

## Research Article

# Least-Squares-Based Iterative Identification Algorithm for Wiener Nonlinear Systems

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This paper focuses on the identification problem of Wiener nonlinear systems. The application of the key-term separation principle provides a simplified form of the estimated parameter model. To solve the identification problem of Wiener nonlinear systems with the unmeasurable variables in the information vector, the least-squares-based iterative algorithm is presented by replacing the unmeasurable variables in the information vector with their corresponding iterative estimates. The simulation results indicate that the proposed algorithm is effective.

## 1. Introduction

Wiener systems are typical nonlinear systems [1], which can represent a nonlinear dynamic system with a dynamic linear block followed by a nonlinear static function. Wiener systems have been used in modeling a glutamate fermentation process [2]. Recently, great attention has been paid to the identification issues for Wiener systems, and many studies have been performed. In much existing work, some assume that the nonlinear part of Wiener systems has an invertible function representation over the operating range of interest [3]. Wang and Ding presented least squares-based and gradient-based iterative identification algorithms for Wiener nonlinear systems [4]; Chen studied identification problems for Wiener systems with saturation and dead-zone nonlinearities [5]. Zhou et al. derived an auxiliary model-based gradient iterative algorithm for Wiener nonlinear output error systems by using the key-term decomposition principle [6].

The least-squares-based iterative algorithm is a class of basic identification algorithms in the field of system identification [7] and used in studying different types of systems, for example, multivariable systems [8–12] and multirate systems [13–16]. To estimate the parameters of systems in which the information vector contains unknown variables, the least-squares-based iterative algorithms are usually used [17–22].

Ding et al. developed a least-squares-based iterative algorithm to estimate the parameters for a multi-input multi-output system with colored ARMA noise from input-output data [23]. On the basis of the work in [6, 24–26], this paper presents a least-squares-based iterative estimation algorithm for Wiener nonlinear systems.

The rest of this paper is organized as follows. Section 2 derives the identification model of Wiener nonlinear systems. Section 3 presents a least squares based iterative algorithm for Wiener nonlinear systems. Section 4 provides an example to illustrate the effectiveness of the proposed algorithm. The conclusions of the paper are summarized in Section 5.

## 2. Problem Formation

Let us firstly introduce some notations [27, 28]. The superscript T denotes the matrix transpose;  $\mathbf{I}$  stands for an identity matrix of appropriate sizes;  $\mathbf{1}_n$  represents an  $n$ -dimensional column vector whose elements are 1; the norm of a matrix  $\mathbf{X}$  is defined by  $\|\mathbf{X}\|^2 := \text{tr}[\mathbf{X}\mathbf{X}^T]$ ;  $\hat{\mathbf{X}}(t)$  stands for the estimate of  $\mathbf{X}$  at time  $t$ .

The Wiener nonlinear system consists of a linear dynamic subsystem followed by a static nonlinear block as shown in

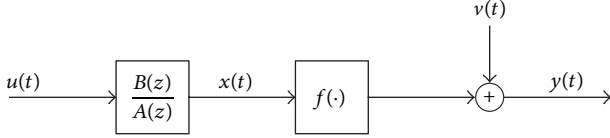


FIGURE 1: The Wiener nonlinear system.

Figure 1 [25, 26]. The linear dynamic subsystem can be given as

$$x(t) = \frac{B(z)}{A(z)} u(t), \quad (1)$$

where  $u(t)$  and  $x(t)$  are the input and the inner output, respectively, and  $A(z)$  and  $B(z)$  are polynomials in  $z^{-1}$ , defined as

$$\begin{aligned} A(z) &:= 1 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_{n_a} z^{-n_a}, \\ B(z) &:= 1 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_{n_b} z^{-n_b}. \end{aligned} \quad (2)$$

The static nonlinear block is generally assumed to be the sum of the nonlinear basis functions of a known basis  $\mathbf{f} := (f_1, f_2, \dots, f_n)$  as follows:

$$\begin{aligned} y(t) &= f(x(t)) \\ &= \gamma_1 f_1(x(t)) + \gamma_2 f_2(x(t)) + \dots + \gamma_{n_\gamma} f_{n_\gamma}(x(t)) \\ &= \mathbf{f}(x(t)) \boldsymbol{\gamma}. \end{aligned} \quad (3)$$

In this paper, we assume that the nonlinear function  $f(\cdot)$  can be expressed by the polynomial of the order  $n_\gamma$  as follows:

$$f(x(t)) = \gamma_1 x(t) + \gamma_2 x^2(t) + \dots + \gamma_{n_\gamma} x^{n_\gamma}(t), \quad (4)$$

and the polynomial orders  $n_\gamma$  are known. Without loss of generality, we introduce a stochastic white noise  $v(t)$  with zero mean and variance  $\sigma^2$  to system output and have

$$y(t) = f(x(t)) + v(t). \quad (5)$$

The linear block output  $x(t)$  is identical with the nonlinear block input. A direct substitution of  $x(t)$  from (1) into (5) would result in a very complex expression containing cross-multiplied parameters and variables. To simplify this problem, the key-term separation principle can be applied [29]. We fix a coefficient of the nonlinear blocks. For example, let the first coefficient of  $\boldsymbol{\gamma}$  be unity; that is,  $\gamma_1 = 1$ . Equation (1) can be rewritten to be

$$x(t) = [1 - A(z)] x(t) + B(z) u(t), \quad (6)$$

and then substituting (6) into (5) for the separated  $x(t)$ , the system output is given in the form

$$y(t) = [1 - A(z)] x(t) + B(z) u(t) + \sum_{i=2}^{n_\gamma} \gamma_i x^i(t) + v(t). \quad (7)$$

Define the information vectors and the parameter vectors

$$\begin{aligned} \boldsymbol{\varphi}_s(t) &:= [-x(t-1), -x(t-2), \dots, -x(t-n_a)], \\ &\quad u(t-1), u(t-2), \dots, u(t-n_b)]^T \in \mathbb{R}^{n_a+n_b}, \\ \boldsymbol{\varphi}_\gamma(t) &:= [x^2(t), x^3(t), \dots, x^{n_\gamma}(t)]^T \in \mathbb{R}^{n_\gamma-1}, \\ \boldsymbol{\varphi}(t) &:= \begin{bmatrix} \boldsymbol{\varphi}_s(t) \\ \boldsymbol{\varphi}_\gamma(t) \end{bmatrix} \in \mathbb{R}^{n_0}, \quad n_0 = n_a + n_b + n_\gamma - 1, \\ \boldsymbol{\theta}_s &:= [a_1, a_2, \dots, a_{n_a}, b_1, b_2, \dots, b_{n_b}]^T \in \mathbb{R}^{n_a+n_b}, \\ \boldsymbol{\theta}_\gamma &:= [\gamma_2, \gamma_3, \dots, \gamma_{n_\gamma}]^T \in \mathbb{R}^{n_\gamma-1}, \\ \boldsymbol{\theta} &:= \begin{bmatrix} \boldsymbol{\theta}_s \\ \boldsymbol{\theta}_\gamma \end{bmatrix} \in \mathbb{R}^{n_0}. \end{aligned} \quad (8)$$

Thus, (6) can be written in a vector form as

$$x(t) = \boldsymbol{\varphi}_s^T(t) \boldsymbol{\theta}_s. \quad (9)$$

Using (9), from (7), we can obtain the following identification model:

$$\begin{aligned} y(t) &= \boldsymbol{\varphi}_s^T(t) \boldsymbol{\theta}_s + \boldsymbol{\varphi}_\gamma^T(t) \boldsymbol{\theta}_\gamma + v(t) \\ &= \boldsymbol{\varphi}^T(t) \boldsymbol{\theta} + v(t). \end{aligned} \quad (10)$$

The objective of this paper is to present a least squares based iterative algorithm to estimate the parameters  $a_i$ ,  $b_i$ ,  $\gamma_i$  for the Wiener nonlinear system.

### 3. The Least-Squares-Based Iterative Algorithm

Based on the methods in [18, 20, 24] for linear systems and Hammerstein nonlinear systems, we derive a least-squares-based iterative algorithm for the Wiener model. Define the stacked output vector  $\mathbf{Y}(L)$ , the stacked information vector  $\boldsymbol{\Phi}(L)$ , and the white noise vector  $\mathbf{V}(L)$  as

$$\begin{aligned} \mathbf{Y}(L) &:= [y(L), y(L-1), \dots, y(1)]^T \in \mathbb{R}^L, \\ \boldsymbol{\Phi}(L) &:= [\boldsymbol{\varphi}(L), \boldsymbol{\varphi}(L-1), \dots, \boldsymbol{\varphi}(1)]^T \in \mathbb{R}^{L \times n_0}, \\ \mathbf{V}(L) &:= [v(L), v(L-1), \dots, v(1)]^T \in \mathbb{R}^L. \end{aligned} \quad (11)$$

Define a quadratic criterion function

$$J(\boldsymbol{\theta}) := \|\mathbf{Y}(L) - \boldsymbol{\Phi}(L) \boldsymbol{\theta}\|. \quad (12)$$

To minimize  $J(\boldsymbol{\theta})$ , letting its partial differential with respect to  $\boldsymbol{\theta}$  be zero gives the least squares estimate of  $\boldsymbol{\theta}$  as follows:

$$\hat{\boldsymbol{\theta}} = [\boldsymbol{\Phi}^T(L) \boldsymbol{\Phi}(L)]^{-1} \boldsymbol{\Phi}^T(L) \mathbf{Y}(L). \quad (13)$$

Since  $\boldsymbol{\Phi}(L)$  in (13) containing unknown inner variable  $x(t)$  leads to a difficulty that the iterative solution  $\hat{\boldsymbol{\theta}}_k$  of  $\boldsymbol{\theta}$  is

impossible to compute. In order to solve this difficulty, the approach here is based on the auxiliary model idea [30–32]. Let  $\hat{x}_k(t)$  be the estimate of  $x(t)$  at iteration  $k$  and  $\hat{\boldsymbol{\varphi}}_k(t)$  the information vector  $\boldsymbol{\varphi}_k(t)$  obtained by replacing  $x(t)$  with  $\hat{x}_k(t)$ ; that is,

$$\hat{\boldsymbol{\varphi}}_k(t) = \begin{bmatrix} \hat{\boldsymbol{\varphi}}_{s,k}(t) \\ \hat{\boldsymbol{\varphi}}_{\gamma,k}(t) \end{bmatrix}, \quad (14)$$

with

$$\begin{aligned} \hat{\boldsymbol{\varphi}}_{s,k}(t) &= [-\hat{x}_{k-1}(t-1), -\hat{x}_{k-1}(t-2), \dots, -\hat{x}_{k-1}(t-n_a), \\ &\quad u(t-1), u(t-2), \dots, u(t-n_b)]^T, \\ \hat{\boldsymbol{\varphi}}_{\gamma,k}(t) &= [\hat{x}_{k-1}^2(t), \hat{x}_{k-1}^3(t), \dots, \hat{x}_{k-1}^{n_\gamma}(t)]^T. \end{aligned} \quad (15)$$

Replacing  $\boldsymbol{\varphi}_s(t)$  and  $\boldsymbol{\theta}_s$  in (9) with  $\hat{\boldsymbol{\varphi}}_{s,k}(t)$  and  $\hat{\boldsymbol{\theta}}_{s,k}$ , respectively, the iterative estimate  $\hat{x}_k(t)$  can be obtained by the following auxiliary model:

$$\hat{x}_k(t) = \hat{\boldsymbol{\varphi}}_{s,k}^T(t) \hat{\boldsymbol{\theta}}_{s,k}, \quad t = 1, 2, \dots, L. \quad (16)$$

Define

$$\hat{\mathbf{\Phi}}_k(L) := [\hat{\boldsymbol{\varphi}}_k(L), \hat{\boldsymbol{\varphi}}_k(L-1), \dots, \hat{\boldsymbol{\varphi}}_k(1)]^T \in \mathbb{R}^{L \times n_0}. \quad (17)$$

Let  $\hat{\boldsymbol{\theta}}_k = \begin{bmatrix} \hat{\boldsymbol{\theta}}_{s,k} \\ \hat{\boldsymbol{\theta}}_{\gamma,k} \end{bmatrix}$  be the estimate of  $\boldsymbol{\theta}_k = \begin{bmatrix} \boldsymbol{\theta}_s \\ \boldsymbol{\theta}_\gamma \end{bmatrix}$  at iteration  $k$ .

Using  $\hat{\mathbf{\Phi}}_k(L)$  in place of  $\mathbf{\Phi}(L)$  in (13), we have

$$\hat{\boldsymbol{\theta}}_k = [\hat{\mathbf{\Phi}}_k^T(L) \hat{\mathbf{\Phi}}_k(L)]^{-1} \hat{\mathbf{\Phi}}_k^T(L) \mathbf{Y}(L). \quad (18)$$

Equations (13)–(18) form the least-squares-based iterative (LSI) identification algorithm for the Wiener nonlinear system, which can be summarized as [1]

$$\hat{\boldsymbol{\theta}}_k = [\hat{\mathbf{\Phi}}_k^T(L) \hat{\mathbf{\Phi}}_k(L)]^{-1} \hat{\mathbf{\Phi}}_k^T(L) \mathbf{Y}(L), \quad (19)$$

$$\hat{\mathbf{\Phi}}_k(L) = [\hat{\boldsymbol{\varphi}}_k(L), \hat{\boldsymbol{\varphi}}_k(L-1), \dots, \hat{\boldsymbol{\varphi}}_k(1)]^T, \quad (20)$$

$$\mathbf{Y}(L) = [y(L), y(L-1), \dots, y(1)]^T, \quad (21)$$

$$\hat{\boldsymbol{\theta}}_k = [\hat{\boldsymbol{\theta}}_{s,k}, \hat{\boldsymbol{\theta}}_{\gamma,k}]^T, \quad (22)$$

$$\hat{\boldsymbol{\varphi}}_k(t) = [\hat{\boldsymbol{\varphi}}_{s,k}(t), \hat{\boldsymbol{\varphi}}_{\gamma,k}(t)]^T,$$

$$\begin{aligned} \hat{\boldsymbol{\varphi}}_{s,k}(t) &= [-\hat{x}_{k-1}(t-1), -\hat{x}_{k-1}(t-2), \dots, \\ &\quad -\hat{x}_{k-1}(t-n_a), u(t-1), \\ &\quad u(t-2), \dots, u(t-n_b)]^T, \end{aligned} \quad (23)$$

$$\hat{\boldsymbol{\varphi}}_{\gamma,k}(t) = [\hat{x}_{k-1}^2(t), \hat{x}_{k-1}^3(t), \dots, \hat{x}_{k-1}^{n_\gamma}(t)]^T,$$

$$\hat{\boldsymbol{\varphi}}_{\gamma,k}(t) = [\hat{x}_{k-1}^2(t), \hat{x}_{k-1}^3(t), \dots, \hat{x}_{k-1}^{n_\gamma}(t)]^T,$$

$$\hat{x}_k(t) = \hat{\boldsymbol{\varphi}}_{s,k}^T(t) \hat{\boldsymbol{\theta}}_{s,k}, \quad t = 1, 2, \dots, L. \quad (24)$$

The flowchart of computing the parameter estimate  $\hat{\boldsymbol{\theta}}_k$  is shown in Figure 2. The steps involved in computing the parameter estimate  $\hat{\boldsymbol{\theta}}_k$  in the LSI algorithm using a fixed data batch with the data length  $L$  are listed as follows.

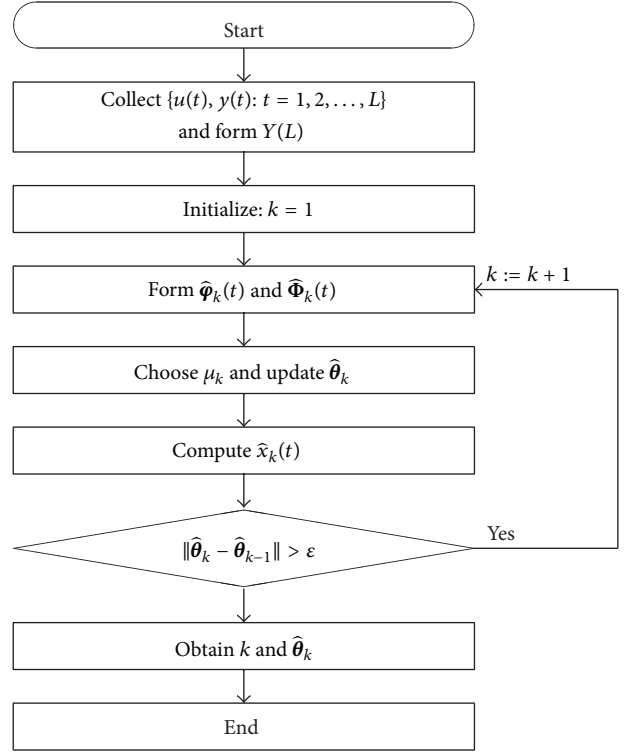


FIGURE 2: The flowchart of computing the LSI estimate  $\hat{\boldsymbol{\theta}}_k$ .

- (1) Collect the input-output data  $\{u(t), y(t) : t = 0, 1, 2, \dots, L\}$  and form  $\mathbf{Y}(L)$  by (21).
- (2) To initialize, let  $k = 1$ ,  $\hat{\boldsymbol{\theta}}_0 = 10^{-6} \mathbf{1}_{n_0}$ , and  $\hat{x}_0(t) = 10^{-6}$  for all  $t$ , and form  $\hat{\boldsymbol{\varphi}}_0(t)$  by (23).
- (3) Form  $\hat{\boldsymbol{\varphi}}_k(t)$  by (23) and  $\hat{\mathbf{\Phi}}_k(t)$  by (20).
- (4) Update the estimate  $\hat{\boldsymbol{\theta}}_k$  by (19).
- (5) Compute  $\hat{x}_k(t)$  by (24).
- (6) For some preset small  $\epsilon$ , if  $\|\hat{\boldsymbol{\theta}}_k - \hat{\boldsymbol{\theta}}_{k-1}\|^2 \leq \epsilon$ , then terminate the procedure and obtain the iterative times  $k$  and estimate  $\boldsymbol{\theta}_k$ ; otherwise, increment  $k$  by 1 and go to step 3.

#### 4. Example

Consider the following Wiener nonlinear system with the linear subsystem

$$x(t) = \frac{B(z)}{A(z)} u(t),$$

$$A(z) = 1 + a_1 z^{-1} + a_2 z^{-2} = 1 - 0.18z^{-1} + 0.44z^{-2}, \quad (25)$$

$$B(z) = 1 + b_1 z^{-1} + b_2 z^{-2} = 1 + 0.58z^{-1} + 0.41z^{-2},$$

and the nonlinearity is given by

$$\begin{aligned} f(x(t)) &= \gamma_1 x(t) + \gamma_2 x^2(t) + \gamma_3 x^3(t) + v(t) \\ &= x(t) - 0.45x^2(t) + 0.25x^3(t) + v(t). \end{aligned} \quad (26)$$

TABLE 1: The parameter estimates  $(a_i, b_i, \gamma_i)$  and their errors ( $\sigma^2 = 0.10^2$ ).

$k$	$a_1$	$a_2$	$b_1$	$b_2$	$\gamma_2$	$\gamma_3$	$\delta$ (%)
1	0.01390	0.05092	0.83611	0.74933	-0.08844	-0.00426	75.27308
2	-0.25018	0.48270	0.60412	0.32843	-0.23432	0.09158	29.18831
5	-0.19276	0.43981	0.58302	0.40929	-0.46012	0.25770	1.73908
10	-0.16452	0.43173	0.58218	0.42255	-0.45226	0.25192	2.37195
20	-0.17048	0.43263	0.58262	0.41964	-0.45268	0.25202	1.76755
30	-0.17101	0.43273	0.58265	0.41937	-0.45271	0.25203	1.71538
40	-0.17106	0.43273	0.58265	0.41935	-0.45271	0.25203	1.71092
50	-0.17106	0.43273	0.58265	0.41935	-0.45271	0.25203	1.71053
True values	-0.18100	0.44000	0.58300	0.40800	-0.45000	0.25000	

TABLE 2: The parameter estimates  $(a_i, b_i, \gamma_i)$  and their errors ( $\sigma^2 = 0.20^2$ ).

$k$	$a_1$	$a_2$	$b_1$	$b_2$	$\gamma_2$	$\gamma_3$	$\delta$ (%)
1	0.01254	0.05414	0.83538	0.75541	-0.08800	-0.00425	75.34758
2	-0.23806	0.47597	0.60817	0.34779	-0.23179	0.08885	28.73714
5	-0.17964	0.43318	0.58596	0.42400	-0.45097	0.24847	1.78072
10	-0.15914	0.42794	0.58447	0.43252	-0.44475	0.24437	3.59012
20	-0.16374	0.42887	0.58476	0.43020	-0.44497	0.24442	3.12454
30	-0.16407	0.42894	0.58478	0.43003	-0.44499	0.24443	3.09113
40	-0.16410	0.42895	0.58478	0.43001	-0.44499	0.24443	3.08873
50	-0.16410	0.42895	0.58478	0.43001	-0.44499	0.24443	3.08856
True values	-0.18100	0.44000	0.58300	0.40800	-0.45000	0.25000	

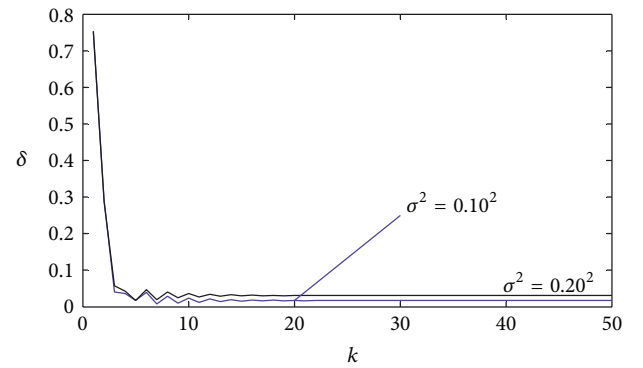
For this example system,  $\{u(t)\}$  are taken as persistent excitation signal sequences with zero mean and unit variance and  $\{v(t)\}$  as a white noise process with zero mean and constant variances  $\sigma^2 = 0.10^2$  and  $\sigma^2 = 0.20^2$ , separately. Taking the data length  $L = 1000$ , we apply the proposed LSI algorithm in (19)–(24) to estimate the parameters  $(a_i, b_i, \gamma_i)$  of this system the parameter estimates and their errors with different noise variances are shown in Tables 1 and 2, and the parameter estimation errors  $\delta$  versus  $k$  are shown in Figure 3, where  $\delta := \|\hat{\theta}_k(t) - \theta\|/\|\theta\|$ .

From Tables 1 and 2 and Figure 3, we can draw the following conclusions.

- (i) The parameter estimation errors given by the proposed approach gradually become smaller as the iteration  $k$  increases; see the error curves of the algorithm in Figure 3 and the estimation errors of the last columns of Tables 1 and 2.
- (ii) As the variance of the noise decreases, the parameter estimation errors given by the proposed approach become smaller; see Tables 1 and 2.

## 5. Conclusions

In this paper, we have presented a least-squares-based iterative identification algorithm for Wiener nonlinear systems. Using the key-term separation principle, we construct the identification model of Wiener nonlinear systems. The proposed algorithm can directly estimate the parameters of the

FIGURE 3: The estimation errors  $\delta$  versus  $k$ .

linear subsystem and nonlinear part of Wiener nonlinear systems. The simulation results verified the effectiveness of the proposed algorithm. The proposed method for Wiener nonlinear systems can combine the auxiliary model identification idea [33–37], the multi-innovation identification theory [38–44], the bias compensation methods [45–48], and the maximum likelihood principle [49–52] to study identification problems of linear or nonlinear systems with colored noise and Hammerstein nonlinear systems [53–59].

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