* (48) *with impulsive controller* (7) *will achieve globally exponential synchronization if*

$$\max_k \|I_n + B_{ik} H\|^2 = \rho_k,$$
$$\rho_k \|P\| \lambda_{\max}(P^{-1}) \leq \eta^2, \quad \eta \geq 1,$$
$$\frac{\ln \eta}{\rho} + \lambda_{\max}(P^{-1}) [K - \lambda_{\min}(D^* \otimes P\Gamma) + \varepsilon\eta \|(C \otimes P\Gamma)\|] < 0. \tag{51}$$

The proof of Corollaries 7 and 8 is the same as that of Theorem 5 and thus omitted here.

3.2. Node Delay. Two networks with node delays and impulsive control can be equivalently expressed as follows:

$$\dot{x}_i(t) = f(t, x_i(t), x_i(t - \tau(t))) + \varepsilon \sum_{j=1}^{N} c_{ij} \Gamma y_j(t),$$
$$y_j(t) = Hx_i(t),$$
$$\dot{\hat{x}}_i(t) = f(t, \hat{x}_i(t), \hat{x}_i(t - \tau(t))) + \varepsilon \sum_{j=1}^{N} c_{ij} \Gamma \hat{y}_j(t) + u_{1i},$$
$$\hat{y}_j(t) = H\hat{x}_i(t), \quad t = t_k, \ k = 1, 2, \ldots,$$
$$\Delta \hat{x}_i = \hat{x}_i(t_k^+) - \hat{x}_i(t_k^-) = B_{ik}(\hat{y}_i - y_i), \quad t = t_k. \tag{52}$$

Accordingly, the synchronization error of two networks with node delays can be equivalently expressed as follows:

$$\dot{e}_i(t) = f(t, \hat{x}_i(t), \hat{x}_i(t - \tau(t))) - f(t, x_i(t), x_i(t - \tau(t))) + \varepsilon \sum_{j=1}^{N} c_{ij} \Gamma He_j(t) + u_{1i},$$
$$e_{yi}(t) = He_i(t), \quad t \neq t_k,$$
$$\Delta e_i = B_{ik} He_i, \quad t = t_k, \ k = 1, 2, \ldots. \tag{53}$$

Then, we obtain the following results.

Theorem 9. *Let* $0 < \rho = \sup\{t_k - t_{k-1}\} < \infty$. *Suppose Assumption 2 holds; the drive network* (4) *and the response network* (5) *with impulsive controller* (7) *will achieve globally exponential synchronization if*

$$\max_k \|I_n + B_{ik} H\|^2 = \rho_k,$$
$$\rho_k \|P\| \lambda_{\max}(P^{-1}) \leq \eta^2, \quad 0 < \eta < 1,$$
$$\frac{\ln \eta}{\rho} + \lambda_{\max}(P^{-1}) \left[K - \lambda_{\min}(D^* \otimes P\Gamma H) + \varepsilon \|(C \otimes P\Gamma H)\| + \frac{L}{\eta^2} \right] < 0. \tag{54}$$

Proof. Take the same Lyapunov candidate function $V(t)$ as in Theorem 5. By the proof of Theorem 5, we can get

$$\begin{aligned}
\dot{V}(t) &= 2 \sum_{i=1}^{N} e_i^T(t) P \Big[f(t, \hat{x}_i(t), \hat{x}_i(t - \tau(t))) - f(t, x_i(t), x_i(t - \tau(t))) + \varepsilon \sum_{j=1}^{N} c_{ij} \Gamma He_j(t) - d_i \Gamma He_i(t) \Big] \\
&\leq 2K \sum_{i=1}^{N} e_i^T(t) e_i(t) + 2L \sum_{i=1}^{N} e_i^T(t - \tau(t)) e_i(t - \tau(t)) - 2d^* \sum_{i=1}^{N} e_i^T(t) P\Gamma He_i(t) + 2\varepsilon e^T(t)(C \otimes P\Gamma H) e(t) \\
&\leq \lambda_{\max}(P^{-1}) \big(2L - 2\lambda_{\min}(D^* \otimes P\Gamma H) + 2\varepsilon \|(C \otimes P\Gamma H)\| \big) V(t) + 2\lambda_{\max}(P^{-1}) LV(t - \tau(t)).
\end{aligned} \tag{55}$$

The rest of the proof of Theorem 9 is the same as that of Theorem 5 and thus omitted here. □

Theorem 10. *Let $\rho = \inf\{t_k - t_{k-1}\} > 0$. Suppose Assumption 2 holds; the drive network* (4) *and the response network* (5) *with impulsive controller* (7) *will achieve globally exponential synchronization if*

$$\max_k \|I_n + B_{ik}H\|^2 = \rho_k,$$
$$\rho_k \|P\| \lambda_{\max}(P^{-1}) \le \eta^2, \quad \eta \ge 1,$$
$$\frac{\ln\eta}{\rho} + \lambda_{\max}(P^{-1})\left[K - \lambda_{\min}(D^* \otimes P\Gamma H) + \varepsilon\|(C \otimes P\Gamma H)\| + \eta^2 L\right] < 0. \tag{56}$$

Similar to the Corollaries 7 and 8, we have the following corollaries.

Corollary 11. *Let $0 < \rho = \sup\{t_k - t_{k-1}\} < \infty$. Suppose Assumption 2 holds, the drive-response network with impulsive controller* (7) *will achieve globally exponential synchronization if*

$$\max_k \|I_n + B_{ik}H\|^2 = \rho_k,$$
$$\rho_k \|P\| \lambda_{\max}(P^{-1}) \le \eta^2, \quad 0 < \eta < 1,$$
$$\frac{\ln\eta}{\rho} + \lambda_{\max}(P^{-1})\left[K - \lambda_{\min}(D^* \otimes P\Gamma) + \varepsilon\|(C \otimes P\Gamma)\| + \frac{L}{\eta^2}\right] < 0. \tag{57}$$

Corollary 12. *Let $\rho = \inf\{t_k - t_{k-1}\} > 0$. Suppose Assumption 2 holds, the drive-response network with impulsive controller* (7) *will achieve globally exponential synchronization if*

$$\max_k \|I_n + B_{ik}H\|^2 = \rho_k,$$
$$\rho_k \|P\| \lambda_{\max}(P^{-1}) \le \eta^2, \quad \eta \ge 1,$$
$$\frac{\ln\eta}{\rho} + \lambda_{\max}(P^{-1})\left[K - \lambda_{\min}(D^* \otimes P\Gamma) + \varepsilon\|(C \otimes P\Gamma)\| + \eta^2 L\right] < 0. \tag{58}$$

Remark 13. It is noted that the configuration matrix C does not need to be symmetric, diffusive, or irreducible. This means that the networks can be undirected or directed networks and may also contain isolated nodes or clusters. Therefore, the network structures here are very general and the results can be applied to great many complex dynamical networks.

Remark 14. In the above theorems and corollaries, the matrix Γ and H can be chosen as $n \times p$ and $p \times n$ matrices, $1 \le p \le n$, based on the method of output coupling; the amount of coupling variables between every two connected nodes is flexible, which can save a lot of channel resources and simplify the network topological structure and is more useful for practical engineering applications.

4. Numerical Results

In this section, numerical simulations are given to verify and demonstrate the effectiveness of the proposed synchronization schemes for synchronizing the drive-response network with time-delayed dynamical nodes or coupling delay. We consider several networks with four nodes, where we will take chaotic systems as the dynamics of nodes. The total synchronization error calculated by $\|e(t)\| = \sqrt{\sum_{i=1}^{N} \|\hat{x}_i(t) - x_i(t)\|^2}$ is used to measure the evolution process.

Example 15. In the first example, we consider diffusively coupled networks with coupling delay. The chaotic Chua's circuit is taken as the node dynamic system of the networks and given by

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} p(y - x - g(x)) \\ x - y + z \\ -qy \end{pmatrix}, \tag{59}$$

where $g(x) = nx + 0.5(m-n)(|x+1| - |x-1|)$, $p = 10$, $q = 14.87$, $n = -0.68$, and $m = -1.27$.

For simplicity, the diffusive coupling configuration matrix $C = (c_{ij})_{N\times N}$ is chosen as

$$C = \begin{pmatrix} -5 & 1 & 3 & 1 \\ 1 & -3 & 2 & 0 \\ 1 & 0 & -2 & 1 \\ 2 & 1 & 0 & -3 \end{pmatrix}. \tag{60}$$

In the numerical simulations, we assume $\varepsilon = 0.01$, $d^* = 50$, $\Gamma = [2 \quad 5 \quad 1]^T$, $H = [1 \quad 0 \quad 0]$, and $B_{ik} = [-0.3 - 0.3 - 0.3]^T$. The coupling delay is $\tau(t) = 2 - e^{-t}$. The initial values are chosen in the real number interval $[-1, 1]$. Let $P = \mathrm{diag}(0.2, 0.5, 0.2)$; we have $(x-y)^T P(f(x) - f(y)) \le [\|PA\| + 0.2(|n| + |m-n|)](x-y)^T(x-y)$; thus, f satisfies Assumption 2 with $K = \|PA\| + 0.2p(|n| + |m-n|) = 6.3888$. After calculations, getting $\|(C \otimes P\Gamma H)\| = 6.6746$, $\max\|I_n + B_{ik}H\|^2 = \rho_k = 1$, one gets $\rho_k\|P\|\lambda_{\max}(P^{-1}) = 2.5 \le \eta^2$. According to Theorem 6, let $\eta = 1.6 > 1$; one gets $\rho > 0.0171$. Thus, taking the impulsive interval $t_{k+1} - t_k = 0.02$. Figure 1 shows the evolution process of the state errors $e_i(t)$. Figure 2 displays the total synchronization error $\|e(t)\|$ of two Chua's circuit networks with the impulsive controller. Figure 3 illustrates the impulsive applied to each state of the observer at instant times. In the early times of the simulation, since state estimation errors are large, the magnitude of the synchronization impulses is larger and as time increases the magnitude of the impulses will decrease. Numerical simulations show that synchronization of two Chua's circuit networks with output coupling can be easily achieved by the simple impulsive control scheme.

Example 16. In this example, we consider nondiffusively coupled networks with node delay. The chaotic delayed Hopfield neural network

$$\dot{x}(t) = Ax(t) + Bf(x(t)) + Cf(x(t-\tau)), \tag{61}$$

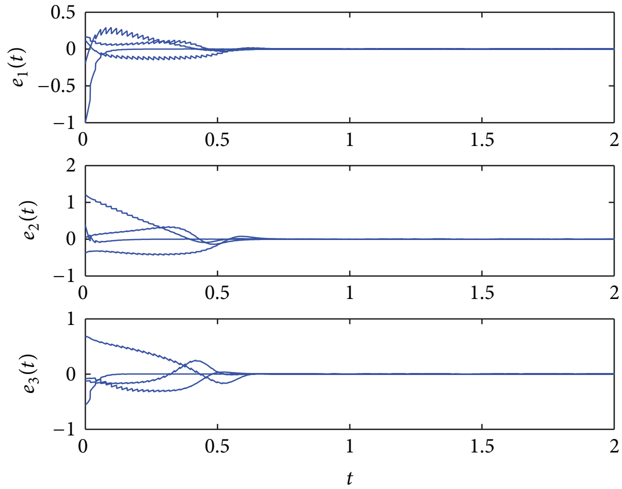

FIGURE 1: Evolution process of synchronization errors of two Chua's circuit networks.

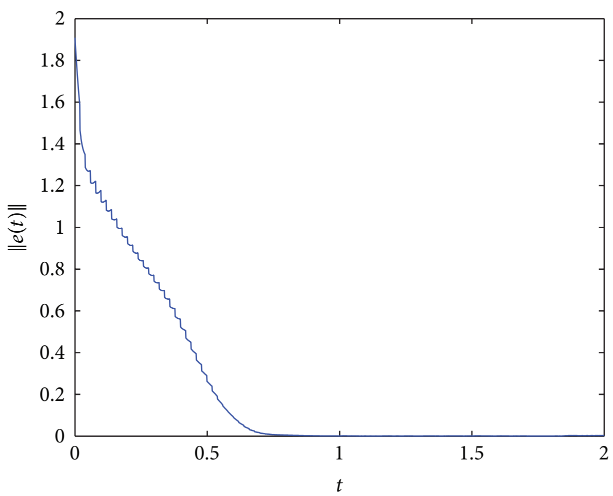

FIGURE 2: Evolution process of total synchronization error $\|e(t)\|$ of two Chua's circuit networks.

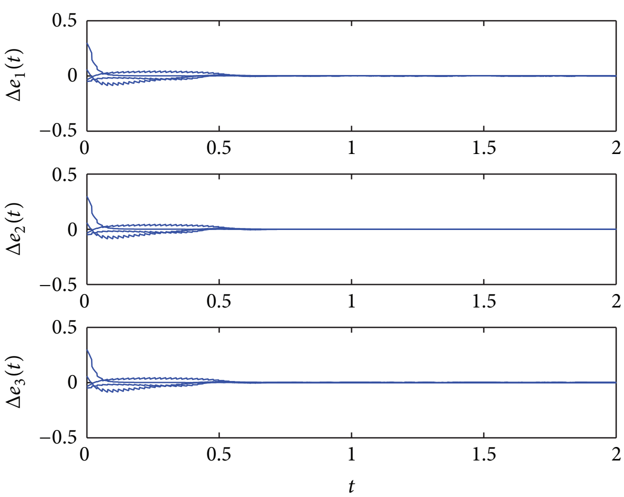

FIGURE 3: Observer estimated states jumps at the impulse times.

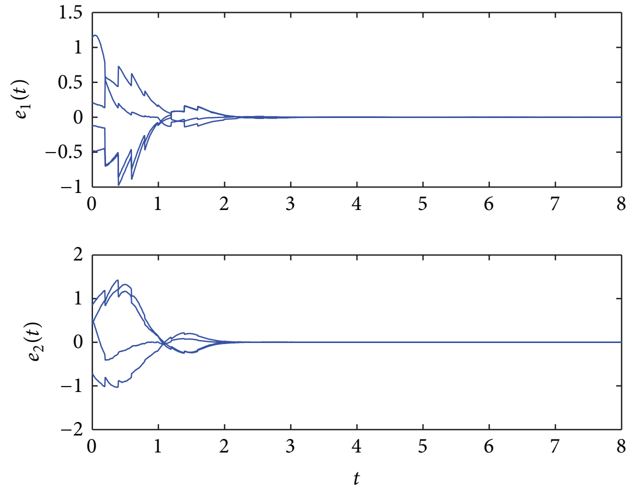

FIGURE 4: Evolution process of synchronization errors of two delayed Hopfield neural networks.

with $x(t) = (x_1(t), x_2(t))^T \in R^2$, $f(x(t)) = (\tanh(x_1(t)), \tanh(x_2(t)))^T \in R^2$, and $A = \begin{pmatrix} -1.0 & 0 \\ 0 & -1.0 \end{pmatrix}$, $B = \begin{pmatrix} 2.1 & -0.1 \\ -5.0 & 3.0 \end{pmatrix}$, $C = \begin{pmatrix} -1.5 & -0.1 \\ -0.2 & -2.5 \end{pmatrix}$, which has a very rich complex dynamical behavior and contains, for example, a double-scroll chaotic attractor for time delay $\tau = 1$, is regarded as the node dynamic system of the networks.

For simplicity, the nondiffusive coupling configuration matrix $C = (c_{ij})_{N\times N}$ is chosen as

$$C = \begin{pmatrix} 5 & 1 & -3 & 1 \\ 1 & 0 & 2 & 0 \\ -1 & 0 & 2 & 1 \\ -2 & 1 & 0 & 3 \end{pmatrix}. \tag{62}$$

In the numerical simulations, we assume $\Gamma = [1 \quad 1]^T$, $H = [1 \quad 1]$, and $B_{ik} = [-0.5 - 0.5]^T$. After calculations, f satisfies Assumption 2 with $K = L = 1$. But the other conditions are chosen to be the same as above; the conditions of Theorem 10 are satisfied; the simulation results are as shown in Figures 4, 5, and 6. From the numerical results, the outer synchronization is achieved by employing impulsive controller. Thus, all numerical simulations illustrate the effectiveness of the proposed synchronization criteria.

5. Conclusion

In this paper, synchronization between the drive network and the response network is investigated based on the impulsive hybrid observer approach. Only by employing the output of the drive network at discrete instant times, the response network would be able to estimate all states. While most of the impulsive synchronization methods need all the states of the drive at the receiver. Based on the stability analysis of impulsive delayed systems and comparison method, sufficient conditions of synchronization between two complex networks are obtained. Numerical simulations have been given to show the effectiveness and the correctness of the theoretical

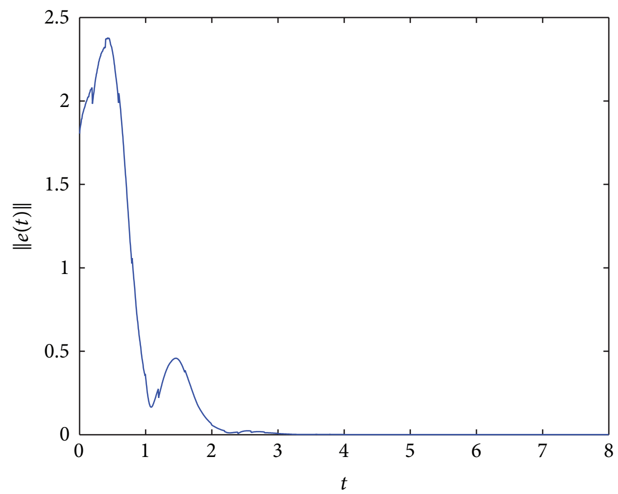

Figure 5: Evolution process of total synchronization error $\|e(t)\|$ of two delayed Hopfield neural networks.

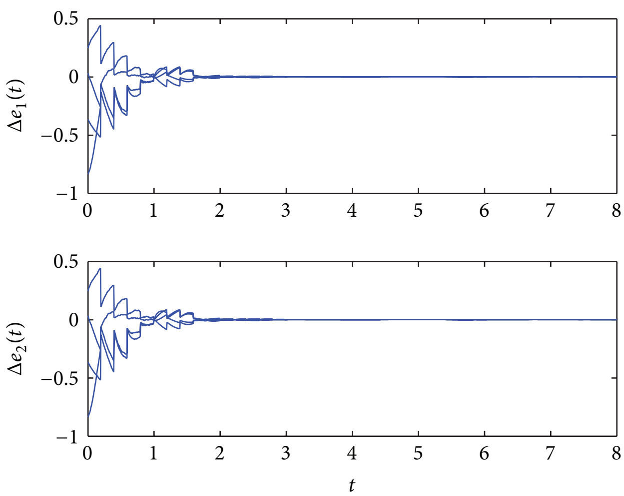

Figure 6: Observer estimated states jumps at the impulse times.

analysis finally. In the near future, it would be of interest to study the impulsive control problem for output coupled complex networks with dynamically switching topologies and time delays.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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