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Research Article

Finite-Time Observer Based Cooperative Tracking Control of Networked Lagrange Systems

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This paper investigates the cooperative tracking control problem for networked uncertain Lagrange systems with a leader-follower structure on digraphs. Since the leader's information is only available to a portion of the followers, finite-time observers are designed to estimate the leader's velocity. Based on the estimated velocity information and the universal approximation ability of fuzzy logic systems, a distributed adaptive fuzzy tracking control protocol is first proposed for the fault-free Lagrange systems. Then, the actuator faults are considered and a distributed fault-tolerant controller is presented. Based on graph theory and Lyapunov theory, the convergence analyses for the proposed algorithms are provided. The development in this paper is suitable for the general directed communication topology. Numerical simulation results are presented to show the closed-loop performance of the proposed control law and illustrate its robustness to actuator faults and external disturbances.

1. Introduction

Recently, consensus problem for multiagent systems has attracted a great deal of attention in many fields such as biology, physics, robotics, and control engineering due to its broad applications in many areas such as mobile robots, unmanned air vehicles, autonomous underwater vehicles, satellites, aircrafts, automated highway systems, and air traffic control. In 1995, Vicsek et al. [1] proposed a simple but interesting discrete-time model of a system of several autonomous agents. It is shown that all agents eventually move in the same direction based on local information without any central controller or leaders. Jadbabaie et al. [2] provided a theoretical explanation for the consensus behavior of the Vicsek model by using the graph theory and the matrix theory. Chen et al. [3] investigated the finite-time distributed consensus problem for multiagent systems using a binary consensus protocol and the pinning control scheme. The readers are referred to [4–7] for more details.

Since a large class of networked mechanical systems can be modeled as a networked Euler-Lagrange system, it is fascinating to study the control problem of networked Euler-Lagrange system. The work in [8] studied the position synchronization problem for a group of Lagrange systems. However, all the robots required the desired common trajectory information. In [9], Ren presented some distributed consensus algorithms for the networked Lagrange systems, where only the undirected communication topologies were considered. The work in [10] proposed a consensus algorithm for networked leaderless Lagrange systems on undirected communication topology. The work in [11] investigated the finite-time cooperative tracking problem for a class of networked Euler-Lagrange systems with a leader-follower structure, where the communication topologies among the followers are undirected. The work in [12] studied the cooperative tracking control problem for a group of Lagrange vehicle systems with directed communication topology. The dynamics of the networked systems, as well as the target system, are all assumed unknown. A neural network (NN) is used at each node to approximate the distributed dynamics [12].

In reality, the actuators of the networked systems may undergo a partial loss of effectiveness or experience bias faults. Actuator faults often cause the control performance to deteriorate and even lead to catastrophic accidents. Thus, fault-tolerant control (FTC) schemes are proposed and used to guarantee system stability and acceptable performance. FTC design may be classified into two types: the passive approaches [13–15] and the active approaches [16–18]. The passive FTC approach designs a fixed controller that is able to tolerate only a limited range of predetermined faults, and, once implemented, it compensates for the anticipated faults without any online fault identification. However, the passive FTC has a very limited fault tolerance capability and is often designed to be conservative. The active FTC compensates for the effects of component faults by synthesizing a new controller online or by selecting a predesigned controller [19]. The active FTC requires a fault detection and diagnosis (FDD) mechanism to detect and identify the faults in real time, and then the controller is reconfigured based on the identified faults. Errors in fault detection may cause the controller to make wrong decisions. In this paper, we present a fault-tolerant control strategy which does not use any fault detection and isolation mechanism to detect, separate, and identify the actuator faults online. Although the fault-tolerant control methods have been used in a class of mechanical systems such as spacecraft [20] and near-space-vehicle [21], few works have focused on the fault-tolerant control of networked Euler-Lagrange systems.

Compared with the aforementioned works [2–12], this paper addresses the distributed fault-tolerant tracking problem for networked unknown Lagrange systems on digraphs. The distributed tracking control problem on digraph is more challenging than that on undirected or balanced graphs. We assume that the inertia matrix, the Coriolis/centrifugal matrix, the friction term, and the gravity term are all unknown for all Lagrange systems. A Lyapunov technique is utilized to design the distributed tracking controllers. By applying the universal approximation ability of fuzzy logic systems [22–24], an adaptive coordinated fuzzy controller is developed firstly when the actuators are fault-free. Then, an active fault-tolerant controller is developed, which can compensate for both the actuator bias faults and the loss of actuator effectiveness. The proposed control law does not require any FDD mechanism to detect the faults, which reduces the computation burden and decreases the response time of the controller. The analysis shows that if the designed nominal controller can ensure the stability of the fault-free distributed system, the proposed fault-tolerant controller guarantees the stability of the distributed system in the presence of faults. Numerical simulation results are given to show the effectiveness of the proposed method.

To our best knowledge, the fault-tolerant cooperative control of networked uncertain Lagrange systems on digraphs had not been fully investigated and it is still a challenging task. We present a solution to this problem in this paper. The main contributions of this research are described as follows.

(1) A novel finite-time observer based cooperative control method is proposed for the networked nonlinear Lagrange systems. The dynamic leader and all the followers have unknown nonidentical dynamics. Compared with the results in [2–12], the proposed method is suitable for the general directed communication topology.

(2) A robust fault-tolerant cooperative control scheme is presented to achieve distributed tracking control even if the actuator bias fault and the loss of actuator effectiveness coexist. In addition to the nominal controller, an auxiliary control input is designed to compensate for the actuator faults. It is proved that the proposed approach guarantees the convergence based on Lyapunov stability theory.

The remainder of this paper is organized as follows. Section 2 introduces the problem formulation and some preliminary results. Section 3 provides the proposed nominal controller and the robust fault-tolerant controller. The simulations are shown in Section 4. Section 5 concludes this paper.

2. Problem Formulation and Preliminaries

2.1. Graph Theory. Let $\mathcal{G} = (\mathcal{V}, \varepsilon, \mathcal{A})$ be a directed graph of order n, where $\mathcal{V} = [v_1, \ldots, v_n]$ is the set of nodes, $\varepsilon \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges, and $\mathcal{A} = [a_{ij}]$ is called the adjacency matrix with weights $a_{ij} > 0$ if $(v_j, v_i) \in \varepsilon$ and $a_{ij} = 0$ otherwise. We assume that the graph is simple; that is, $(v_i, v_i) \notin \varepsilon \ \forall i$, with no self-loops. Thus, $a_{ii} = 0$. Define the in-degree of node v_i as the ith row sum of A; that is, $d_{in}(v_i) = \sum_{j=1}^{n} a_{ij}$. Define the diagonal in-degree matrix $D = \operatorname{diag}\{d_{in}(v_i)\}$ and the graph Laplacian matrix $L = D - A$. The set of neighbors of a node v_i is $N_i = \{v_j : (v_j, v_i) \in \varepsilon\}$, which is the set of nodes with edges incoming to v_i. A directed path is a sequence of nodes $v_1, v_2, \ldots, v_r$ such that $(v_{i+1}, v_i) \in \varepsilon$, $i \in \{1, 2, \ldots, r-1\}$. A directed tree is a directed graph, where every node, except the root, has exactly one parent. A spanning tree of a digraph is a directed tree that connects all the nodes of the graph.

2.2. Fuzzy Logic Systems. A fuzzy logic system (FLS) consists of four parts: the knowledge base, the fuzzifier, the fuzzy inference engine, and the defuzzifier. The knowledge base for FLS comprises a collection of fuzzy if-then rules of the following form:

$$\begin{gathered} R^l\text{: If } x_1 \text{ is } A_1^l,\ x_2 \text{ is } A_2^l \ldots,\ x_n \text{ is } A_n^l, \\ \text{Then } y \text{ is } B^l, \quad l = 1, 2, \ldots, M, \end{gathered} \tag{1}$$

where $\hat{x} = [x_1, x_2, \ldots, x_n]^T \in U \subset R^n$ and y denote the FLS input and output, respectively. A_i^l, $i = 1, 2, \ldots, n$ and $l = 1, 2, \ldots, M$, are fuzzy sets and B^l is the fuzzy singleton for the output in the lth rule. Fuzzy sets A_i^l and B^l are, respectively, associated with the membership functions $\mu_{A_i^l}(x_i) = \exp(-((x_i - a_i^l)/b_i^l)^2)$ and $\mu_{B^l}(y^l) = 1$, where a_i^l is the center of the receptive field and b_i^l denotes the width of the Gaussian function. M is the rules number. By applying singleton function, center average defuzzification, and product inference, the FLS can be expressed as

$$y(x) = \frac{\left\{\sum_{l=1}^{M} y^l \left(\prod_{i=1}^{n} \mu_{A_i^l}(x_i)\right)\right\}}{\left\{\sum_{l=1}^{M} \left(\prod_{i=1}^{n} \mu_{A_i^l}(x_i)\right)\right\}}, \tag{2}$$

where $y^l = \max_{y\in R}\mu_{B^l}$. Define the fuzzy basis functions as

$$\xi_l(x) = \frac{\left[\prod_{l=1}^{n}\mu_{A_i^l}(x_i)\right]}{\left[\sum_{l=1}^{M}\left(\prod_{i=1}^{n}\mu_{A_i^l}(x_i)\right)\right]}, \tag{3}$$

and let $\theta^T = [y^1, y^2, \ldots, y^l] = [\theta_1, \theta_2, \ldots, \theta_M]$ and $\xi(x) = [\xi_1(x), \xi_2(x), \ldots, \xi_M(x)]^T$. Then, the FLS (2) can be rewritten as

$$y(x,\theta) = \theta^T \xi(x). \tag{4}$$

Lemma 1 (see [23]). *Let $f(x)$ be a continuous function defined on a compact set Ω. Then, for any constant $\varepsilon > 0$, there exists an FLS* (4) *such that*

$$\sup_{x\in\Omega}\left|f(x) - \theta^T\xi(x)\right| \le \varepsilon. \tag{5}$$

By Lemma 1, FLSs are universal approximators; that is, they can approximate any smooth function on a compact space. Due to this approximation ability, we can assume that the nonlinear term $f(x)$ is approximated as

$$f(x,\theta) = \theta^T\xi(x). \tag{6}$$

Define the optimal parameter vector $\theta^* = \arg\min_{\theta\in\Omega}\sup_{x\in U}|f(x) - \theta^T\xi(x)| \le \varepsilon$, where Ω and U are compact regions for θ and x, respectively. The FLS minimum approximation error is defined as

$$\varepsilon = f(x) - \theta^{*T}\xi(x). \tag{7}$$

2.3. Problem Formulation. In this paper, the networked system consists of $n+1$ Lagrange systems, where the subsystem indexed by zero is assigned as the leader or target system and the other systems indexed by $1, 2, \ldots, n$ are referred to as the followers. The Euler-Lagrange equations of motion for the leader vehicle and the followers are described as

$$M_0(q_0)\ddot{q}_0 + C_0(q_0,\dot{q}_0)\dot{q}_0 + H_0(\dot{q}_0) + G_0(q_0) = \tau_0, \tag{8}$$

$$M_i(q_i)\ddot{q}_i + C_i(q_i,\dot{q}_i)\dot{q}_i + H_i(\dot{q}_i) + G_i(q_i) = \tau_i + \delta_i, \quad i = 1, 2, \ldots, n, \tag{9}$$

respectively, where $q_i \in R^m$ $(i = 0, 1, \ldots, n)$ is the generalized configuration coordinate, $M_i(q_i) \in R^{m\times m}$ is the inertia matrix, $C_i(q_i,\dot{q}_i) \in R^{m\times m}$ is the Coriolis/centrifugal matrix, $H_i(\dot{q}_i) \in R^m$ is the friction term, $G_i(q_i) \in R^m$ is the vector of gravitational torques, τ_i is the vector of control input torques, and δ_i represents the input disturbance. The inertia, Coriolis, friction, and gravity terms are all assumed unknown.

Note that the actuators of the Lagrange systems in (9) are assumed to be fault-free. They are called the nominal system. The case with actuator faults will be discussed in this paper. We will consider two types of actuator faults simultaneously, namely, the bias fault denoted by $f_i \in R^m$ and the loss of effectiveness of the actuators represented by a multiplicative matrix $\sigma_i \in R^{m\times m}$. Hence, the dynamic model given by (9) can be rewritten as

$$M_i(q_i)\ddot{q}_i + C_i(q_i,\dot{q}_i)\dot{q}_i + H_i(\dot{q}_i) + G_i(q_i) = \sigma_i\tau_i + f_i + \delta_i. \tag{10}$$

Assumption 2. For the bias fault, f_i, there exists a positive continuous function $k_i(t)$ satisfying $|f_{ij}| \le k_{ij}(t)$ for each $j \in \{1, \ldots, m\}$. The actuator effectiveness matrix σ_i satisfies $0 < \overline{\sigma}_{ij} < \sigma_{ij} \le 1$ $(j \in \{1, \ldots, m\})$ for some constants $\overline{\sigma}_{ij}$.

Assumption 3. The input disturbance δ_i is bounded; that is, there exists a fixed bound δ_0 such that $\|\delta_i\| \le \delta_0$.

Assumption 4. The acceleration of the leader system is bounded by a positive scalar μ_0 such that $\|\ddot{q}_0(t)\|_\infty \le \mu_0 < \infty$.

Property 1. The inertia matrix $M_i(q_i)$ is symmetric and positive definite such that $M_m I \le M_i(q_i) \le M_M I$ for some constants $M_m > 0$, $M_M > 0$.

Property 2. The Coriolis/centrifugal matrix can always be selected so that the matrix $(\dot{M}_i(q_i) - 2C_i(q_i,\dot{q}_i))$ is skew symmetric. Therefore, $x^T(\dot{M}_i(q_i) - 2C_i(q_i,\dot{q}_i))\,x = 0$ for all vectors x.

The distributed tracking control problem confronted here is as follows: we need to design the control protocols by using only local information such that the states of the follower systems synchronize to the states of the leader system; that is, one requires $q_i(t) \to q_0(t)$ and $\dot{q}_i(t) \to \dot{q}_0(t)$.

3. Main Results

3.1. Finite-Time Observer Design. In this paper, we focus on the general case that the information of the leader system is only available to a portion of the followers. Observer based control methods are used in this paper; that is, finite-time observers are used to estimate the leader's information. We first design a velocity observer for each follower as follows:

$$\dot{z}_i = -\gamma\,\mathrm{sgn}\left\{\sum_{j\in N_i} a_{ij}(z_i - z_j) + b_i(z_i - \dot{q}_0)\right\}, \quad i = 1, 2, \ldots, n, \tag{11}$$

where $\gamma > \mu_0 > 0$ is a positive constant and z_i denotes the estimate of $\dot{q}_0$ on the ith Lagrange system.

Let $D^+V(t,x)$ be the upper right-hand Dini derivative of $V(t,x)$ with respect to t; that is,

$$D^+V(t,x) = \limsup_{h\to 0^+}\frac{V(t+h, x(t+h)) - V(t, x(t))}{h}. \tag{12}$$

Lemma 5 (see [25]). *Let $I_0 = \{1, 2, \ldots, n\}$ and let, for each $i \in I_0$, $V_i(t,x) : R\times R^m \to R$ $(i = 1, \ldots, n)$ be c^1. Define $V(t,x) = \max_{i\in I_0}V_i(t,x)$. Let $I(t) = \{i \in I_0 : V_i(t,x) = V(t,x)\}$ be the set of indices where the maximum is reached at time t. Then $D^+V(t,x)$ satisfies $D^+V(t,x) = \max_{i\in I(t)}\dot{V}_i(t,x)$.*

Lemma 6. *Let the communication graph G contain a spanning tree with the root node being the leader system* (8). *For the* *observers (11), it holds that $z_i \to \dot{q}_0(t)$ $(i = 1, \ldots, n)$ in finite time. The finite setting time T_0 satisfies*

$$T_0 \leq \frac{\max_k \left\{\max_j \left(z_{jk}(0)\right) - \min_j \left(z_{jk}(0)\right)\right\}}{\gamma - \mu_0}, \quad (13)$$

with $k \in \{1, \ldots, m\}$ and $j \in \{0, 1, \ldots, n\}$.

Proof. Let $z_0 = \dot{q}_0$ and $a_{i0} = b_i$. Then, (11) can be rewritten as

$$\dot{z}_{ik} = -\gamma \operatorname{sign}\left\{\sum_{j=0}^{n} a_{ij}\left(z_{ik} - z_{jk}\right)\right\}, \quad i = 1, \ldots, n, \quad k = 1, \ldots, m. \quad (14)$$

Define $e_{ik} = z_{ik} - \dot{q}_{0k}$. Thus,

$$\dot{e}_{ik} = -\gamma \operatorname{sign}\left\{\sum_{j=0}^{n} a_{ij}\left(e_{ik} - e_{jk}\right)\right\} - \ddot{q}_{0k}. \quad (15)$$

Choose the Lyapunov function

$$V_k = \max_i \{e_{ik}\} - \min_i \{e_{ik}\}. \quad (16)$$

It is obvious that V_k is a positive definite, Lipschitz continuous, and regular functions. Let $N_{\max}$ denote the set of indices i for which $e_{ik} = \max\{e_{ik} : i \in \{0, 1, \ldots, n\}\}$. Let $N_{\min}$ denote the set of indices i for which $e_{ik} = \min\{e_{ik} : i \in \{0, 1, \ldots, n\}\}$. In light of Lemma 5, one has

$$D^+ V_k = \max_{i \in N_{\max}} \{\dot{e}_{ik}\} - \min_{i \in N_{\min}} \{\dot{e}_{ik}\}. \quad (17)$$

If $D^+ V_k = 0$, then $\max_{i \in N_{\max}}\{e_{ik}\} = \min_{i \in N_{\min}}\{e_{ik}\}$; that is, $z_{ik} = \dot{q}_{0k}$. If $D^+ V_k > 0$, then $N_{\max} \neq \phi$ and $N_{\min} \neq \phi$. Since the communication graph G contains a spanning tree, there must exist a node $l \in \{0, 1, \ldots, n\}/N_{\max}$ (or $l \in \{0, 1, \ldots, n\}/N_{\min}$) such that a path from l to the node in $N_{\max}$ (or the node in $N_{\min}$) exists; that is,

$$\max_{i \in N_{\max}} \{\dot{e}_{ik}\} = -\gamma - \ddot{q}_{0k}, \quad (18)$$

or

$$\min_{i \in N_{\min}} \{\dot{e}_{ik}\} = \gamma - \ddot{q}_{0k}. \quad (19)$$

Thus,

$$D^+ V_k \leq -\gamma - \ddot{q}_{0k} \leq -\gamma + \left|\ddot{q}_{0k}\right| \leq -\left(\gamma - \mu_0\right). \quad (20)$$

Integrating both sides of the above equation between the time $t = 0$ and $t = T_0$ yields

$$T_0 \leq \frac{V_k(0)}{\gamma - \mu_0} \leq \frac{\max_k \left\{\max_j \left(z_{jk}(0)\right) - \min_j \left(z_{jk}(0)\right)\right\}}{\gamma - \mu_0}. \quad (21)$$

Thus the claim follows. □

3.2. Nominal Controller Design for the Fault-Free Lagrange Systems

Definition 7. The local neighborhood position errors for subsystem i are defined as

$$s_i = \sum_{j \in N_i} a_{ij}\left(q_i - q_j\right) + b_i\left(q_i - q_0\right). \quad (22)$$

Definition 8 (see [12]). The position and velocity tracking errors for each Lagrange system are said to be cooperatively uniformly ultimately bounded if there exist compact sets $\Phi_1 \in R^m$ and $\Phi_2 \in R^m$ such that, for all $q_i(t_0) \in \Phi_1$ and $\dot{q}_i(t_0) \in \Phi_2$, there exist bounds B_1 and B_2 and a time $T(B_1, B_2, q_i(t_0), \dot{q}_i(t_0))$ such that $\|q_i(t) - q_0(t)\| \leq B_1$ and $\|\dot{q}_i(t) - \dot{q}_0(t)\| \leq B_2$ for all $t \geq t_0 + T$.

Define the sliding-mode error

$$e_i = \dot{q}_i - z_i + s_i. \quad (23)$$

For the follower system i, the error dynamics are given by

$$M_i \dot{e}_i + C_i e_i = M_i \ddot{q}_i - M_i \dot{z}_i + M_i \dot{s}_i + C_i e_i. \quad (24)$$

In light of (9), one has

$$M_i \dot{e}_i + C_i e_i = \left(\tau_i + \delta_i\right) + F_i\left(x_i\right) - M_i \dot{z}_i, \quad (25)$$

where

$$F_i\left(x_i\right) = -G_i - H_i - C_i \dot{q}_i + M_i \dot{s}_i + C_i e_i. \quad (26)$$

The function $F_i(x_i)$ is unknown. In light of the approximation property of fuzzy logic systems, there exist weights θ_i such that

$$F_i\left(x_i\right) = \theta_i^T \xi_i\left(x_i\right) + \varepsilon_i. \quad (27)$$

For further analysis, we need the following assumptions.

Assumption 9. On a compact set Γ, the fuzzy logic system approximation error ε_i in (27) is bounded; that is, there exists a fixed constant bound ε_{i0} such that $\|\varepsilon_i\| \leq \varepsilon_{i0}$.

Assumption 10. There exists a constant θ_{i0} such that the weight matrix θ_i is bounded by $\|\theta_i\|_F \leq \theta_0$, $\forall i$.

Now we get one of the main results.

Theorem 11. *Consider the networked Lagrange systems* (8) *and* (9). *Assume that the directed communication graph G has a spanning tree. Make Assumptions 3–10. Take the distributed adaptive controller as*

$$\tau_i = -K_i e_i - \widehat{\theta}_i^T \xi_i - \frac{3 e_i}{4 \mu_i}, \quad (28)$$

with $K_i = k_i I$, $(k_i > 0)$ and $\mu_i > 0$, and the distributed adaptive tuning law as

$$\dot{\widehat{\theta}}_i = \beta_i \xi_i e_i^T - \eta_i \widehat{\theta}_i \quad (29)$$

with $\beta_i > 0$ and $\eta_i > 0$. The position and velocity tracking errors are cooperatively uniformly ultimately bounded (UUB). The ultimate bounds can be made as small as possible.

Proof. Consider the candidate Lyapunov function

$$V = \frac{1}{2}\sum_{i=1}^{n} e_i^T M_i e_i + \sum_{i=1}^{n} \frac{1}{2\beta_i} \operatorname{tr}\left\{\widetilde{\theta}_i^T \widetilde{\theta}_i\right\}, \quad (30)$$

with $\widetilde{\theta}_i = \theta_i - \widehat{\theta}_i$. Taking the time derivative on both sides of (30) yields

$$\dot{V} = \sum_{i=1}^{n}\left(e_i^T M_i\left(\ddot{q}_i - \dot{z}_i + \dot{s}_i\right) + \frac{1}{2} e_i^T \dot{M}_i e_i\right) - \sum_{i=1}^{n} \frac{1}{\beta_i} \operatorname{tr}\left\{\widetilde{\theta}_i^T \dot{\widehat{\theta}}_i\right\}. \quad (31)$$

By substituting (9) into (31), we have

$$\begin{aligned}\dot{V} &= \sum_{i=1}^{n}\left(e_i^T M_i\left(M_i^{-1}\left(\tau_i + \delta_i - G_i - H_i - C_i\dot{q}_i\right) - \dot{z}_i + \dot{s}_i\right) + \frac{1}{2} e_i^T \dot{M}_i e_i\right) - \sum_{i=1}^{n} \frac{1}{\beta_i} \operatorname{tr}\left\{\widetilde{\theta}_i^T \dot{\widehat{\theta}}_i\right\} \\ &= \sum_{i=1}^{n}\left(e_i^T\left(\tau_i + \delta_i - G_i - H_i - C_i\dot{q}_i - M_i\dot{z}_i + M_i\dot{s}_i\right) + \frac{1}{2} e_i^T \dot{M}_i e_i\right) - \sum_{i=1}^{n} \frac{1}{\beta_i} \operatorname{tr}\left\{\widetilde{\theta}_i^T \dot{\widehat{\theta}}_i\right\}.\end{aligned} \quad (32)$$

In order to apply Property 2, we first add the term $e_i^T C_i e_i$ and then subtract it. Thus

$$\dot{V} = \sum_{i=1}^{n}\left(e_i^T\left(\tau_i + \delta_i - G_i - H_i - C_i\dot{q}_i - M_i\dot{z}_i + M_i\dot{s}_i\right) + e_i^T C_i e_i\right) - \sum_{i=1}^{n} \frac{1}{\beta_i} \operatorname{tr}\left\{\widetilde{\theta}_i^T \dot{\widehat{\theta}}_i\right\}. \quad (33)$$

Define

$$F_i\left(x_i\right) = -G_i - H_i - C_i\dot{q}_i + M_i\dot{s}_i + C_i e_i. \quad (34)$$

According to (27), we have

$$\dot{V} = \sum_{i=1}^{n} e_i^T\left(\tau_i + \theta_i^T \xi_i\left(x_i\right) + \varepsilon_i + \delta_i - M_i\dot{z}_i\right) - \sum_{i=1}^{n} \frac{1}{\beta_i} \operatorname{tr}\left\{\widetilde{\theta}_i^T \dot{\widehat{\theta}}_i\right\}. \quad (35)$$

In light of Young's inequality, one has

$$\begin{aligned} e_i^T \varepsilon_i + e_i^T \delta_i - e_i^T M_i \dot{z}_i &\le \left\|e_i\right\|\left(\left\|\varepsilon_i\right\| + \left\|\delta_i\right\| + \left\|M_i\dot{z}_i\right\|\right) \\ &\le \frac{3\left\|e_i\right\|^2}{4\mu_i} + \mu_i\left\|\delta_i\right\|^2 + \mu_i\left\|\varepsilon_i\right\|^2 + \mu_i\left\|M_i\dot{z}_i\right\|^2 \\ &\le \frac{3\left\|e_i\right\|^2}{4\mu_i} + \mu_i\left(\delta_{i0}^2 + \varepsilon_{i0}^2 + m\gamma^2 M_M^2\right).\end{aligned} \quad (36)$$

Substituting (28) and (36) into (35) yields

$$\dot{V} \le \sum_{i=1}^{n} e_i^T\left(-K_i e_i + \widetilde{\theta}_i^T \xi_i\left(x_i\right)\right) - \sum_{i=1}^{n} \frac{1}{\beta_i} \operatorname{tr}\left\{\widetilde{\theta}_i^T \dot{\widehat{\theta}}_i\right\} + \sum_{i=1}^{n} \mu_i\left(\delta_{i0}^2 + \varepsilon_{i0}^2 + m\gamma^2 M_M^2\right). \quad (37)$$

From (29), we have

$$\dot{V} \le -\sum_{i=1}^{n} e_i^T K_i e_i + \sum_{i=1}^{n} \frac{\eta_i}{\beta_i} \operatorname{tr}\left\{\widetilde{\theta}_i^T \widehat{\theta}_i\right\} + \sum_{i=1}^{n} \mu_i\left(\delta_{i0}^2 + \varepsilon_{i0}^2 + m\gamma^2 M_M^2\right). \quad (38)$$

Note that

$$\operatorname{tr}\left\{\widetilde{\theta}_i^T \widehat{\theta}_i\right\} \le \left\|\widetilde{\theta}_i\right\|_F \left\|\theta_i\right\|_F - \left\|\widetilde{\theta}_i\right\|_F^2 \le -\frac{1}{2}\left(\left\|\widetilde{\theta}_i\right\|_F^2 - \left\|\theta_i\right\|_F^2\right). \quad (39)$$

Thus,

$$\dot{V} \le -\sum_{i=1}^{n}\left(k_i e_i^T e_i + \frac{\eta_i}{2\beta_i} \operatorname{tr}\left\{\widetilde{\theta}_i^T \widetilde{\theta}_i\right\}\right) + \sum_{i=1}^{n}\left(\mu_i\left(\delta_{i0}^2 + \varepsilon_{i0}^2 + m\gamma^2 M_M^2\right) + \frac{\eta_i}{2\beta_i}\left\|\theta_i\right\|_F^2\right). \quad (40)$$

Let $d_0 = \sum_{i=1}^{n}(\mu_i(\delta_{i0}^2 + \varepsilon_{i0}^2 + m\gamma^2 M_M^2) + (\eta_i/2\beta_i)\|\theta_i\|_F^2)$. Thus,

$$\dot{V} \le -\sum_{i=1}^{n}\left(\frac{k_i}{M_m} e_i^T M_i e_i + \frac{\eta_i}{2\beta_i} \operatorname{tr}\left\{\widetilde{\theta}_i^T \widetilde{\theta}_i\right\}\right) + d_0. \quad (41)$$

Moreover,

$$\dot{V} \le -c_0 V + d_0 \quad (42)$$

with $c_0 = \min\{(2k_i/M_m), \eta_i\}$.

Let $e = (e_1^T, e_2^T, \ldots, e_n^T)^T$. From (42), one has

$$\|e\|^2 \le \frac{2V(0)}{M_m} e^{-c_0 t} + \frac{2d_0}{M_m c_0}. \quad (43)$$

with M_m defined in Property 1.

Note that $z_i = \dot{q}_0$ as $t > T_0$. Thus, we have

$$\dot{S} = -\left(\left(L + B\right) \otimes I_m\right) S + \left(\left(L + B\right) \otimes I_m\right) e, \quad t > T_0, \quad (44)$$

with $S = (s_1^T, s_2^T, \ldots, s_n^T)^T$. Since $L + B$ is a positive stable matrix, there exists a symmetric positive matrix P_0 such that $Q = P_0(L + B) + (L + B)^T P_0$ is symmetric positive definite. Choose the Lyapunov function

$$V_1 = S^T\left(P_0 \otimes I_m\right) S. \quad (45)$$

The time derivative of V_1 is

$$\begin{aligned}\dot{V}_1 &= 2S^T\left(P_0\otimes I_m\right)\left(-\left((L+B)\otimes I_m\right)S + \left((L+B)\otimes I_m\right)e\right)\\ &= S^T\Big(-\left(P_0\left(L+B\right)\right)\otimes I_m - \left((L+B)^T P_0\right)\otimes I_m\Big)S\\ &\quad + 2S^T\left(\left(P_0\left(L+B\right)\right)\otimes I_m\right)e\\ &\le -\lambda_{\min}(Q)\|S\|^2\\ &\quad + 2\|S\|\,\sigma_{\max}(P_0)\,\sigma_{\max}(L+B)\,\|e\|;\end{aligned} \tag{46}$$

$\dot{V}_1$ is negative as long as

$$\|S\| > \frac{2\sigma_{\max}(P_0)\,\sigma_{\max}(L+B)}{\lambda_{\min}(Q)}\|e\|. \tag{47}$$

In light of the fact that

$$\left\|q_i - q_0\right\| \le \frac{\|S\|}{\sigma_{\min}(L+B)}, \tag{48}$$

we have

$$\left\|q_i - q_0\right\| \le \frac{2\sigma_{\max}(P_0)\,\sigma_{\max}(L+B)}{\lambda_{\min}(Q)\,\sigma_{\min}(L+B)}\sqrt{\frac{2V(0)}{M_m}e^{-c_0 t} + \frac{2d_0}{c_0 M_m}}. \tag{49}$$

Moreover,

$$\begin{aligned}\left\|\dot{q}_i - \dot{q}_0\right\| &\le \left\|e_i\right\| + \left\|s_i\right\|\\ &\le \left(1 + \frac{2\sigma_{\max}(P_0)\,\sigma_{\max}(L+B)}{\lambda_{\min}(Q)}\right)\\ &\quad\times\sqrt{\frac{2V(0)}{M_m}e^{-c_0 t} + \frac{2d_0}{c_0 M_m}}.\end{aligned} \tag{50}$$

From (49) and (50), we can see that the position and velocity tracking errors are cooperatively UUB and the ultimate bounds can be made as small as possible by appropriately tuning the design parameters. □

Remark 12. The work in [11] requires that the communication topologies among the followers are undirected. We extend these results to the case that the communication graph among the followers can be a directed graph, which is a more general case. The works in [22–24] considered the centralized fuzzy logic systems or the centralized tracking problem. Different from these works [22–24], we considered the decentralized systems, where the controller design and the systems analysis are more involved.

3.3. Fault-Tolerant Controller Design under Actuator Faults. From the above analysis, the proposed control law in (28) can guarantee arbitrary small tracking errors for the closed-loop system with fault-free actuators. This controller is called the nominal controller and denoted by $\tau_{\text{nor}(i)}$. To perform tracking control with actuator faults as defined by (10), an auxiliary controller $\tau_{\text{aux}(i)}$ is developed in addition to the nominal controller $\tau_{\text{nor}(i)}$, to compensate for the actuator faults. Consequently, the control input τ_i to the system shown in (10) is designed as

$$\tau_i = \tau_{\text{nor}(i)} + \tau_{\text{aux}(i)}, \tag{51}$$

where $\tau_{\text{nor}(i)}$ is given by (28) and the fault-tolerant controller $\tau_{\text{aux}(i)}$ is synthesized as

$$\tau_{\text{aux}(i)} = -\begin{pmatrix}\dfrac{k_{i1}\tanh\left(k_{i1}e_{i1}/\varsigma_{i1}\right)}{\overline{\sigma}_{i1}}\\ \vdots\\ \dfrac{k_{im}\tanh\left(k_{im}e_{im}/\varsigma_{im}\right)}{\overline{\sigma}_{im}}\end{pmatrix} - \begin{pmatrix}\dfrac{\tau_{\text{nor}(i1)}\tanh\left(\tau_{\text{nor}(i1)}e_{i1}/\upsilon_{i1}\right)}{\overline{\sigma}_{i1}}\\ \vdots\\ \dfrac{\tau_{\text{nor}(im)}\tanh\left(\tau_{\text{nor}(im)}e_{im}/\upsilon_{im}\right)}{\overline{\sigma}_{im}}\end{pmatrix}, \tag{52}$$

with the positive design parameters ς_{ij} and υ_{ij}, $j = 1, 2, \ldots, m$.

Theorem 13. *Consider the networked Lagrange systems with the bias actuator faults and the partial loss of actuator effectiveness as defined by* (10). *Under the control law provided in* (51) *and the adaptive law given in* (29), *the cooperative tracking errors can be made as small as possible by suitably tuning the controller parameters.*

Proof. Substituting the fault-tolerant controller (51) into (25) gives

$$\begin{aligned}M_i\dot{e}_i + C_i e_i &= \left(\sigma_i\tau_i + f_i + \delta_i\right) + F_i\left(x_i\right) - M_i\dot{z}_i\\ &= \tau_{\text{nor}(i)} + \sigma_i\tau_{\text{aux}(i)} - \rho_i\tau_{\text{nor}(i)} + f_i\\ &\quad + \delta_i + F_i\left(x_i\right) - M_i\dot{z}_i,\end{aligned} \tag{53}$$

where $\rho_i = I - \sigma_i$. The inequality $0 \le \|\rho_i\|_\infty < 1$ always holds.

The candidate Lyapunov function V defined in Theorem 11 is also used here. Substituting (53) into the time derivative of V yields

$$\begin{aligned}\dot{V} &= \sum_{i=1}^{n}\left(e_i^T M_i\dot{e}_i + e_i^T C_i e_i\right) - \sum_{i=1}^{n}\frac{1}{\beta_i}\operatorname{tr}\left\{\widetilde{\theta}_i^T\dot{\widehat{\theta}}_i\right\}\\ &= \sum_{i=1}^{n}e_i^T\left(\tau_{\text{nor}(i)} + \sigma_i\tau_{\text{aux}(i)} - \rho_i\tau_{\text{nor}(i)}\right.\\ &\quad\left. + f_i + \delta_i + F_i\left(x_i\right) - M_i\dot{z}_i\right)\\ &\quad - \sum_{i=1}^{n}\frac{1}{\beta_i}\operatorname{tr}\left\{\widetilde{\theta}_i^T\dot{\widehat{\theta}}_i\right\}.\end{aligned} \tag{54}$$

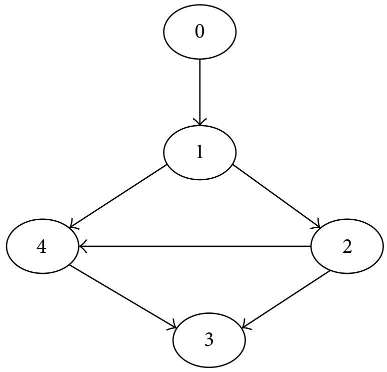


Figure 1: The communication topology.

Applying the same analysis as that of Theorem 11, one has

$$\begin{aligned}\dot{V} &\leq -c_0 V + d_0 + \sum_{i=1}^{n} e_i^T \left(\sigma_i \tau_{\mathrm{aux}(i)} - \rho_i \tau_{\mathrm{nor}(i)} + f_i\right) \\ &\leq -c_0 V + d_0 \\ &\quad + \sum_{i=1}^{n}\sum_{j=1}^{m} e_{ij} \left(\sigma_{ij} \tau_{\mathrm{aux}(ij)} - \rho_{ij} \tau_{\mathrm{nor}(ij)} + f_{ij}\right).\end{aligned} \tag{55}$$

In light of $|\rho_{ij}| < 1$ and Assumption 2, we have

$$\begin{aligned}\dot{V} &\leq -c_0 V + d_0 + \sum_{i=1}^{n}\sum_{j=1}^{m} e_{ij}\sigma_{ij}\tau_{\mathrm{aux}(ij)} \\ &\quad + \sum_{i=1}^{n}\sum_{j=1}^{m} \left|e_{ij}\right| \left|\tau_{\mathrm{nor}(ij)}\right| + \sum_{i=1}^{n}\sum_{j=1}^{m} \left|e_{ij}\right| \left|k_{ij}(t)\right|.\end{aligned} \tag{56}$$

Furthermore,

$$\begin{aligned}\dot{V} &\leq -c_0 V + d_0 + \sum_{i=1}^{n}\sum_{j=1}^{m} e_{ij}\sigma_{ij}\tau_{\mathrm{aux}(ij)} \\ &\quad + \sum_{i=1}^{n}\sum_{j=1}^{m} e_{ij}\tau_{\mathrm{nor}(ij)} \tanh\left(\frac{e_{ij}\tau_{\mathrm{nor}(ij)}}{\upsilon_{ij}}\right) \\ &\quad + \sum_{i=1}^{n}\sum_{j=1}^{m} \upsilon_{ij} + \sum_{i=1}^{n}\sum_{j=1}^{m} e_{ij}k_{ij} \tanh\left(\frac{e_{ij}k_{ij}}{\varsigma_{ij}}\right) \\ &\quad + \sum_{i=1}^{n}\sum_{j=1}^{m} \varsigma_{ij}.\end{aligned} \tag{57}$$

In light of (52), we have

$$\begin{aligned}\dot{V} &\leq -c_0 V + d_0 + \sum_{i=1}^{n}\sum_{j=1}^{m} e_{ij}\sigma_{ij} \\ &\quad \times \left(-\frac{k_{ij}\tanh\left(e_{ij}k_{ij}/\varsigma_{ij}\right)}{\overline{\sigma}_{ij}} - \frac{\tau_{\mathrm{nor}(ij)}\tanh\left(e_{ij}\tau_{\mathrm{nor}(ij)}/\upsilon_{ij}\right)}{\overline{\sigma}_{ij}}\right) \\ &\quad + \sum_{i=1}^{n}\sum_{j=1}^{m} e_{ij}\tau_{\mathrm{nor}(ij)} \tanh\left(\frac{e_{ij}\tau_{\mathrm{nor}(ij)}}{\upsilon_{ij}}\right) \\ &\quad + \sum_{i=1}^{n}\sum_{j=1}^{m} \upsilon_{ij} + \sum_{i=1}^{n}\sum_{j=1}^{m} e_{ij}k_{ij} \tanh\left(\frac{e_{ij}k_{ij}}{\varsigma_{ij}}\right) \\ &\quad + \sum_{i=1}^{n}\sum_{j=1}^{m} \varsigma_{ij}.\end{aligned} \tag{58}$$

In light of Assumption 2, we have

$$\begin{aligned}\dot{V} &\leq -c_0 V + d_0 + \sum_{i=1}^{n}\sum_{j=1}^{m} \upsilon_{ij} + \sum_{i=1}^{n}\sum_{j=1}^{m} \varsigma_{ij} \\ &\leq -c_0 V + d_1,\end{aligned} \tag{59}$$

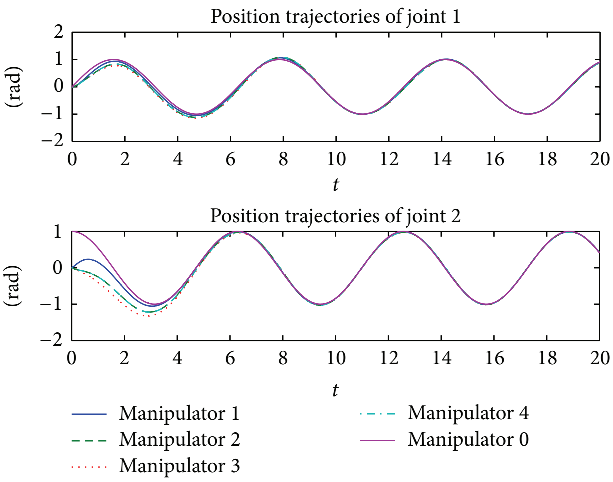
with $d_1 = d_0 + \sum_{i=1}^{n}\sum_{j=1}^{m} \varsigma_{ij} + \sum_{i=1}^{n}\sum_{j=1}^{m} \upsilon_{ij}$.

Following the same procedure as that of Theorem 11, we get that the local tracking errors are cooperatively UUB. □

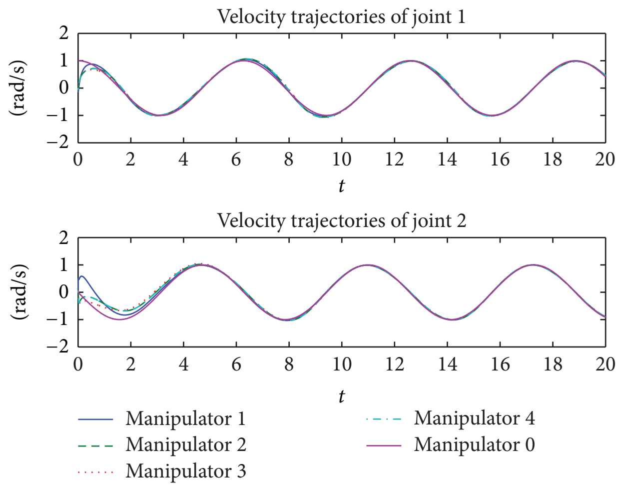
Remark 14. The works in [1–12] do not consider the actuator faults. We consider the actuator faults and present a fault-tolerant control scheme. The proposed fault-tolerant controller consists of the nominal controller (28) and the auxiliary controller (52). Different from the works [16–19], the fault detection and diagnosis mechanism is not required here. Thus, the proposed algorithm reduces the computation burden and decreases the response time of the controller. Compared with the work in [20], the proposed control law does not use the sign function. It avoids the chattering phenomenon.

4. Simulation Results

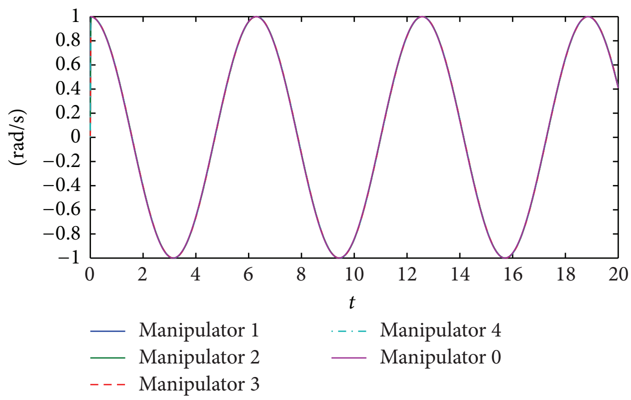
In this section, we present the simulation results to examine the performance of the proposed control scheme. The networked systems considered here consist of four standard two-DOF manipulators interconnected on a directed graph containing a spanning tree (see Figure 1). The manipulator



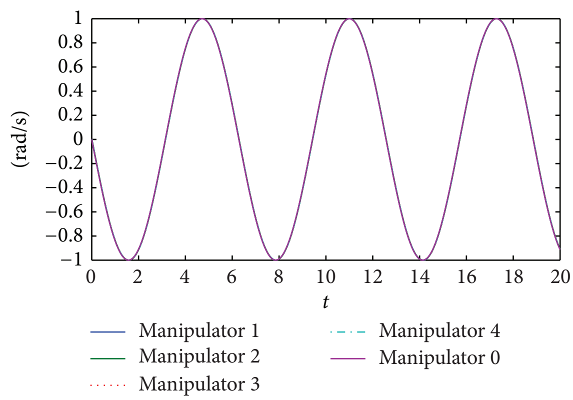
(a) The desired and actual position trajectories of the manipulators



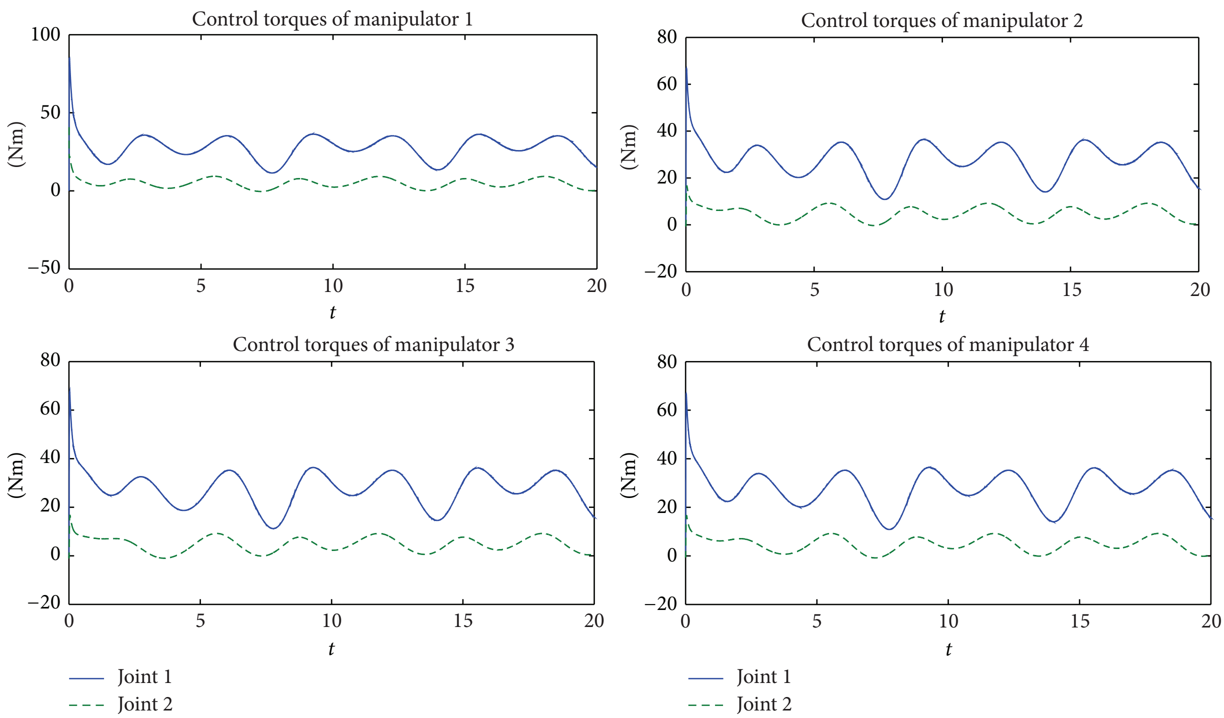
(b) The desired and actual velocity trajectories of the manipulators



(c) The estimated values for $\dot{q}_0$ at joint 1

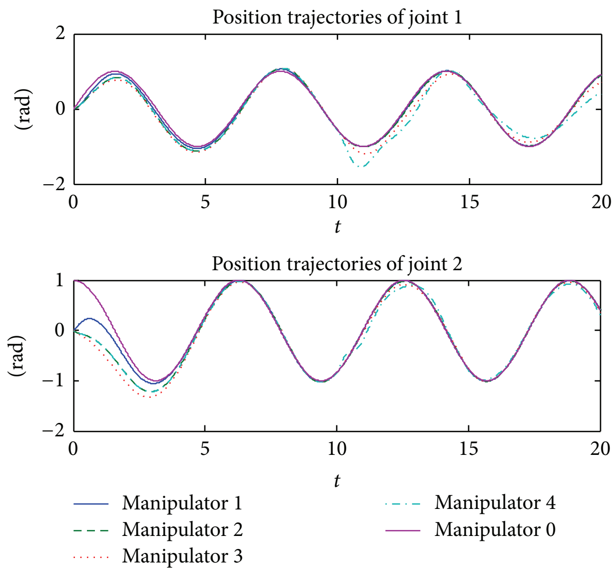


(d) The estimated values for $\dot{q}_0$ at joint 2

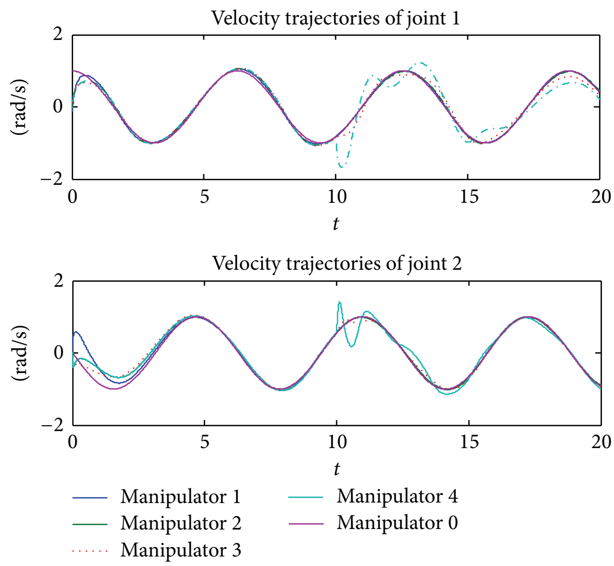


(e) The control inputs of each manipulator

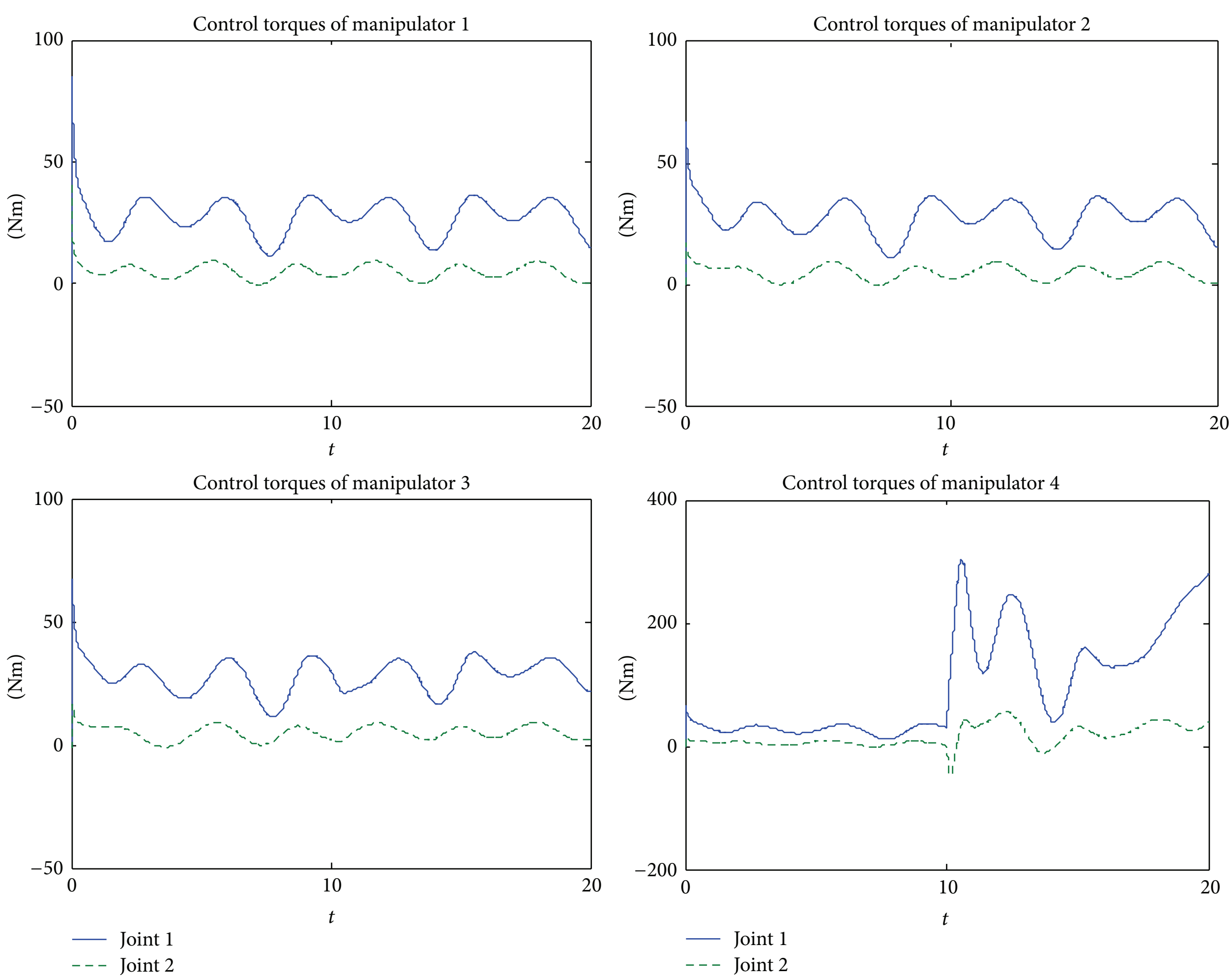
FIGURE 2: Simulation results with healthy actuators.



(a) The desired and actual position trajectories of the manipulators

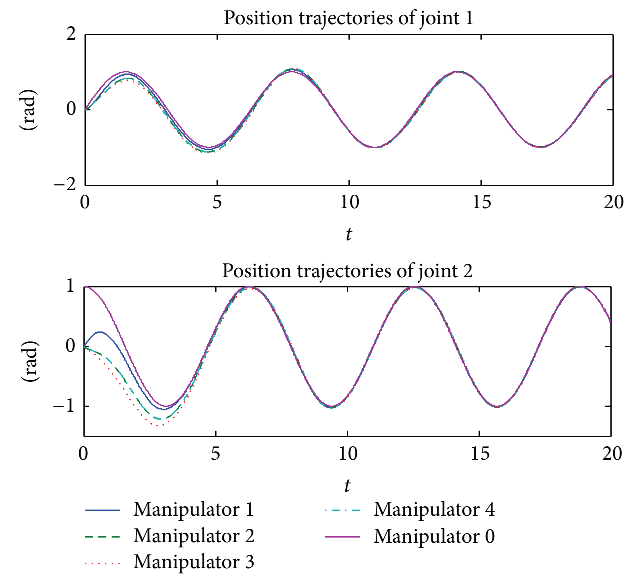


(b) The desired and actual velocity trajectories of the manipulators

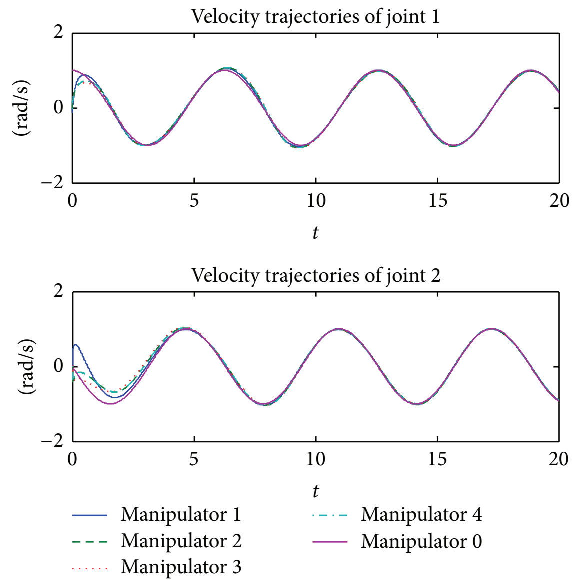


(c) The control inputs of each manipulator

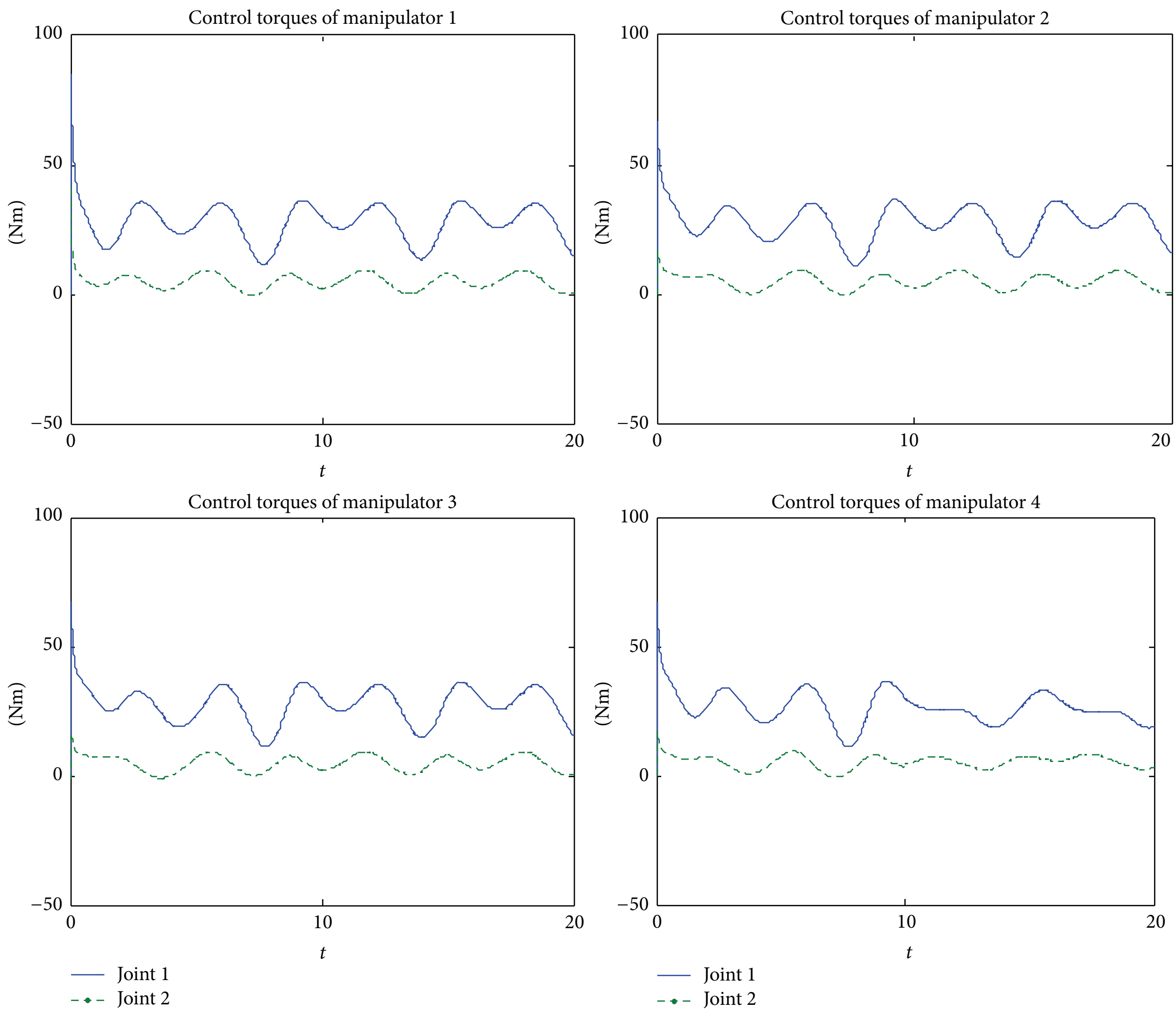
FIGURE 3: Simulation results with the actuator faults under the nominal controller.



(a) The desired and actual position trajectories of the manipulators



(b) The desired and actual velocity trajectories of the manipulators



(c) The control inputs of each manipulator

FIGURE 4: Simulation results with the actuator faults under the FTC.

consists of two serially connected links. The dynamics are described by (9) with

$$M_i(q_i) = \begin{bmatrix} p_1 + p_2 + 2p_3\cos(q_{i2}) & p_2 + p_3\cos(q_{i2}) \\ p_2 + p_3\cos(q_{i2}) & p_2 \end{bmatrix},$$
$$C_i(q_i,\dot{q}_i) = \begin{bmatrix} -p_3\dot{q}_{i2}\sin(q_{i2}) & -p_3(\dot{q}_{i1}+\dot{q}_{i2})\sin(q_{i2}) \\ -p_3\dot{q}_{i1}\sin(q_{i2}) & 0 \end{bmatrix},$$
$$G_i(q_i) = \begin{bmatrix} p_4 g\cos(q_{i1}) + p_5 g\cos(q_{i1}+q_{i2}) \\ p_5 g\cos(q_{i1}+q_{i2}) \end{bmatrix}, \quad (60)$$

where $p = [p_1, p_2, p_3, p_4, p_5] = [2.9, 0.76, 0.87, 0.34, 0.87]$.

Assume that the dynamics of the manipulators are unknown. Fuzzy logic systems are used to approximate the unknown dynamics. Choose the fuzzy membership functions as

$$\mu_{A_i^1}(x_i) = \exp\left(-(x_i+1)^2\right), \qquad \mu_{A_i^2}(x_i) = \exp\left(-x_i^2\right),$$
$$\mu_{A_i^3}(x_i) = \exp\left(-(x_i-1)^2\right), \quad (61)$$

where x_i is a variable which denotes the position or velocity of the manipulator.

The design parameters for the controller and adaptive laws are chosen as $K_i = \begin{bmatrix} 80 & 0 \\ 0 & 40 \end{bmatrix}$, $\beta_i = 500$, $\eta_i = 0.01$, and $\mu_i = 1$. The initial conditions are set as $q_i(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $\dot{q}_i(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, and $\widehat{\theta}_i(0) = 0$.

Case 1: *Healthy Actuators*. This case simulates the ideal situation when no actuator faults occur. Figure 2 provides the simulation results for the nominal controller (NC) given by (28). Figures 2(c) and 2(d) show the finite-time convergence of the velocity observers. Figures 2(a) and 2(b) demonstrate that the high control precision and an acceptable system performance have been achieved.

Case 2: *Faulty Actuators*. This example represents the severe case in which both the bias faults and the partial loss of actuator effectiveness occur. Assume that the actuators of the networked systems undergo a partial loss of effectiveness and a bias fault at $t = 10s$. Let $\overline{\sigma}_i = 0.02$, $k_{ij}(t) = 2$, and $v_{ij} = \varsigma_{ij} = 0.5$ $(i = 1, 2, 3, 4, j = 1, 2)$. In the simulation, the partial loss coefficient and the bias faults are set as

$$\sigma_{im} = \begin{cases} 1 & t < 10s \\ 0.15 + 0.1\sin(0.5t) & t \geq 10s \end{cases} \quad (m = 1, 2),$$
$$f_{im} = \begin{cases} 0 & t < 10s \\ 1 + 0.5\sin(0.2t) & t \geq 10s \end{cases} \quad (m = 1, 2), \quad (62)$$

respectively.

Figures 3 and 4 show the results using the two different control laws based on the same simulation conditions. From Figures 3(a) and 3(b), we see that the effects of the actuator faults propagate to the Lagrangian dynamics and cannot be compensated by the nominal controllers. Significant degradation of the control performance and system instability after the faults can be observed. In contrast, Figures 4(a) and 4(b) show that the FTC controller can achieve the objective of fault-tolerant control and does succeed in compensating for the fault simultaneously.

In summary, for the healthy case, the proposed controller achieves the desired control performance of the closed-loop Lagrange system. For the cases with actuator faults, the proposed method can compensate for the effects of the faults. As the faults become more severe, the proposed controller still guarantees system stability.

5. Conclusion

A distributed fault-tolerant tracking control scheme was presented for networked uncertain Euler-Lagrange systems which may experience the bias faults and the partial loss of actuator effectiveness. Using finite-time observer based method and Lyapunov analysis we have derived stable adaptive protocol which consists of the nominal control input and the auxiliary control input, where the former achieved the distributed tracking control of healthy systems while the latter compensated for the effect of actuator faults. The proposed scheme was suitable for the general directed communication topology. The simulation results presented in this paper showed that the control scheme can successfully handle the model uncertainties and the unknown actuators faults.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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