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Letter to the Editor

He's Max-Min Approach to a Nonlinear Oscillator with Discontinuous Terms

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Recently, the max-min approach was systematically studied in the review article (Ji-Huan, 2012). This paper concludes that He's max-min approach is also a very much effective method for nonlinear oscillators with discontinuous terms.

The ancient Chinese mathematics revives modern applications [1–8]; hereby, we show that He's max-min approach [1, 9–11] is also very effective for nonlinear oscillators with discontinuous terms.

The max-min approach was first proposed in 2008 based on an ancient Chinese mathematics, and it has become a well-known method for nonlinear oscillators; see, for example, [12–14].

To illustrate the basic idea of the max-min approach [1], we consider the following nonlinear oscillator:

$$u'' + \beta u^3 + \varepsilon u\,|u| = 0, \quad u(0) = A, \quad u'(0) = 0. \tag{1}$$

By a similar treatment as given in [1], we have

$$0 < \omega^2 < \beta A^2 + \varepsilon A, \tag{2}$$

where ω is the unknown frequency.

According to an ancient Chinese inequality [1, 8, 10, 11], we have

$$\begin{aligned} \omega^2 &= \frac{n\left(\beta A^2 + \varepsilon A\right)}{m+n} \\ &= k\left(\beta A^2 + \varepsilon A\right), \quad k = \frac{n}{(m+n)}, \end{aligned} \tag{3}$$

where m, n, and k are constants.

According to He's max-min approach, we set

$$\begin{gathered} \int_0^{T/4} \left(k\left(\beta A^2 + \varepsilon A\right) u - \beta u^3 - \varepsilon u\,|u|\right) \cos \omega t dt = 0, \\ T = \frac{2\pi}{\omega}, \end{gathered} \tag{4}$$

or

$$\begin{gathered} \int_0^{T/4} \left(k\left(\beta A^2 + \varepsilon A\right) A \cos \omega t - \beta A^3 \cos^3 \omega t \right. \\ \left. -\varepsilon A \cos \omega t\,|A \cos \omega t|\right) \cos \omega t dt = 0, \end{gathered} \tag{5}$$

from which the frequency ω can be determined approximately as

$$\omega = \sqrt{\frac{3}{4}\beta A^2 + \frac{8}{3\pi}\varepsilon A}, \tag{6}$$

which is the same as that obtained by the homotopy perturbation method [15].

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