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Research Article

Exact CS Reconstruction Condition of Undersampled Spectrum-Sparse Signals

Ying Luo,[1] Qun Zhang,[1,2] Guozheng Wang,[3] and Youqing Bai[3]

[1] *Institute of Information and Navigation, Air Force Engineering University, Xi'an 710077, China*
[2] *Key Laboratory for Information Science of Electromagnetic Waves, Ministry of Education, Fudan University, Shanghai 200433, China*
[3] *Institute of Science, Air Force Engineering University, Xi'an 710051, China*

Correspondence should be addressed to Qun Zhang; zhangqunnus@gmail.com

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Compressive sensing (CS) reconstruction of a spectrum-sparse signal from undersampled data is, in fact, an ill-posed problem. In this paper, we mathematically prove that, in certain cases, the exact CS reconstruction of a spectrum-sparse signal from undersampled data is impossible. Then we present the exact CS reconstruction condition of undersampled spectrum-sparse signals, which is valuable for digital signal compression.

1. Introduction

In digital signal processing, the Nyquist sampling theorem indicates that the sampling rate must be twice as large as the bandwidth of the analog signal at least for acquiring the intact information of the signal. Restricted by the theorem, it is a challenge to digitize ultrawide bandwidth (UWB) signals because of the unfeasible high sampling rate requirement for the analog-to-digital converter (ADC). On the other hand, the mass sampling data have to be compressed to save the storage, which means that many data are abandoned in the compression processing. Hence, why not to obtain the compressed data of signals directly rather than to sample signal with ultrahigh rate and then abandon most of the samplings?

The emerging *compressive sensing* (CS) theory [1] provides an effective approach to solve this problem, which has attracted much attention recently [2–7]. Consider a signal $\mathbf{x} \in \mathbf{C}^{N\times 1}$ and assume it is sparse on an orthogonal basis $\mathbf{\Psi} = \{\psi_i\}$ with K-sparse representation ($K \ll N$) as $\mathbf{x} = \mathbf{\Psi}\boldsymbol{\theta}$, where $\boldsymbol{\theta}$ is an $N \times 1$ column vector with K nonzero elements. Let $\mathbf{\Phi}$ denote a measurement matrix and let $\mathbf{y}$ be the measurements vector of signal $\mathbf{x}$; it can be expressed as $\mathbf{y} = \mathbf{\Phi}\mathbf{x} = \mathbf{\Phi}\mathbf{\Psi}\boldsymbol{\theta}$, where $\mathbf{\Phi}$ is a $M \times N$ matrix, M denotes the number of measurements, and $K < M \ll N$. Therefore, the sampling rate is reduced significantly compared with Nyquist rate. Generally, recovery of the signal $\mathbf{x}$ from the measurements $\mathbf{y}$ is ill-posed because $M \ll N$ [8]. However, the CS theory demonstrates that if $\mathbf{\Phi}\mathbf{\Psi}$ has the *Restricted Isometry Property* (RIP), then it is indeed possible to recover the K largest $\theta_x(i)$ when M is large enough [3, 9]. It is difficult to validate if a measurement matrix satisfies the RIP constraints given in [9] directly, but fortunately, the RIP is closely related to an incoherency between $\mathbf{\Phi}$ and $\mathbf{\Psi}$, where the rows of $\mathbf{\Phi}$ do not provide a sparse representation of the columns of $\mathbf{\Psi}$ and vice versa [4]. Furthermore, to ensure exact reconstruction, two different K-sparse signals may not be projected by a measurement matrix into the same sampling ensemble [1, 2, 10].

When $\mathbf{x}$ is sparse in spectrum, the N-dimensional inverse discrete Fourier transform (IDFT) matrix ($\mathbf{D}_N^{-1}$) can be chosen as the sparse representation matrix ($\mathbf{\Psi}$). In this case, an easy way to obtain the compressed data of the signal is to undersample the signal with lower sampling rate than the Nyquist rate [2]. Therefore the measurement matrix is in fact a partial unit matrix [11]. It is important to investigate the mathematical properties of compressive sensing reconstruction for this kind of undersampled spectrum-sparse signal. In the following, we mathematically prove that the exact CS reconstruction of a spectrum-sparse signal from undersampled data is impossible under certain conditions. In order to reconstruct a spectrum-sparse signal from undersampled data exactly, the corresponding exact CS reconstruction condition is presented, which is valuable for digital signal compression.

2. Inexact CS Reconstruction Cases

When a signal $\mathbf{x}$ is sparse in spectrum, the IDFT matrix $\mathbf{D}_N^{-1}$ and partial unit matrix can be chosen as the sparse representation matrix and the measurement matrix, respectively. By defining the downsampling rate r to be the ratio between the Nyquist rate and the undersampling rate, an inexact CS reconstruction case can be depicted as the following theorem.

Theorem 1. *Suppose* $\mathbf{x} \in \mathbf{C}^{N\times 1}$ *with Nyquist sampling rate* f_s *is* K*-sparse in spectrum domain;* $\mathbf{y} = \{x_{mr+b} \mid mr + b \le N,\ m > 0,\ m \in \mathbb{N}\}$ *is an arbitrary subset of* $\mathbf{x}$*, where* r *is the downsampling rate,* $r \ge 2$*, and* $r \in \mathbb{N}$*,* b *is a constant and* $b \in \mathbb{N}$*, and* $\mathbb{N}$ *is the set of all natural numbers.* $\mathbf{x}$ *cannot be exactly reconstructed from* $\mathbf{y}$ *by CS.*

Proof. According to the expression of $\mathbf{y}$, we have

$$\mathbf{y}_{M\times 1} = \mathbf{\Phi}_{M\times N}\mathbf{x}_{N\times 1} = \mathbf{\Phi}_{M\times N}\mathbf{\Psi}_{N\times N}\boldsymbol{\theta}_{N\times 1}, \tag{1}$$

where $\boldsymbol{\theta}$ is K-sparse; $\mathbf{\Phi}$ is the measurement matrix with size $M \times N$:

$$\begin{array}{c} \text{Column index} \quad 1 \qquad\qquad r+1 \qquad\qquad 2r+1 \quad (M-1)r+1 \\ \downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow \end{array}$$

$$\mathbf{\Phi}_{M\times N} = \begin{bmatrix} 1 & 0 & \cdots & & & & & & & 0 \\ 0 & 0 & \cdots & 0 & 1 & 0 & \cdots & & & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 1 & \\ \vdots & \vdots & \vdots & & & & & & & \\ 0 & 0 & \cdots & & & & & & 0 \ \cdots & 1 \end{bmatrix},$$

$$\mathbf{\Psi}_{N\times N} = \mathrm{D}_N^{-1} = \frac{1}{N}\begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & W_N^{-1} & W_N^{-2} & \cdots & W_N^{(N-1)} \\ 1 & W_N^{-2} & W_N^{-4} & \cdots & W_N^{-2(N-1)} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & W_N^{-(N-1)} & W_N^{-2(N-1)} & \cdots & W_N^{-(N-1)^2} \end{bmatrix},$$

$$W_N = \exp\left(-j\frac{2\pi}{N}\right). \tag{2}$$

Equation (1) is equivalent to

$$\boldsymbol{\theta}'_{M\times 1} = \mathbf{D}_M\mathbf{y}_{M\times 1} = \mathbf{D}_M\mathbf{\Phi}_{M\times N}\mathbf{D}_N^{-1}\boldsymbol{\theta}_{N\times 1}, \tag{3}$$

where $\mathbf{D}_M$ is the M-dimensional DFT matrix. Because $\mathbf{y}$ is isometrically downsampled from $\mathbf{x}$, $\boldsymbol{\theta}'$ is also sparse. In (3), we define a new measurement matrix $\mathbf{\Phi}_1 = \mathbf{D}_M\mathbf{\Phi}_{M\times N}$ and it can be expressed as

$$\begin{array}{c} \text{Column index} \quad 1 \qquad\quad r+1 \qquad\qquad 2r+1 \qquad\qquad (M-1)r+1 \\ \downarrow \qquad\quad \downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow \end{array}$$

$$\mathbf{\Phi}_1 = \begin{bmatrix} 1 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 & 1 \\ 1 & 0 & & 0 & W_M & 0 & & 0 & W_M^2 & 0 & & 0 & W_M^{M-1} \\ 1 & 0 & & 0 & W_M^2 & 0 & & 0 & W_M^4 & 0 & & 0 & W_M^{2(M-1)} \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1 & 0 & \cdots & 0 & W_M^{M-1} & 0 & \cdots & 0 & W_M^{2(M-1)} & 0 & \cdots & 0 & W_M^{(M-1)^2} \end{bmatrix}. \tag{4}$$

According to CS theory, if $\mathbf{\Phi}_1\mathbf{D}_N^{-1}$ satisfies the RIP, $\boldsymbol{\theta}$ can be exactly reconstructed. Let $\boldsymbol{\phi}_{1a}$ be the ath row of $\mathbf{\Phi}_1$; we have

$$\begin{aligned} \boldsymbol{\phi}_{1a}^T &= \mathbf{D}_N^{-1}\mathbf{D}_N\boldsymbol{\phi}_{1a}^T = \mathbf{D}_N^{-1}\left(\mathbf{D}_N\boldsymbol{\phi}_{1a}^T\right) \\ &= \mathbf{D}_N^{-1}\left[0, \frac{1 - W_N^{Mr}}{1 - W_M^{a-1}W_N^{r}}, \dots, \frac{1 - W_N^{(N-1)Mr}}{1 - W_M^{a-1}W_N^{(N-1)r}}\right]^T, \end{aligned} \tag{5}$$

where $W_M = \exp(-j2\pi/M)$. Because $\mathrm{Rank}(\mathbf{D}_N^{-1}) = N$, according to *Cramer's Rule*, the equation $\mathbf{D}_N^{-1}\boldsymbol{\alpha} = \boldsymbol{\phi}_{1a}^T$ has unique solution; that is,

$$\boldsymbol{\alpha} = \left[0, \frac{1 - W_N^{Mr}}{1 - W_M^{a-1}W_N^{r}}, \dots, \frac{1 - W_N^{(N-1)Mr}}{1 - W_M^{a-1}W_N^{(N-1)r}}\right]^T. \tag{6}$$

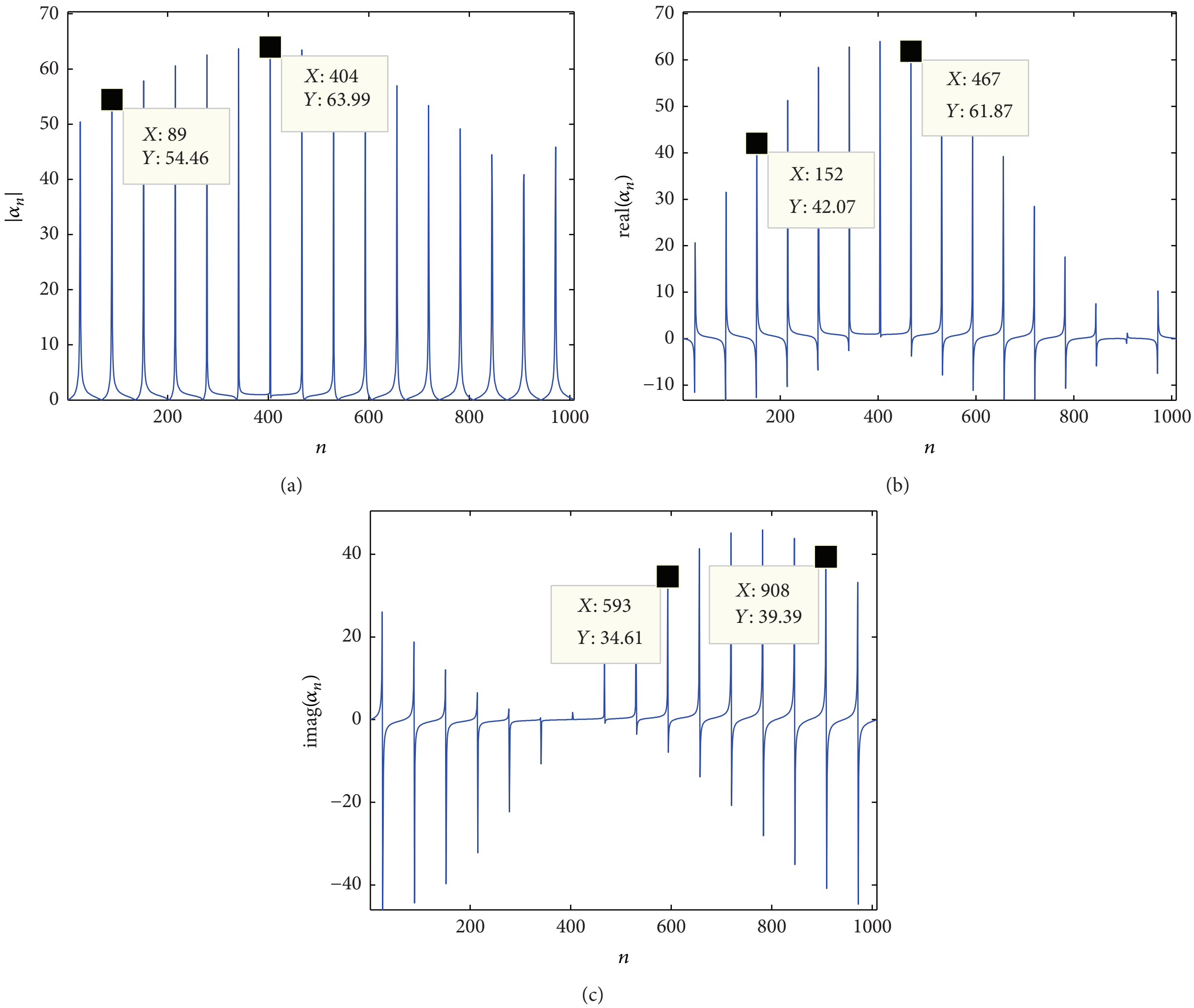

Figure 1: The sparsity of α_n when $N = 1024$, $M = 64$, $r = 16$, and $a = 40$. (a) $\{|\alpha_n|\}$; (b) $\{\text{real}(\alpha_n)\}$; (c) $\{\text{imag}(\alpha_n)\}$.

Let α_n be the nth element of $\boldsymbol{\alpha}$; it can be obtained

$$|\alpha_n| = \left|\frac{1 - W_N^{(n-1)Mr}}{1 - W_M^{a-1} W_N^{(n-1)r}}\right| = \frac{1 - \cos((2\pi/N)(n-1)Mr)}{1 - \cos(2\pi((a-1)/M + ((n-1)r)/N))}. \quad (7)$$

It peaks at

$$\frac{(n-1)r}{N} = -\frac{a-1}{M} + l, \quad l \in \mathbb{Z}, \quad (8)$$

where $\mathbb{Z}$ is set of all nonnegative integers. Because $N = (M-1)r + 1$, we have

$$n = -a + 2 + \frac{(r-1)(a-1)}{Mr} + l\left(M - 1 + \frac{1}{r}\right) \quad (9)$$

and n is a natural number; hence

$$n = \left[-a + 2 + \frac{(r-1)(a-1)}{Mr} + l\left(M - 1 + \frac{1}{r}\right)\right], \quad (10)$$

where $[(\cdot)]$ returns the round of $(\cdot)$. When n does not satisfy (10), $|\alpha_n|$ is relatively quite small. It indicates that $\{|\alpha_n|\}$ is sparse. Similarly, it can be proved that both $\{\text{real}(\alpha_n)\}$ and $\{\text{imag}(\alpha_n)\}$ are also sparse. Therefore, $\boldsymbol{\phi}_{1a}$ can be sparsely represented by the columns of $\mathbf{D}_N^{-1}$; that is, $\boldsymbol{\Phi}_1\mathbf{D}_N^{-1}$ does not satisfy the RIP, and $\boldsymbol{\theta}$ cannot be exactly reconstructed. □

Figure 1 shows the values of $\{|\alpha_n|\}$, $\{\text{real}(\alpha_n)\}$, and $\{\text{imag}(\alpha_n)\}$ when $N = 1024$, $M = 64$, $r = 16$, and $a = 40$. It is obvious that $\{|\alpha_n|\}$, $\{\text{real}(\alpha_n)\}$, and $\{\text{imag}(\alpha_n)\}$ are sparse and the locations of peaks agree well with the theoretical values determined by (10).

By Theorem 1 and its proof, we can also obtain the following corollary.

Corollary 2. *Suppose* $\mathbf{x} \in \mathbf{C}^{N\times 1}$ *with Nyquist sampling rate* f_s *is K-sparse in spectrum domain;* $\mathbf{x}' = \{x_{mr+b} \mid mr + b \le N,\ m > 0,\ m \in \mathbb{N}\}$ *is an arbitrary subset of* $\mathbf{x}$*, where* r *is the downsampling rate,* $r \ge 2$*, and* $r \in \mathbb{N}$ *and* b *is a constant and*

$b \in \mathbb{N}$. *Let* $\mathbf{y}$ *be an arbitrary subset of* $\mathbf{x}'$; *then* $\mathbf{x}$ *cannot be exactly reconstructed from* $\mathbf{y}$ *by CS.*

Proof. Assume the length of $\mathbf{x}'$ is M and we have

$$\mathbf{y} = \mathbf{\Phi}_2\mathbf{x}' = \mathbf{\Phi}_2\mathbf{\Phi}\mathbf{D}_N^{-1}\boldsymbol{\theta}, \tag{11}$$

where $\mathbf{\Phi}_2$ is a partial unit matrix. Assuming the size of $\mathbf{\Phi}_2$ is $L\times M$ $(L \le M)$, the solution set of (1) is a subset of the solution set of (11). According to Theorem 1, the solutions of (1) are not determined; therefore, $\boldsymbol{\theta}$ in (11) cannot be determined and $\mathbf{x}$ cannot be exactly reconstructed. □

Corollary 2 indicates that, if the set of undersamplings is only a subset of the set of signals' samplings with sampling rate lower than the Nyquist rate, the signal cannot be exactly reconstructed from these undersamplings by CS. Therefore, when designing the ADC with random sampling space, the ADC should better possess the capability with the minimum sampling space of $1/f_s$, where f_s is the Nyquist rate. Taken in this sense, the high sampling rate requirement for ADC is indeed not suppressed even though the CS theory is utilized.

3. Exact CS Reconstruction Condition

In the following, we present the exact CS reconstruction condition of undersampled spectrum-sparse signals.

Theorem 3. *Suppose* $\mathbf{x} \in \mathbf{C}^{N\times 1}$ *with Nyquist sampling rate* f_s *is K-sparse in spectrum domain; and the frequency indexes of K nonzero points in spectrum are* f_h $(h = 1, 2, \ldots, K)$. *Undersampling* $\mathbf{x}$ *with rate* $f_s/r_1, f_s/r_2, \ldots, f_s/r_i, \ldots, f_s/r_I$ $(i = 1, 2, \ldots, I,\ r_i \ge 2,$ *and* $r_i \in \mathbb{N})$, *respectively, all the samples consist of* $\mathbf{y}_{M\times 1}$. *The necessary and sufficient condition for* $\mathbf{x}$ *exactly reconstructed from* $\mathbf{y}$ *by CS is*

$$\begin{aligned}
&\left\{f_h + \frac{k_1 f_s}{r_1} \mid h = 1, 2, \ldots, K; k_1 \in \mathbb{Z}; \left|f_h + \frac{k_1 f_s}{r_1}\right| \le \frac{f_s}{2}\right\} \\
&\quad \cap \left\{f_h + \frac{k_2 f_s}{r_2} \mid h = 1, 2, \ldots, K; k_2 \in \mathbb{Z}; \left|f_h + \frac{k_2 f_s}{r_2}\right| \le \frac{f_s}{2}\right\} \\
&\quad \cap \cdots \\
&\quad \cap \left\{f_h + \frac{k_I f_s}{r_I} \mid h = 1, 2, \ldots, K; k_I \in \mathbb{Z}; \left|f_h + \frac{k_I f_s}{r_I}\right| \le \frac{f_s}{2}\right\} \\
&\quad = \left\{f_h \mid h = 1, 2, \ldots, K\right\}.
\end{aligned} \tag{12}$$

Proof. Reconstructing $\mathbf{x}$ from $\mathbf{y}$ is in fact to solve the following underdetermined equation system:

$$\begin{aligned}
\mathbf{y}_1 &= \mathbf{\Phi}_1\mathbf{x} = \mathbf{\Phi}_1\mathbf{D}_N^{-1}\boldsymbol{\theta}, \\
\mathbf{y}_2 &= \mathbf{\Phi}_2\mathbf{x} = \mathbf{\Phi}_2\mathbf{D}_N^{-1}\boldsymbol{\theta}, \\
&\ \ \vdots \\
\mathbf{y}_I &= \mathbf{\Phi}_I\mathbf{x} = \mathbf{\Phi}_I\mathbf{D}_N^{-1}\boldsymbol{\theta},
\end{aligned} \tag{13}$$

where $\mathbf{\Phi}_i$ is the measurement matrix according to the undersampling rate f_s/r_i; $\mathbf{y}_i$ is composed of the undersamplings of $\mathbf{x}$ with rate f_s/r_i. Assume the frequency indexes of the solution set of the ith equation are $\{\widehat{f}_{hi}\}$; because $\mathbf{y}_i$ is the isometric downsampling from $\mathbf{x}$, we have

$$\begin{aligned}
&\left\{\widehat{f}_{hi}\right\} \\
&\quad \subseteq \left\{f_h + \frac{k_i f_s}{r_i} \mid h = 1, 2, \ldots, K; k_i \in \mathbb{Z}; \left|f_h + \frac{k_i f_s}{r_i}\right| \le \frac{f_s}{2}\right\}.
\end{aligned} \tag{14}$$

Hence, the solution set of (13) is

$$\begin{aligned}
\cap\left\{\widehat{f}_{hi}\right\} \subseteq \cap &\left\{f_h + \frac{k_i f_s}{r_i} \mid h = 1, 2, \cdots, K; k_i \in \mathbb{Z};\right. \\
&\left.\left|f_h + \frac{k_i f_s}{r_i}\right| \le \frac{f_s}{2}\right\}.
\end{aligned} \tag{15}$$

Only if (12) holds true, under the constraint of K-sparse, it yields $\cap\{\widehat{f}_{hi}\} = \{f_h \mid h = 1, 2, \ldots, K\}$; therefore, $\mathbf{x}$ is exactly reconstructed. □

Theorem 3 indicates that it is possible to reconstruct a spectrum-sparse signal exactly from its multirate downsamplings. It means that, when condition (12) is satisfied, the multirate downsampling can be used to compress the data of digital signals. The conclusion is valuable to the design of the ADC when the analog signal is a priori known sparse in spectrum.

In order to validate Theorem 3, an experiment is given as follows. In the experiment, the signal $x(t)$ with Nyquist sampling rate $f_s = 1\,\text{Hz}$ is expressed as $x(t) = \exp(0.2 \times 2\pi t) + 0.8\exp(0.45 \times 2\pi t) + 0.6\exp(0.3 \times 2\pi t)$, $t \in [0, 1023]$. The sparsity of $x(t)$ in spectrum domain is 3 and $f_h = \{0.2, 0.45, 0.3\}$. The spectrum of $x(t)$ is shown in Figure 2(a). Let $r_1 = 7$ and $r_2 = 10$. Therefore, we have

$$\begin{aligned}
&\left\{f_h + \frac{k_1 f_s}{r_1} \mid h = 1, 2, \cdots, K; k_1 \in \mathbb{Z}; \left|f_h + \frac{k_1 f_s}{r_1}\right| \le \frac{f_s}{2}\right\} \\
&\quad = \left\{0.2 + \frac{k_1}{7} \mid k_1 \in \mathbb{Z}; \left|0.2 + \frac{k_1}{7}\right| \le \frac{1}{2}\right\} \\
&\quad \cap \left\{0.45 + \frac{k_1}{7} \mid k_1 \in \mathbb{Z}; \left|0.45 + \frac{k_1}{7}\right| \le \frac{1}{2}\right\} \\
&\quad \cap \left\{0.3 + \frac{k_1}{7} \mid k_1 \in \mathbb{Z}; \left|0.3 + \frac{k_1}{7}\right| \le \frac{1}{2}\right\} \\
&\quad = \left\{0.2, 0.2 + \frac{1}{7}, 0.2 + \frac{2}{7}, 0.2 - \frac{1}{7}, 0.2 - \frac{2}{7},\right. \\
&\qquad 0.2 - \frac{3}{7}, 0.2 - \frac{4}{7}, 0.45, 0.45 - \frac{1}{7}, \\
&\qquad 0.45 - \frac{2}{7}, 0.45 - \frac{3}{7}, 0.45 - \frac{4}{7}, 0.45 - \frac{5}{7}, \\
&\qquad 0.45 - \frac{6}{7}, 0.3, 0.3 + \frac{1}{7}, 0.3 - \frac{1}{7}, 0.3 - \frac{2}{7}, \\
&\qquad \left.0.3 - \frac{3}{7}, 0.3 - \frac{4}{7}, 0.3 - \frac{5}{7}\right\},
\end{aligned}$$

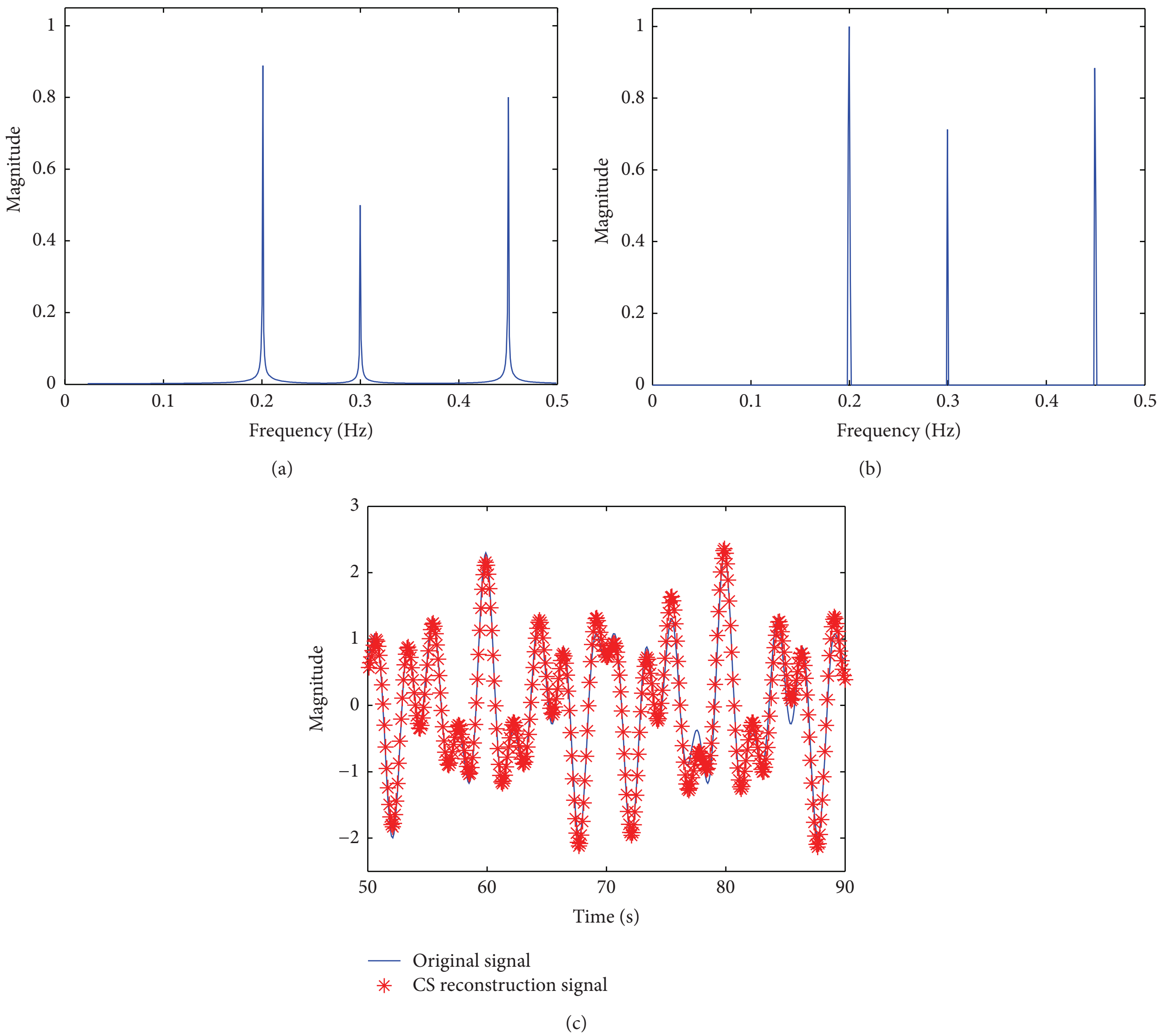

Figure 2: Experiment for validation of Theorem 3. (a) The spectrum of the signal $x(t)$ with Nyquist sampling rate; (b) the spectrum of the reconstructed signal by CS; (c) the comparison of $x(t)$ and the reconstructed signal in time domain.

$$\begin{aligned}
&\left\{f_h + \frac{k_2 f_s}{r_2} \mid h = 1, 2, \ldots, K; k_2 \in \mathbb{Z}; \left| f_h + \frac{k_2 f_s}{r_2} \right| \le \frac{f_s}{2}\right\} \\
&\quad = \left\{0.2 + \frac{k_2}{10} \mid k_2 \in \mathbb{Z}; \left| 0.2 + \frac{k_2}{10} \right| \le \frac{1}{2}\right\} \\
&\quad \cap \left\{0.45 + \frac{k_2}{10} \mid k_2 \in \mathbb{Z}; \left| 0.45 + \frac{k_2}{10} \right| \le \frac{1}{2}\right\} \\
&\quad \cap \left\{0.3 + \frac{k_2}{10} \mid k_2 \in \mathbb{Z}; \left| 0.3 + \frac{k_2}{10} \right| \le \frac{1}{2}\right\} \\
&\quad = \{-0.5, -0.4, -0.3, -0.2, -0.1, 0, 0.1, 0.2, 0.3, \\
&\qquad 0.4, 0.5, 0.45, 0.35, 0.25, 0.15, 0.05, -0.05, \\
&\qquad -0.15, -0.25, -0.35, -0.45\}.
\end{aligned} \tag{16}$$

Obviously, the intersection of (16) is $f_h = \{0.2, 0.45, 0.3\}$. Hence, $x(t)$ can be exactly reconstructed from the downsamplings. In the experiment, we choose the orthogonal matching pursuit (OMP) algorithm [12] to reconstruct $x(t)$ from the downsamplings. The spectrum of the reconstructed signal by CS is shown in Figure 2(b), which is very close to that in Figure 2(a). The comparison of the original signal $x(t)$ with Nyquist sampling rate and the reconstructed signal in time domain is also given in Figure 2(c). From the figure, it can be found that the reconstructed signal is close to the original signal, which validates the correctness of Theorem 3.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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