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Research Article

LMI Approach to Exponential Stability and Almost Sure Exponential Stability for Stochastic Fuzzy Markovian-Jumping Cohen-Grossberg Neural Networks with Nonlinear p-Laplace Diffusion

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The robust exponential stability of delayed fuzzy Markovian-jumping Cohen-Grossberg neural networks (CGNNs) with nonlinear p-Laplace diffusion is studied. Fuzzy mathematical model brings a great difficulty in setting up LMI criteria for the stability, and stochastic functional differential equations model with nonlinear diffusion makes it harder. To study the stability of fuzzy CGNNs with diffusion, we have to construct a Lyapunov-Krasovskii functional in non-matrix form. But stochastic mathematical formulae are always described in matrix forms. By way of some variational methods in $W^{1,p}(\Omega)$, *Itô* formula, Dynkin formula, the semi-martingale convergence theorem, Schur Complement Theorem, and LMI technique, the LMI-based criteria on the robust exponential stability and almost sure exponential robust stability are finally obtained, the feasibility of which can efficiently be computed and confirmed by computer MatLab LMI toolbox. It is worth mentioning that even corollaries of the main results of this paper improve some recent related existing results. Moreover, some numerical examples are presented to illustrate the effectiveness and less conservatism of the proposed method due to the significant improvement in the allowable upper bounds of time delays.

1. Introduction

It is well known that in 1983, Cohen-Grossberg [1] proposed originally the Cohen-Grossberg neural networks (CGNNs). Since then the CGNNs have found their extensive applications in pattern recognition, image and signal processing, quadratic optimization, and artificial intelligence [2–6]. However, these successful applications are greatly dependent on the stability of the neural networks, which is also a crucial feature in the design of the neural networks. In practice, time delays always occur unavoidably due to the finite switching speed of neurons and amplifiers [2–8], which may cause undesirable dynamic network behaviors such as oscillation and instability. Besides delay effects, stochastic effects also exist in real systems. In fact, many dynamical systems have variable structures subject to stochastic abrupt changes, which may result from abrupt phenomena such as sudden environment changes, repairs of the components, changes in the interconnections of subsystems, and stochastic failures. (see [9] and references therein). The stability problems for stochastic systems, in particular the Ito-type stochastic systems, become important in both continuous-time case and discrete-time case [10]. In addition, neural networks with Markovian jumping parameters have been extensively studied due to the fact that systems with Markovian jumping parameters are useful in modeling abrupt phenomena, such as random failures, operating in different points of a nonlinear plant, and changing in the interconnections of subsystems [11–15].

Remark 1. Deterministic system is only the simple simulation for the real system. Indeed, to model a system realistically, a degree of randomness should be incorporated into the model due to various inevitable stochastic factors. For example, in real nervous systems, synaptic transmission is a noisy process brought on by random fluctuations from the release of neurotransmitters and other probabilistic causes. It is showed that the above-mentioned stochastic factors likewise cause undesirable dynamic network behaviors and possibly lead to instability. So it is of significant importance to consider stochastic effects for neural networks. In recent years, the stability of stochastic neural networks has become a hot study topic [3, 16–21].

On the other hand, diffusion phenomena cannot be unavoidable in real world. Usually diffusion phenomena are simply simulated by linear Laplace diffusion in much of the previous literature [2, 22–24]. However, diffusion behavior is so complicated that the nonlinear reaction-diffusion models were considered in several papers [3, 25–28]. Very recently, the nonlinear p-Laplace diffusion ($p > 1$) is applied to the simulation of some diffusion behaviors [3]. But almost all of the above mentioned works were focused on the traditional neural networks models without fuzzy logic. In the factual operations, we always encounter some inconveniences such as the complicity, the uncertainty and vagueness. As far as we know, vagueness is always opposite to exactness. To a certain degree, vagueness cannot be avoided in the human way of regarding the world. Actually, vague notations are often applied to explain some extensive detailed descriptions. As a result, fuzzy theory is regarded as the most suitable setting to taking vagueness and uncertainty into consideration. In 1996, Yang and his coauthor [29] originally introduced the fuzzy cellular neural networks integrating fuzzy logic into the structure of traditional neural networks and maintaining local connectedness among cells. Moreover, the fuzzy neural network is viewed as a very useful paradigm for image processing problems since it has fuzzy logic between its template input and/or output besides the sum of product operation. In addition, the fuzzy neural network is a cornerstone in image processing and pattern recognition. And hence, investigations on the stability of fuzzy neural networks have attracted a great deal of attention [30–37]. Note that stochastic stability for the delayed p-Laplace diffusion stochastic fuzzy CGNNs have never been considered. Besides, the stochastic exponential stability always remains the key factor of concern owing to its importance in designing a neural network, and such a situation motivates our present study. Moreover, the robustness result is also a matter of urgent concern [10, 38–46], for it is difficult to achieve the exact parameters in practical implementations. So in this paper, we will investigate the stochastic global exponential robust stability criteria for the nonlinear reaction-diffusion stochastic fuzzy Markovian-jumping CGNNs by means of linear matrix inequalities (LMIs) approach.

Both the non-linear p-Laplace diffusion and fuzzy mathematical model bring a great difficulty in setting up LMI criteria for the stability, and stochastic functional differential equations model with nonlinear diffusion makes it harder. To study the stability of fuzzy CGNNs with diffusion, we have to construct a Lyapunov-Krasovskii functional in non-matrix form (see, e.g., [4]). But stochastic mathematical formulae are always described in matrix forms. Note that there is no stability criteria for fuzzy CGNNs with p-Laplace diffusion, let alone Markovian-jumping stochastic fuzzy CGNNs with p-Laplace diffusion. Only the exponential stability of $It\hat{o}$-type stochastic CGNNs with p-Laplace diffusion was studied by one literature [3] in 2012. Recently, Ahn use the passivity approach to derive a learning law to guarantee that Takagi-Sugeno fuzzy delayed neural networks are passive and asymptotically stable (see, e.g., [47, 48] and related literature [49–57]). Especially, LMI optimization approach for switched neural networks (see, e.g., [53]) may bring some new edificatory to our studying the stability criteria of Markovian jumping CGNNs. Muralisankar, Gopalakrishnan, Balasubramaniam, and Vembarasan investigated the LMI-based robust stability for Takagi-Sugeno fuzzy neural networks [36, 38–41]. Mathiyalagan et al. studied robust passivity criteria and exponential stability criteria for stochastic fuzzy systems [10, 37, 42–46]. Motivated by some recent related works ([9, 10, 36–57], and so on), particularly, Zhu and Li [4], Zhang et al. [2], Pan and Zhong [58], we are to investigate the exponential stability and robust stability of $It\hat{o}$-type stochastic Markovian jumping fuzzy CGNNs with p-Laplace diffusion. By way of some variational methods in $W^{1,p}(\Omega)$ (Lemma 6), $It\hat{o}$ formula, Dynkin formula, the semi-martingale convergence theorem, Schur Complement Theorem, and LMI technique, the LMI-based criteria on the (robust) exponential stability and almost sure exponential (robust) stability are finally obtained, the feasibility of which can efficiently be computed and confirmed by computer matlab LMI toolbox. When $p = 2$, or ignoring some fuzzy or stochastic effects, the simplified system may be investigated by existing literature (see, e.g., [2–4, 58]). Another purpose of this paper is to verify that some corollaries of our main results improve some existing results in the allowable upper bounds of time delays, which may be illustrated by numerical examples (see, e.g., Examples 30 and 36).

The rest of this paper is organized as follows. In Section 2, the new p-Laplace diffusion fuzzy CGNNs models are formulated, and some preliminaries are given. In Section 3, new LMIs are established to guarantee the stochastic global exponential stability and almost sure exponential stability of the above-mentioned CGNNs. Particularly in Section 4, the robust exponential stability criteria are given. In Section 5, Examples 28, 30, 32, 35, 36, and 38 are presented to illustrate that the proposed methods improve significantly the allowable upper bounds of delays over some existing results ([4, Theorem 1], [4, Theorem 3], [58, Theorem 3.1], [58, Theorem 3.2]). Finally, some conclusions are presented in Section 6.

2. Model Description and Preliminaries

In 2012, Zhu and Li [4] consider the following stochastic fuzzy Cohen-Grossberg neural networks:

$$dx_i(t) = \Bigg\{ -a_i(x_i(t)) \Bigg[b_i(x_i(t)) - \bigwedge_{j=1}^{n} \hat{c}_{ij} f_j(x_j(t)) - \bigvee_{j=1}^{n} \check{c}_{ij} f_j(x_j(t))$$

$$
\begin{aligned}
& - \bigwedge_{j=1}^{n} \widehat{d}_{ij} g_j \left(x_j \left(t - \tau \right) \right) \\
& \left. \left. - \bigvee_{j=1}^{n} \breve{d}_{ij} g_j \left(x_j \left(t - \tau \right) \right) \right] \right\} dt \\
& + \sum_{j=1}^{n} \sigma_{ij} \left(x_j \left(t \right), x_j \left(t - \tau \right) \right) dw_j \left(t \right), \\
& x_i \left(t \right) = \phi_i \left(t \right), \quad -\tau \leqslant t \leqslant 0,
\end{aligned} \tag{1}
$$

where each $w_j(t)$ is scalar standard Brownian motion defined on a complete probability space $(\boldsymbol{\Omega}, \mathscr{F}, \mathscr{P})$ with a natural filtration $\{\mathscr{F}_t\}_{t\geqslant 0}$. The noise perturbation $\sigma_{ij} : R \times R \to R$ is a Borel measurable function. $\bigwedge$ and $\bigvee$ denote the fuzzy **AND** and **OR** operation, respectively. Under several inequalities conditions and the following five similar assumptions on System (1), some exponential stability results are obtained in [4]. Of course, in this paper, we may present the following conditions which are more flexible than those of [4].

(A1) There exists a positive definite diagonal matrix $\overline{A} = \operatorname{diag}(\overline{a}_1, \overline{a}_2, \ldots, \overline{a}_n)$ such that

$$
0 < a_i \left(r \right) \leqslant \overline{a}_i, \tag{2}
$$

for all $r \in R$, $i = 1, 2, \ldots, n$.

(A2) There exist positive definite diagonal matrix $B = \operatorname{diag}(B_1, B_2, \ldots, B_n)$ such that

$$
\frac{a_j \left(r \right) b_j \left(r \right)}{r} \geqslant B_j, \quad \forall j = 1, 2, \ldots, n, \ 0 \neq r \in R. \tag{3}
$$

(A3) For any given $j = 1, 2, \ldots, n$, f_j is locally Lipschitz continuous, and there exists a constant F_j such that $|f_j'(r)| \leqslant |F_j|$ for all $r \in R$ at which f_j is differentiable; g_j is locally Lipschitz continuous, and there exists a constant G_j such that $|g_j'(r)| \leqslant |G_j|$ at which g_j is differentiable.

(A4) There exist nonnegative matrices $\boldsymbol{\mathscr{U}} = (\mu_{ij})_{n\times n}$ and $\boldsymbol{\mathscr{V}} = (\nu_{ij})_{n\times n}$ such that

$$
\operatorname{trace} \left[\sigma^T \left(u, v \right) \sigma \left(u, v \right) \right] \leqslant u^T \boldsymbol{\mathscr{U}} u + v^T \boldsymbol{\mathscr{V}} v, \tag{4}
$$

where $u, v \in R^n$, $\sigma(u, v) = (\sigma_{ij}(u, v))_{n\times n}$.

(A5) $b_j(0) = f_j(0) = g_j(0) = 0$, $\sigma_{ij}(0, 0) \equiv 0$, $i, j = 1, 2, \ldots, n$.

Remark 2. The condition (A3) is different from that of some existing literature (e.g., [2–4]). In those previous literature, f_j and g_j are always assumed to be globally Lipschitz continuous. Here, we relax this assumption, for f_j and g_j are only the local Lipschitz continuous functions. From Rademacher's theorem [59], a locally Lipschitz continuous function $f : R^n \to R^n$ is differentiable almost everywhere. Let $\mathfrak{D}_f$ be the set of those points where f is differentiable, then $f'(x)$ is the Jacobian of f at $x \in \mathfrak{D}_f$ and the set $\mathfrak{D}_f$ is dense in R^n. The generalized Jacobian $\partial f(x)$ of a locally Lipschitz continuous function $f : R^n \to R^n$ is a set of matrices defined by

$$
\begin{aligned}
\partial f \left(x \right) = \operatorname{co} \Big\{ & \mathfrak{w} \mid \text{there exists a sequence} \\
& \left\{ x^k \right\} \subset \mathfrak{D}_f \text{ with } \lim_{x^k \to x} f' \left(x^k \right) = \mathfrak{w} \Big\},
\end{aligned} \tag{5}
$$

where $\operatorname{co}(\cdot)$ denotes the convex hull of a set.

Remark 3. The conditions (A1) and (A2) relax the corresponding ones in some previous literature (e.g., [2–4]).

The condition (A5) guarantees zero-solution is an equilibrium of stochastic fuzzy system (1). Throughout this paper, we always assume that all assumptions (A1)–(A5) hold. In addition, we assume that $\boldsymbol{\mathscr{U}}$ and $\boldsymbol{\mathscr{V}}$ are symmetric matrices in consideration of LMI-based criteria presented in this paper.

Besides delays, stochastic effects, the complexity, the vagueness and diffusion behaviors always occur in real nervous systems. So in this paper, we are to consider the following delays stochastic fuzzy Markovian-jumping Cohen-Grossberg neural networks with nonlinear p-Laplace diffusion ($p > 1$):

$$
\begin{aligned}
& dv_i \left(t, x \right) \\
& \quad = \Bigg\{ \sum_{k=1}^{m} \frac{\partial}{\partial x_k} \left(\mathscr{D}_{ik} \left(t, x, v \right) \left| \nabla v_i \left(t, x \right) \right|^{p-2} \frac{\partial v_i}{\partial x_k} \right) \\
& \qquad - a_i \left(v_i \left(t, x \right) \right) \\
& \qquad \times \Bigg[b_i \left(v_i \left(t, x \right) \right) \\
& \qquad\qquad - \bigwedge_{j=1}^{n} \widehat{c}_{ij} \left(r \left(t \right) \right) f_j \left(v_j \left(t, x \right) \right) \\
& \qquad\qquad - \bigvee_{j=1}^{n} \breve{c}_{ij} \left(r \left(t \right) \right) f_j \left(v_j \left(t, x \right) \right) \\
& \qquad\qquad - \bigwedge_{j=1}^{n} \widehat{d}_{ij} \left(r \left(t \right) \right) g_j \left(v_j \left(t - \tau, x \right) \right) \\
& \qquad\qquad - \bigvee_{j=1}^{n} \breve{d}_{ij} \left(r \left(t \right) \right) g_j \left(v_j \left(t - \tau, x \right) \right) \Bigg] \Bigg\} dt \\
& \qquad + \sum_{j=1}^{n} \sigma_{ij} \left(v_j \left(t, x \right), v_j \left(t - \tau, x \right) \right) dw_j \left(t \right), \\
& \qquad\qquad \forall t \geqslant t_0, \ x \in \Omega, \\
& v \left(\theta, x \right) = \phi \left(\theta, x \right), \quad \left(\theta, x \right) \in \left[-\tau, 0 \right] \times \Omega
\end{aligned} \tag{6}
$$

with boundary condition

$$\mathfrak{B}\left[v_i(t,x)\right]=0,\quad (t,x)\in[-\tau,+\infty)\times\partial\Omega,\quad i=1,2,\ldots,n, \tag{6a}$$

where $p>1$ is a given scalar, $\Omega\in R^m$ is a bounded domain with a smooth boundary $\partial\Omega$ of class $\mathscr{C}^2$ by Ω, $v(t,x)=(v_1(t,x),v_2(t,x),\ldots,v_n(t,x))^T\in R^n$, and $v_i(t,x)$ is the state variable of the ith neuron and the jth neuron at time t and in space variable x. The smooth nonnegative functions $\mathscr{D}_{jk}(t,x,v)$ are diffusion operators. Time delay $\tau\geqslant 0$. $a_j(v_j(t,x))$ represents an amplification function, and $b_j(v_j(t,x))$ is an appropriately behavior function. $f_j(v_j(t,x))$, $g_j(v_j(t,x))$ are neuron activation functions of the jth unit at time t and in space variable x. $\{r(t),t\geqslant 0\}$ is a right-continuous Markov process on the probability space which takes values in the finite space $S=\{1,2,\ldots,N\}$ with generator $\Pi=\{\pi_{ij}\}$ given by

$$\mathscr{P}\left(r(t+\delta)=j\mid r(t)=i\right)=\begin{cases}\pi_{ij}\delta+o(\delta), & j\neq i,\\ 1+\pi_{ij}\delta+o(\delta), & j=i,\end{cases} \tag{7}$$

where $\pi_{ij}\geqslant 0$ is transition probability rate from i to j $(j\neq i)$ and $\pi_{ii}=-\sum_{j=1,j\neq i}^{s}\pi_{ij}$, $\delta>0$ and $\lim_{\delta\to 0}o(\delta)/\delta=0$. In mode $r(t)=k$, we denote $\widehat{c}_{ij}(r(t))=\widehat{c}_{ij}^{(k)}$, $\widehat{d}_{ij}(r(t))=\widehat{d}_{ij}^{(k)}$, $\breve{c}_{ij}(r(t))=\breve{c}_{ij}^{(k)}$ and $\breve{d}_{ij}(r(t))=\breve{d}_{ij}^{(k)}$, which imply the connection strengths of the ith neuron on the jth neuron, respectively.

The boundary condition (6a) is called Dirichlet boundary condition if $\mathfrak{B}[v_i(t,x)]=v_i(t,x)$, and Neumann boundary condition if $\mathfrak{B}[v_i(t,x)]=\partial v_i(t,x)/\partial\nu$, where $\partial v_i(t,x)/\partial\nu=(\partial v_i(t,x)/\partial x_1,\partial v_i(t,x)/\partial x_2,\ldots,\partial v_i(t,x)/\partial x_m)^T$ denotes the outward normal derivative on $\partial\Omega$. It is well known that the stability of neural networks with Neumann boundary condition has been widely studied. The Dirichlet boundary conditions describe the situation where the space is totally surrounded by a region in which the states of the neuron equal zero on the boundary. And the stability analysis of delayed reaction-diffusion neural networks with the Dirichlet boundary conditions is very important in theories and applications, and also has attracted much attention [2, 3, 29, 58]. So in this paper, we consider the CGNNs under Neumann boundary condition and Dirichlet boundary condition, respectively.

If the complexity and the vagueness of CGNNs are ignored, the stochastic fuzzy system (6) is simplified to the following stochastic system:

$$\begin{aligned}dv(t,x)=&\Big\{\nabla\cdot\big(\mathscr{D}(t,x,v)\circ\nabla_p v(t,x)\big)\\&-A(v(t,x))\,[B(v(t,x))\\&\qquad-C(r(t))\,f(v(t,x))\\&\qquad-D(r(t))\,g(v(t-\tau,x))]\Big\}dt\\&+\sigma(t,v(t,x),v(t-\tau,x))\,dw(t),\\&\qquad\forall t\geqslant t_0,\ x\in\Omega,\\v(\theta,x)=&\,\phi(\theta,x),\qquad(\theta,x)\in[-\tau,0]\times\Omega,\end{aligned} \tag{8}$$

where matrices $C_r=(c_{ij}^{(r)})_{n\times n}$, $D_r=(d_{ij}^{(r)})_{n\times n}$. In 2012, Wang et al. [3] studied the stability of System (8) without Markovian-jumping.

Finally, we consider the global robust exponential stability for the following uncertain fuzzy CGNNs with p-Laplace diffusion:

$$\begin{aligned}dv_i(t,x)=&\Bigg\{\sum_{k=1}^{m}\frac{\partial}{\partial x_k}\left(\mathscr{D}_{ik}(t,x,v)\left|\nabla v_i(t,x)\right|^{p-2}\frac{\partial v_i}{\partial x_k}\right)\\&-a_i(v_i(t,x))\\&\times\Bigg[b_i(v_i(t,x))\\&\qquad-\bigwedge_{j=1}^{n}\widehat{c}_{ij}(r(t),t)\,f_j\big(v_j(t,x)\big)\\&\qquad-\bigvee_{j=1}^{n}\breve{c}_{ij}(r(t),t)\,f_j\big(v_j(t,x)\big)\\&\qquad-\bigwedge_{j=1}^{n}\widehat{d}_{ij}(r(t),t)\,g_j\big(v_j(t-\tau,x)\big)\\&\qquad-\bigvee_{j=1}^{n}\breve{d}_{ij}(r(t),t)\,g_j\big(v_j(t-\tau,x)\big)\Bigg]\Bigg\}dt\\&+\sum_{j=1}^{n}\sigma_{ij}\big(v_j(t,x),v_j(t-\tau,x)\big)\,dw_j(t),\\&\qquad\forall t\geqslant t_0,\ x\in\Omega,\\v(\theta,x)=&\,\phi(\theta,x),\qquad(\theta,x)\in[-\tau,0]\times\Omega.\end{aligned} \tag{9}$$

For any mode $r\in S$, we denote $\widehat{c}_{ij}(r(t),t)$, $\breve{c}_{ij}(r(t),t)$, $\widehat{d}_{ij}(r(t),t)$, $\breve{d}_{ij}(r(t),t)$ by $\widehat{c}_{ij}^{(r)}(t)$, $\breve{c}_{ij}^{(r)}(t)$, $\widehat{d}_{ij}^{(r)}(t)$, $\breve{d}_{ij}^{(r)}(t)$, and matrices $\widehat{C}_r(t)=(\widehat{c}_{ij}^{(r)}(t))_{n\times n}$, $\breve{C}_r(t)=(\breve{c}_{ij}^{(r)}(t))_{n\times n}$, $\widehat{D}_r(t)=(\widehat{d}_{ij}^{(r)}(t))_{n\times n}$, $\breve{D}_r(t)=(\breve{d}_{ij}^{(r)}(t))_{n\times n}$. Assume

$$\begin{aligned}&\widehat{C}_r(t)=\widehat{C}_r+\Delta\widehat{C}_r(t); &&\widehat{D}_r(t)=\widehat{D}_r+\Delta\widehat{D}_r(t);\\&\breve{C}_r(t)=\breve{C}_r+\Delta\breve{C}_r(t); &&\breve{D}_r(t)=\breve{D}_r+\Delta\breve{D}_r(t).\end{aligned} \tag{10}$$

The $\Delta\widehat{C}_r(t)$, $\Delta\widehat{D}_r(t)$, $\Delta\breve{C}_r(t)$, and $\Delta\breve{D}_r(t)$ are parametric uncertainties, satisfying

$$\begin{pmatrix}\Delta\widehat{C}_r(t) & \Delta\breve{C}_r(t)\\ \Delta\widehat{D}_r(t) & \Delta\breve{D}_r(t)\end{pmatrix}=\begin{pmatrix}E_{1r}\\ E_{2r}\end{pmatrix}\mathscr{F}(t)\begin{pmatrix}\mathscr{N}_{1r} & \mathscr{N}_{2r}\end{pmatrix}, \tag{11}$$

where $\mathscr{F}(t)$ is an unknown matrix with $|\mathscr{F}^T(t)||\mathscr{F}(t)|\leqslant I$, and E_{1r}, E_{2r}, $\mathscr{N}_{1r}$, $\mathscr{N}_{2r}$ are known real constant matrices for all $r\in S$.

Throughout this paper, we denote matrices $A(v(t,x))=\operatorname{diag}(a_1(v_1(t,x)),a_2(v_2(t,x)),\ldots,a_n(v_n(t,x)))$,

$B(v(t,x)) = (b_1(v_1(t,x)),\ b_2(v_2(t,x)),\dots,b_n(v_n(t,x)))^T$, $f(v(t,x)) = (f_1(v_1(t,x)),\ f_2(v_2(t,x)),\dots,f_n(v_n(t,x)))^T$, $g(v(t,x)) = (g_1(v_1(t,x)),\dots,g_n(v_n(t,x)))^T$. For the sake of simplicity, let $\sigma(t) = \sigma(t, v(t,x), v(t-\tau,x))$, and $w(t) = (w_1(t),\ w_2(t),\dots,w_n(t))^T$. Matrix $\mathscr{D}(t,x,v) = (\mathscr{D}_{jk}(t,x,v))_{n\times m}$ satisfies $\mathscr{D}_{jk}(t,x,v) \geqslant 0$ for all j, k, (t,x,v). Denote $\nabla_p v = (\nabla_p v_1,\dots,\nabla_p v_n)^T$ with $\nabla_p v_i = (|\nabla v_i|^{p-2}(\partial v_i/\partial x_1),\dots,|\nabla v_i|^{p-2}(\partial v_i/\partial x_m))^T$. And $\mathscr{D}(t,x,v)\circ\nabla_p v = (\mathscr{D}_{jk}(t,x,v)|\nabla v_i|^{p-2}(\partial v_i/\partial x_k))_{n\times m}$ denotes the Hadamard product of matrix $\mathscr{D}(t,x,v)$ and $\nabla_p v$ (see, [60] or [3]).

For convenience's sake, we need introduce some standard notations.

(i) $L^2(R\times\Omega)$: The space of real Lebesgue measurable functions of $R\times\Omega$, it is a Banach space for the 2-norm $\|v(t)\|_2 = (\sum_{i=1}^n \|v_i(t)\|)^{1/2}$ with $\|v_i(t)\| = (\int_\Omega |v_i(t,x)|^2 dx)^{1/2}$, where $|v_i(t,x)|$ is Euclid norm.

(ii) $L^2_{\mathscr{F}_0}([-\tau,0]\times\Omega;R^n)$: The family of all $\mathscr{F}_0$-measurable $C([-\tau,0]\times\Omega;R^n)$-value random variable $\xi=\{\xi(\theta,x): -\tau\leqslant\theta\leqslant 0, x\in\Omega\}$ such that $\sup_{-\tau\leqslant\theta\leqslant 0}\mathbb{E}\|\xi(\theta)\|_2^2<\infty$, where $\mathbb{E}\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure $\mathscr{P}$.

(iii) $Q=(q_{ij})_{n\times n}>0$ (<0): A positive (negative) definite matrix, that is, $y^TQy>0$ (<0) for any $0\neq y\in R^n$.

(iv) $Q=(q_{ij})_{n\times n}\geqslant 0$ ($\leqslant 0$): A semi-positive (semi-negative) definite matrix, that is, $y^TQy\geqslant 0$ ($\leqslant 0$) for any $y\in R^n$.

(v) $Q_1\geqslant Q_2$ ($Q_1\leqslant Q_2$): This means Q_1-Q_2 is a semi-positive (semi-negative) definite matrix.

(vi) $Q_1>Q_2$ ($Q_1<Q_2$): This means Q_1-Q_2 is a positive (negative) definite matrix.

(vii) $\lambda_{\max}(\Phi),\lambda_{\min}(\Phi)$ denotes the largest and smallest eigenvalue of matrix Φ, respectively.

(viii) Denote $|C|=(|c_{ij}|)_{n\times n}$ for any matrix $C=(c_{ij})_{n\times n}$; $|u(t,x)|=(|u_1(t,x)|,|u_2(t,x)|,\dots,|u_n(t,x)|)^T$ for any $u(t,x)=(u_1(t,x),u_2(t,x),\dots,u_n(t,x))^T$.

(ix) I: Identity matrix with compatible dimension.

(x) The symmetric terms in a symmetric matrix are denoted by $*$.

(xi) The Sobolev space $W_{1,p}(\Omega)=\{u\in L^p: Du\in L^p\}$ (see [61] for detail). Particularly in the case of $p=2$, then $W^{1,p}(\Omega)=H^1(\Omega)$.

(xii) Denote by λ_1 the lowest positive eigenvalue of the boundary value problem

$$\begin{aligned} -\Delta\varphi(t,x) &= \lambda\varphi(t,x), \quad x\in\Omega, \\ \mathfrak{B}[\varphi(t,x)] &= 0, \quad x\in\partial\Omega. \end{aligned} \tag{12}$$

Let $v(t,x;\phi,i_0)$ be the state trajectory from the initial condition $r(0)=i_0$, $v(\theta,x;\phi)=\phi(\theta,x)$ on $-\tau\leqslant\theta\leqslant 0$ in $L^2_{\mathscr{F}_0}([-\tau,0]\times\Omega;R^n)$. Below, we always assume $(v(t,x;\phi,i_0)$ is a solution of System (6).

Definition 4. For any given scalar $p>1$, the null solution of system (6) is said to be stochastically globally exponentially stable in the mean square if for every initial condition $\phi\in L^2_{\mathscr{F}_0}([-\tau,0]\times\Omega;R^n)$, $r(0)=i_0$, there exist scalars $\beta>0$ and $\gamma>0$ such that for any solution $v(t,x;\phi,i_0)$,

$$\mathbb{E}\left(\|v(t;\phi,i_0)\|_2^2\right)\leqslant\gamma e^{-\beta t}\left[\sup_{-\tau\leqslant\theta\leqslant 0}\mathbb{E}\left(\|\phi(\theta)\|_2^2\right)\right],\quad t\geqslant t_0. \tag{13}$$

Definition 5. The null solution of System (6) is said to be almost sure exponentially stable if for every $\phi\in L^2_{\mathscr{F}_0}([-\tau,0]\times\Omega;R^n)$, there exists a positive scalar $\lambda>0$ such that the following inequality holds:

$$\limsup_{t\to\infty}\log\left(\|v(t)\|_2^2\right)\leqslant-\lambda,\quad \mathscr{P}-\text{a.s.} \tag{14}$$

Lemma 6. *Let $P=\operatorname{diag}(p_1,p_2,\dots,p_n)$ be a positive definite matrix, and v be a solution of system* (6) *with the boundary condition* (6a). *Then one has*

$$\begin{aligned} &\int_\Omega v^TP\left(\nabla\cdot\left(\mathscr{D}(t,x,v)\circ\nabla_p v\right)\right)dx \\ &\quad=-\sum_{k=1}^m\sum_{j=1}^n\int_\Omega p_j\mathscr{D}_{jk}(t,x,v) \\ &\qquad\times\left|\nabla v_j\right|^{p-2}\left(\frac{\partial v_j}{\partial x_k}\right)^2dx \\ &\quad=\int_\Omega\left(\nabla\cdot\left(\mathscr{D}(t,x,v)\circ\nabla_p v\right)\right)^TPv\,dx\ . \end{aligned} \tag{15}$$

Proof. Since v is a solution of system (6), we can derive it by Guass formula and the boundary condition (6a) that

$$\begin{aligned} &\int_\Omega v^TP\left(\nabla\cdot\left(D(t,x,v)\circ\nabla_p v\right)\right)dx \\ &\quad=\int_\Omega v^TP\left(\sum_{k=1}^m\frac{\partial}{\partial x_k}\left(D_{1k}|\nabla v_1|^{p-2}\frac{\partial v_1}{\partial x_k}\right),\dots,\right. \\ &\qquad\left.\sum_{k=1}^m\frac{\partial}{\partial x_k}\left(D_{nk}|\nabla v_n|^{p-2}\frac{\partial v_n}{\partial x_k}\right)\right)^Tdx \\ &\quad=\int_\Omega\sum_{j=1}^n p_jv_j\sum_{k=1}^m\frac{\partial}{\partial x_k}\left(D_{jk}\left|\nabla v_j\right|^{p-2}\frac{\partial v_j}{\partial x_k}\right)dx \\ &\quad=-\sum_{k=1}^m\sum_{j=1}^n\int_\Omega p_jD_{jk}\left|\nabla v_j\right|^{p-2}\left(\frac{\partial v_j}{\partial x_k}\right)^2dx. \end{aligned} \tag{16}$$

Then the other three equalities can be proved similarly. □

Remark 7. Lemma 9 actually generalizes the conclusion of [62, Lemma 3.1] from Hilbert space $H^1(\Omega)$ to Banach space $W^{1,p}(\Omega)$.

Lemma 8 (nonnegative semi-martingale convergence theorem [63]). *Let $A(t)$ and $U(t)$ be two continuous adapted increasing processes on $t \geqslant 0$ with $A(0) = U(0) = 0$, a.s. Let $M(t)$ be a real-valued continuous local martingale with $M(0) = 0$, a.s. Let ξ be a nonnegative $\mathscr{F}_0$-measurable random variable with $E\xi < \infty$. Define*

$$X(t) = \xi + A(t) - U(t) + M(t) \tag{17}$$

for $t \geqslant 0$. If $X(t)$ is nonnegative, then

$$\begin{aligned} \left\{\lim_{t\to\infty} A(t) < \infty\right\} \subset \left\{\lim_{t\to\infty} X(t) < \infty\right\} \\ \cap \left\{\lim_{t\to\infty} U(t) < \infty\right\}, \quad a.s., \end{aligned} \tag{18}$$

where $B \subset D$ a.s. means $P(B \cup D^c) = 0$. In particular, if $\lim_{t\to\infty} A(t) < \infty$ a.s., then for almost all $\omega \in \mathbf{\Omega}$, $\lim_{t\to\infty} X(t) < \infty$ and $\lim_{t\to\infty} U(t) < \infty$, that is, both $X(t)$ and $U(t)$ converge to finite random variables.

Lemma 9 (see [64]). *Let $f : R^n \to R^n$ be locally Lipschitz continuous. For any given $x, y \in R^n$, there exists an element $\mathfrak{w}$ in the union $\cup_{z\in[x,y]}\partial f(z)$ such that*

$$f(y) - f(x) = \mathfrak{w}(y - x), \tag{19}$$

where $[x, y]$ denotes the segment connecting x and y.

Lemma 10 (see [65]). *Let $\varepsilon > 0$ be any given scalar, and $\mathscr{M}$, $\mathfrak{E}$ and $\mathscr{K}$ be matrices with appropriate dimensions. If $\mathscr{K}^T\mathscr{K} \leqslant I$, then one has*

$$\mathscr{M}\mathscr{K}\mathfrak{E} + \mathfrak{E}^T\mathscr{K}^T\mathscr{M}^T \leqslant \varepsilon^{-1}\mathscr{M}\mathscr{M}^T + \varepsilon\mathfrak{E}^T\mathfrak{E}. \tag{20}$$

3. Main Results

Theorem 11. *Assume that $p > 1$. In addition, there exist a sequence of positive scalars $\overline{\alpha}_r$ ($r \in S$) and positive definite diagonal matrices $P_r = \operatorname{diag}(p_{r1}, p_{r2}, \ldots, p_{rn})$ ($r \in S$) and $Q = \operatorname{diag}(q_1, q_2, \ldots, q_n)$ such that the following LMI conditions hold:*

$$\Theta_r \triangleq -\begin{pmatrix} \mathscr{A}_r & P_r\overline{A}\left(\left|\widehat{D}_r\right| + \left|\check{D}_r\right|\right)G \\ * & -e^{-\lambda\tau}Q + \overline{\alpha}_r\mathscr{V} \end{pmatrix} > 0, \quad r \in S, \tag{21}$$

$$P_r < \overline{\alpha}_r I, \quad r \in S, \tag{22}$$

where matrices $\widehat{C}_r = (\widehat{c}_{ij}^{(r)})_{n\times n}$, $\widehat{D}_r = (\widehat{d}_{ij}^{(r)})_{n\times n}$, $\check{C}_r = (\check{c}_{ij}^{(r)})_{n\times n}$, $\check{D}_r = (\check{d}_{ij}^{(r)})_{n\times n}$, and

$$\begin{aligned} \mathscr{A}_r = {} & \lambda P_r - 2P_rB + P_r\overline{A}\left(\left|\widehat{C}_r\right| + \left|\check{C}_r\right|\right)F \\ & + F\left(\left|\widehat{C}_r^T\right| + \left|\check{C}_r^T\right|\right)\overline{A}P_r + \overline{\alpha}_r\boldsymbol{\mathscr{U}} \\ & + Q + \sum_{j\in S}\pi_{rj}P_j\ , \end{aligned} \tag{23}$$

then the null solution of Markovian jumping stochastic fuzzy system (6) is stochastically exponentially stable in the mean square.

Proof. Consider the Lyapunov-Krasovskii functional:

$$\begin{aligned} V(t, v(t), r) = {} & e^{\lambda t}\int_\Omega \sum_{i=1}^n p_{ri}v_i^2(t, x)\,dx \\ & + \int_\Omega \int_{t-\tau}^t e^{\lambda s}\sum_{i=1}^n q_i v_i^2(s, x)\,ds\,dx, \\ & \forall r \in S, \end{aligned} \tag{24}$$

where $v(t, x) = (v_1(t, x), v_2(t, x), \ldots, v_n(t, x))^T$ is a solution for stochastic fuzzy system (6). Sometimes we may denote $v(t, x)$ by v, $v_i(t, x)$ by v_i, and $\sigma(v(t, x), v(t-\tau, x))$ by $\sigma(t)$ for simplicity.

Let $\mathscr{L}$ be the weak infinitesimal operator. Then it follows by Lemma 6 that

$$\begin{aligned} & \mathscr{L}V(t, v(t), r) \\ & = \lambda e^{\lambda t}\int_\Omega v^TP_rv\,dx - 2e^{\lambda t} \\ & \quad \times \sum_{k=1}^m\sum_{i=1}^n\int_\Omega p_{ri}\mathscr{D}_{ik}(t, x, v) \\ & \quad \times \left|\nabla v_i\right|^{p-2}\left(\frac{\partial v_i}{\partial x_k}\right)^2 dx \\ & \quad - 2e^{\lambda t}\sum_{i=1}^n\int_\Omega p_{ri}v_i \\ & \quad \times \Bigg\{a_i(v_i)\Bigg[b_i(v_i) - \bigwedge_{j=1}^n\widehat{c}_{ij}^{(r)}f_j(v_j) \\ & \quad - \bigvee_{j=1}^n\check{c}_{ij}^{(r)}f_j(v_j) \\ & \quad - \bigwedge_{j=1}^n\widehat{d}_{ij}^{(r)}g_j(v_j(t-\tau, x)) \\ & \quad - \bigvee_{j=1}^n\check{d}_{ij}^{(r)} \\ & \quad \times g_j(v_j(t-\tau, x))\Bigg]\Bigg\}dx \\ & \quad + e^{\lambda t}\int_\Omega v^T\sum_{j\in S}\pi_{rj}P_jv\,dx \\ & \quad + e^{\lambda t}\int_\Omega \operatorname{trace}\left(\sigma^T(t)P_r\sigma(t)\right)dx \\ & \quad + \int_\Omega \Big(e^{\lambda t}v^TQv - e^{\lambda(t-\tau)} \\ & \quad \times v^T(t-\tau, x)Qv(t-\tau, x)\Big)dx. \end{aligned} \tag{25}$$

Moreover, we get by A4 and A5

$$\begin{aligned}
&\mathscr{L}V(t,v(t),r)\\
&\quad\leqslant e^{\lambda t}\Bigg\{\int_\Omega v^T\Bigg(\lambda P_r+\sum_{j\in S}\pi_{rj}P_j\Bigg)v\,dx\\
&\qquad+0-2\sum_{i=1}^n\int_\Omega p_{ri}b_iv_i^2\,dx\\
&\qquad+2\sum_{i=1}^n\int_\Omega\Bigg[p_{ri}|v_i|\overline{a}_i\bigwedge_{j=1}^n\left|\widehat{c}_{ij}^{(r)}\right|\\
&\qquad\qquad\times\left|f_j(v_j)-f_j(0)\right|\\
&\qquad\qquad+p_{ri}|v_i|\overline{a}_i\bigvee_{j=1}^n\left|\check{c}_{ij}^{(r)}\right|\\
&\qquad\qquad\times\left|f_j(v_j)-f_j(0)\right|\\
&\qquad\qquad+p_{ri}|v_i|\overline{a}_i\bigwedge_{j=1}^n\left|\widehat{d}_{ij}^{(r)}\right|\\
&\qquad\qquad\times\left|g_j(v_j(t-\tau,x))-g_j(0)\right|\\
&\qquad\qquad+p_{ri}|v_i|\overline{a}_i\bigvee_{j=1}^n\left|\check{d}_{ij}^{(r)}\right|\\
&\qquad\qquad\times\left|g_j(v_j(t-\tau,x))-g_j(0)\right|\Bigg]dx\\
&\qquad+\overline{\alpha}_r\int_\Omega\Big(v^T\boldsymbol{\mathscr{U}}v+v^T(t-\tau,x)\\
&\qquad\qquad\times\boldsymbol{\mathscr{V}}v(t-\tau,x)\Big)dx\Bigg\}\\
&\quad+\int_\Omega\Big(e^{\lambda t}v^TQv-e^{\lambda(t-\tau)}\\
&\qquad\times v^T(t-\tau,x)Qv(t-\tau,x)\Big)dx.
\end{aligned}\tag{26}$$

From A3 and Lemma 9, we know

$$\begin{aligned}
&\left|f(v(t,x))-f(0)\right|=|\mathfrak{F}|\cdot|v(t,x)-0|\leqslant F|v(t,x)|;\\
&\left|g(v(t-\tau,x))-g(0)\right|=|\mathfrak{G}|\cdot|v(t-\tau,x)-0|\\
&\qquad\leqslant G|v(t-\tau,x)|,
\end{aligned}\tag{27}$$

where $\mathfrak{F}\in\cup_{z\in[0,v(t,x)]}\partial f(z)$, and $\mathfrak{G}\in\cup_{z\in[0,v(t-\tau,x)]}\partial f(z)$.

So it follows by A1–A5 that

$$\begin{aligned}
&\mathscr{L}V(t,v(t),r)\\
&\quad\leqslant e^{\lambda t}\Bigg\{\int_\Omega v^T\Bigg(\lambda P_r+\sum_{j\in S}\pi_{rj}P_j\Bigg)v\,dx\\
&\qquad-2\sum_{i=1}^n\int_\Omega p_{ri}b_iv_i^2\,dx\\
&\qquad+2\sum_{i=1}^n\int_\Omega\Bigg[p_{ri}|v_i|\overline{a}_i\sum_{j=1}^n\left|\widehat{c}_{ij}^{(r)}\right|F_j\left|v_j\right|\\
&\qquad\qquad+p_{ri}|v_i|\overline{a}_i\sum_{j=1}^n\left|\check{c}_{ij}^{(r)}\right|F_j\left|v_j\right|\\
&\qquad\qquad+p_{ri}|v_i|\overline{a}_i\sum_{j=1}^n\left|\widehat{d}_{ij}^{(r)}\right|\\
&\qquad\qquad\times G_j\left|v_j(t-\tau,x)\right|\\
&\qquad\qquad+p_{ri}|v_i|\overline{a}_i\sum_{j=1}^n\left|\check{d}_{ij}^{(r)}\right|\\
&\qquad\qquad\times G_j\left|v_j(t-\tau,x)\right|\Bigg]dx\\
&\qquad+\overline{\alpha}_r\int_\Omega\Big(v^T\boldsymbol{\mathscr{U}}v+v^T(t-\tau,x)\\
&\qquad\qquad\times\boldsymbol{\mathscr{V}}v(t-\tau,x)\Big)dx\Bigg\}\\
&\quad+\int_\Omega\Big(e^{\lambda t}v^TQv-e^{\lambda(t-\tau)}\\
&\qquad\times v^T(t-\tau,x)Qv(t-\tau,x)\Big)dx,
\end{aligned}\tag{28}$$

or

$$\begin{aligned}
&\mathscr{L}V(t,v(t),r)\\
&\quad\leqslant e^{\lambda t}\Bigg\{\lambda\int_\Omega\left|v^T\right|\Bigg(\lambda P_r+\sum_{j\in S}\pi_{rj}P_j\Bigg)|v|\,dx\\
&\qquad-2\int_\Omega\left|v^T\right|P_rB|v|\,dx\\
&\qquad+2\int_\Omega\Big[\left|v^T\right|P_r\overline{A}\left(\left|\widehat{C}_r\right|+\left|\check{C}_r\right|\right)F|v|\\
&\qquad\qquad+\left|v^T\right|P_r\overline{A}\left(\left|\widehat{D}_r\right|+\left|\check{D}_r\right|\right)\\
&\qquad\qquad\times G|v(t-\tau,x)|\Big]dx
\end{aligned}$$

$$+\int_\Omega \overline{\alpha}_r\left(\left|v^T\right|\mathscr{U}|v|+\left|v^T(t-\tau,x)\right|\times\mathscr{V}|v(t-\tau,x)|\right)dx+\int_\Omega\left(\left|v^T\right|Q|v|-e^{-\lambda\tau}\times\left|v^T(t-\tau,x)\right|Q|v(t-\tau,x)|\right)dx\Bigg\}. \tag{29}$$

□

Remark 12. In (28), we employ a new method, which is different from that of [4, (3)]. Therefore, our LMI conditions in Theorem 11 may be more feasible and effective than [4, Theorem 1] to some extent, which may be illustrated by a numerical example below (see, Example 30).

Denote $\zeta^T(t,x)=(|v^T(t,x)|,|v^T(t-\tau,x)|)$. Then we get by (21)

$$\mathscr{L}V(t,v(t),r)\leqslant-\int_\Omega\zeta^T(t,x)\Theta_r\zeta(t,x)\,dx\leqslant 0,\qquad r\in S. \tag{30}$$

Then we can obtain by the Dynkin formula

$$\mathbb{E}V(t,v(t),r)-\mathbb{E}V(0,v(0),r)=\mathbb{E}\int_0^t\mathscr{L}V(s,v(s),r)\,ds\leqslant 0,\qquad r\in S. \tag{31}$$

Hence, we have

$$\begin{aligned}\mathbb{E}\int_\Omega e^{\lambda t}\sum_{i=1}^n p_{ri}v_i^2(t,x)\,dx&\leqslant\mathbb{E}V(t,v(t),r)\\&\leqslant\mathbb{E}V(0,v(0),r)\\&=\mathbb{E}\int_\Omega\sum_{i=1}^n p_{ri}v_i^2(0,x)\,dx\\&\quad+\mathbb{E}\int_\Omega\int_{-\tau}^0 e^{\lambda s}\sum_{i=1}^n q_iv_i^2(s,x)\,ds\,dx\\&\leqslant\left[\max_{r\in S}\left(\max_{1\leqslant j\leqslant n}p_{rj}+\tau\max_{1\leqslant j\leqslant n}q_j\right)\right]\sup_{-\tau\leqslant s\leqslant 0}\mathbb{E}\|\phi(s)\|_2^2.\end{aligned}\tag{32}$$

On the other hand,

$$\mathbb{E}\int_\Omega e^{\lambda t}\sum_{i=1}^n p_{ri}v_i^2(t,x)\,dx\geqslant e^{\lambda t}\min_{r\in S}\left(\min_{1\leqslant j\leqslant n}p_{rj}\right)\mathbb{E}\|v(t)\|_2^2. \tag{33}$$

Combining the above two inequalities, we obtain

$$\mathbb{E}\|v(t)\|_2^2\leq\frac{\max_{r\in S}\left(\max_{1\leqslant j\leqslant n}p_{rj}+\tau\max_{1\leqslant j\leqslant n}q_j\right)}{\min_{r\in S}\left(\min_{1\leqslant j\leqslant n}p_{rj}\right)}e^{-\lambda t}\times\sup_{-\tau\leqslant s\leqslant 0}\mathbb{E}\|\phi(s)\|_2^2. \tag{34}$$

Therefore, we can see it by Definition 4 that the null solution of stochastic fuzzy system (6) is globally stochastically exponentially stable in the mean square.

Corollary 13. *If there exist a positive scalar $\overline{\alpha}$ and positive definite diagonal matrices P and Q such that the following LMI conditions hold:*

$$\Theta\triangleq-\begin{pmatrix}\mathscr{A} & P\overline{A}\left(\left|\widehat{D}\right|+\left|\check{D}\right|\right)G\\ * & -e^{-\lambda\tau}Q+\overline{\alpha}\mathscr{V}\end{pmatrix}>0,\qquad P<\overline{\alpha}I, \tag{35}$$

where matrices $\widehat{C}=(\hat{c}_{ij})_{n\times n}$, $\widehat{D}=(\hat{d}_{ij})_{n\times n}$, $\check{C}=(\check{c}_{ij})_{n\times n}$, $\check{D}=(\check{d}_{ij})_{n\times n}$, and

$$\mathscr{A}=\lambda P-2PB+P\overline{A}\left(\left|\widehat{C}\right|+\left|\check{C}\right|\right)F+F\left(\left|\widehat{C}^T\right|+\left|\check{C}^T\right|\right)\overline{A}P+\overline{\alpha}\mathscr{U}+Q, \tag{36}$$

then the null solution of stochastic fuzzy system (1) *is stochastically exponentially stable in the mean square.*

Remark 14. It is obvious from Remark 12 that our Corollary 13 is more feasible and effective than [4, Theorem 1]. In addition, the LMI-based criterion of Corollary 13 has its practical value in real work, for it is available to computer matlab calculation.

Corollary 15. *Assume that $p>1$. In addition, there exist a sequence of positive scalars $\overline{\alpha}_r$ $(r\in S)$ and positive definite diagonal matrices P_r $(r\in S)$ and Q such that the following LMI conditions hold:*

$$\Theta_r\triangleq-\begin{pmatrix}\mathscr{B}_r & P_r\overline{A}\left|D_r\right|G\\ * & -e^{-\lambda\tau}Q+\overline{\alpha}_r\mathscr{V}\end{pmatrix}>0,\quad r\in S,\qquad P_r<\overline{\alpha}_rI,\quad r\in S, \tag{37}$$

where

$$\mathscr{B}_r=\lambda P_r-2P_rB+P_r\overline{A}\left|C_r\right|F+F\left|C_r^T\right|\overline{A}P_r+\overline{\alpha}_r\mathscr{U}+Q+\sum_{j\in S}\pi_{rj}P_j, \tag{38}$$

then the null solution of Markovian jumping stochastic system (8) *is stochastically exponentially stable in the mean square.*

Particularly for the case of $p=2$, we get from the Poincaré inequality (see, e.g., [58, Lemma 2.4]) that

$$\lambda_1\int_\Omega\left|v_i(t,x)\right|^2dx\leqslant\int_\Omega\left|\nabla v_i(t,x)\right|^2dx. \tag{39}$$

Denote $\mathscr{D} = \min_{j,k}(\inf_{t,x,v}\mathscr{D}_{jk}(t,x,v))$. Then Lemma 6 derives that

$$\begin{aligned}&\int_\Omega v^T P_r \left(\nabla\cdot\left(\mathscr{D}(t,x,v)\circ\nabla_p v\right)\right)dx\\&\quad= -\sum_{k=1}^{m}\sum_{j=1}^{n}\int_\Omega p_{rj}\mathscr{D}_{jk}(t,x,v)\left|\nabla v_j\right|^{2-2}\left(\frac{\partial v_j}{\partial x_k}\right)^2 dx\\&\quad\leqslant -\lambda_1\mathscr{D}\underline{\alpha}_r\|v\|_2^2,\end{aligned}\tag{40}$$

where $P_r = \operatorname{diag}(p_{r1},p_{r2},\ldots,p_{rn}) > 0$, and $\underline{\alpha}_r$ is a positive scalar, satisfying

$$\underline{\alpha}_r I < P_r,\quad \forall r\in S.\tag{41}$$

Moreover, one can conclude the following Corollary from (40) and the proof of Theorem 11.

Corollary 16. *Assume that $p = 2$. In addition, there exist a sequence of positive scalars $\overline{\alpha}_r$, $\underline{\alpha}_r$ $(r\in S)$ and positive definite diagonal matrices P_r $(r\in S)$ and Q such that the following LMI conditions hold:*

$$\begin{aligned}\widetilde{\Theta}_r &\triangleq -\begin{pmatrix}\mathscr{B}_r - 2\lambda_1\mathscr{D}\underline{\alpha}_r I & P_r\overline{A}\left|D_r\right|G\\ * & -e^{-\lambda\tau}Q+\overline{\alpha}_r\mathscr{V}\end{pmatrix} > 0,\quad r\in S,\\ &P_r < \overline{\alpha}_r I,\quad r\in S,\\ &\underline{\alpha}_r I < P_r,\quad \forall r\in S,\end{aligned}\tag{42}$$

where $\mathscr{B}_r$ satisfies (38), then the null solution of Markovian jumping stochastic system (8) with $p = 2$ is stochastically exponentially stable in the mean square.

Remark 17. Corollary 16 not only extends [58, Theorem 3.2] into the case of Markovian jumping, but also improves its complicated conditions by presenting the efficient LMI-based criterion.

Below, we denote $\nu = \max_{i,j}\nu_{ij}$ for convenience's sake.

Theorem 18. *Assume $p > 1$. The null solution of Markovian jumping stochastic fuzzy system (6) is almost sure exponentially stable if there exist positive scalars λ, $\overline{\alpha}_r$ $(r\in S)$, β and positive definite matrices $P_r = \operatorname{diag}(p_{r1},p_{r2},\ldots,p_{rn})$, $(r\in S)$ such that*

$$\begin{aligned}\widehat{\Theta}_r &\triangleq -\begin{pmatrix}\widehat{\mathscr{A}}_r & P_r\overline{A}\left(\left|\widehat{D}_r\right|+\left|\check{D}_r\right|\right)G\\ * & -\overline{\alpha}_r\beta\mathscr{V}\end{pmatrix} > 0,\quad r\in S,\\ &P_r < \overline{\alpha}_r I,\quad r\in S,\end{aligned}\tag{43}$$

where

$$\begin{aligned}\widehat{\mathscr{A}}_r &= \lambda P_r - 2P_rB + P_r\left(\left|\widehat{C}_r\right|+\left|\check{C}_r\right|\right)F\\&\quad+F\left(\left|\widehat{C}_r^T\right|+\left|\check{C}_r^T\right|\right)\overline{A}P_r+\overline{\alpha}_r\mathscr{U}\\&\quad+n\nu e^{\lambda\tau}\overline{\alpha}_r(1+\beta)I+\sum_{j\in S}\pi_{rj}P_j.\end{aligned}\tag{44}$$

Proof. Consider the Lyapunov-Krasovskii functional:

$$\mathscr{V}(t,v(t),r) = e^{\lambda t}\int_\Omega\sum_{i=1}^{n}p_{ri}v_i^2(t,x)\,dx,\quad r\in S.\tag{45}$$

By applying $It\hat{o}$ formula (see, e.g., [3, (2.7)]) and Lemma 6, we can get

$$\begin{aligned}&\mathscr{V}(t,v(t),r)-\mathscr{V}(0,v(0),r)\\&\quad=\int_0^t\lambda e^{\lambda s}\int_\Omega v^T(s,x)P_rv(s,x)\,dx\,ds\\&\qquad-2\int_0^t e^{\lambda s}\sum_{k=1}^{m}\sum_{i=1}^{n}\int_\Omega p_i\mathscr{D}_{ik}(s,x,v)\times\left|\nabla v_i(s,x)\right|^{p-2}\times\left(\frac{\partial v_i(s,x)}{\partial x_k}\right)^2dx\,ds\\&\qquad-2\int_0^t e^{\lambda s}\sum_{i=1}^{n}\int_\Omega p_{ri}v_i(s,x)\times\Bigg\{a_i(v_i(s,x))\times\Bigg[b_i(v_i(s,x))-\bigwedge_{j=1}^{n}\widehat{c}_{ij}f_j(v_j(s,x))-\bigvee_{j=1}^{n}\check{c}_{ij}f_j(v_j(s,x))\\&\qquad\qquad-\bigwedge_{j=1}^{n}\widehat{d}_{ij}g_j(v_j(s-\tau,x))-\bigvee_{j=1}^{n}\check{d}_{ij}\times g_j(v_j(s-\tau,x))\Bigg]\Bigg\}dx\,ds\\&\qquad+\int_0^t e^{\lambda s}\int_\Omega v^T(s,x)\sum_{j\in S}\pi_{rj}P_jv(s,x)\,dx\,ds\\&\qquad+2\int_0^t e^{\lambda s}\sum_{i=1}^{n}\int_\Omega p_{ri}v_i(s,x)\sum_{j=1}^{n}\sigma_{ij}(s)\,dw_j(s)\,dx\\&\qquad+\int_0^t e^{\lambda s}\int_\Omega\operatorname{trace}\left(\sigma^T(s)P_r\sigma(s)\right)dx\,ds,\end{aligned}\tag{46}$$

where $\sigma_{ij}(s) = \sigma_{ij}(v_j(s,x),v_j(s-\tau,x))$, and $\sigma(s) = (\sigma_{ij}(s))_{n\times n}$.

Similarly as (26)–(29), we can derive by A1–A5 and Lemma 9

$$\begin{aligned}
&\mathscr{V}(t, v(t), r) \\
&\quad \leqslant \int_\Omega \sum_{i=1}^{n} p_{ri} v_i^2(0,x)\,dx \\
&\quad + 2\int_0^t e^{\lambda s} \int_\Omega \Big\{ \left|v^T(s,x)\right| \\
&\qquad \times \Big(\lambda P_r + \sum_{j\in S} \pi_{rj} P_j\Big) |v(s,x)| \\
&\qquad - \left|v^T(s,x)\right| P_r B |v(s,x)| \\
&\qquad + \left|v^T(s,x)\right| P_r \overline{A} \left(\left|\widehat{C}_r\right| + \left|\check{C}_r\right|\right) \\
&\qquad \times F |v(s,x)| \\
&\qquad + \left|v^T(s,x)\right| P_r \overline{A} \left(\left|\widehat{D}_r\right| + \left|\check{D}_r\right|\right) \\
&\qquad \times G |v(s-\tau,x)| \\
&\qquad + \frac{1}{2}\left|v^T(s,x)\right| \overline{\alpha}_r \boldsymbol{\mathscr{U}} |v(s,x)| \Big\}\,dx\,ds \\
&\quad + \int_0^t e^{\lambda s} \int_\Omega v^T(s-\tau,x)\,\overline{\alpha}_r \boldsymbol{\mathscr{V}} v(s-\tau,x)\,dx\,ds \\
&\quad + 2\int_0^t e^{\lambda s} \sum_{i=1}^{n} \int_\Omega p_{ri} v_i(s,x) \sum_{j=1}^{n} \sigma_{ij}(s)\,dw_j(s)\,dx.
\end{aligned} \tag{47}$$

On the other hand,

$$\begin{aligned}
&\int_0^t e^{\lambda s} \int_\Omega v^T(s-\tau,x)\,\overline{\alpha}_r \boldsymbol{\mathscr{V}} v(s-\tau,x)\,dx\,ds \\
&\quad = \overline{\alpha}_r \int_0^t e^{\lambda s} \int_\Omega \sum_{i=1}^{n}\sum_{j=1}^{n} v_{ij} v_i(s-\tau,x)\, v_j(s-\tau,x)\,dx\,ds \\
&\quad \leqslant \frac{1}{2}\overline{\alpha}_r \int_0^t e^{\lambda s} \int_\Omega \sum_{i=1}^{n}\sum_{j=1}^{n} v_{ij} \\
&\qquad \times \big(v_i^2(s-\tau,x) \\
&\qquad\quad + v_j^2(s-\tau,x)\,dx\,ds\big) \\
&\quad \leqslant n\nu\overline{\alpha}_r \int_0^t e^{\lambda s} \int_\Omega v^T(s-\tau,x)\, v(s-\tau,x)\,dx\,ds \\
&\quad = n\nu\overline{\alpha}_r \int_{-\tau}^{t-\tau} e^{\lambda(\theta+\tau)} \int_\Omega v^T(\theta,x)\, v(\theta,x)\,dx\,d\theta \\
&\quad = n\nu\overline{\alpha}_r \Big(\int_{-\tau}^{t} e^{\lambda(\theta+\tau)} \int_\Omega v^T(\theta,x)\, v(\theta,x)\,dx\,d\theta \\
&\qquad - \int_{t-\tau}^{t} e^{\lambda(\theta+\tau)} \int_\Omega v^T(\theta,x)\, v(\theta,x)\,dx\,d\theta \Big) \\
&\quad \leqslant n\nu\overline{\alpha}_r \int_{-\tau}^{t} e^{\lambda(s+\tau)} \int_\Omega \left|v^T(s,x)\right| |v(s,x)|\,dx\,ds.
\end{aligned} \tag{48}$$

Denote $v_\tau = v(s-\tau,x)$ for convenience's sake. Then we have

$$\begin{aligned}
&\int_0^t e^{\lambda s} \int_\Omega v^T(s-\tau,x)\,\overline{\alpha}_r \boldsymbol{\mathscr{V}} v(s-\tau,x)\,dx\,ds \\
&\quad = [-\beta + (1+\beta)] \int_0^t e^{\lambda s} \int_\Omega v_\tau^T \overline{\alpha}_r \boldsymbol{\mathscr{V}} v_\tau\,dx\,ds \\
&\quad \leqslant -\beta \int_0^t e^{\lambda s} \int_\Omega v_\tau^T \overline{\alpha}_r \boldsymbol{\mathscr{V}} v_\tau\,dx \\
&\qquad + n\nu\overline{\alpha}_r (1+\beta) \\
&\qquad \times \int_{-\tau}^{t} e^{\lambda(s+\tau)} \int_\Omega \left|v^T(s,x)\right| |v(s,x)|\,dx\,ds.
\end{aligned} \tag{49}$$

Combining (47) and (49) results in

$$\begin{aligned}
&\mathscr{V}(t, v(t), r) \\
&\quad \leqslant \int_\Omega \sum_{i=1}^{n} p_{ri} v_i^2(0,x)\,dx \\
&\quad + \int_0^t e^{\lambda s} \int_\Omega \Big\{ \left|v^T(s,x)\right| \\
&\qquad \times \Big(\lambda P_r + \sum_{j\in S} \pi_{rj} P_j - 2P_r B \\
&\qquad\quad + 2P_r \overline{A} \left(\left|\widehat{C}\right| + \left|\check{C}\right|\right) F + \alpha \boldsymbol{\mathscr{U}} \\
&\qquad\quad + n\nu e^{\lambda\tau} (1+\beta)\,\overline{\alpha}_r I \Big) |v(s,x)| \\
&\qquad\quad + 2\left|v^T(s,x)\right| P_r \overline{A} \left(\left|\widehat{D}\right| + \left|\check{D}\right|\right) \\
&\qquad\quad \times G |v(s-\tau,x)| \Big\}\,dx\,ds \\
&\quad - \beta \int_0^t e^{\lambda s} \int_\Omega v^T(s-\tau,x)\,\overline{\alpha}_r \boldsymbol{\mathscr{V}} v(s-\tau,x)\,dx\,ds \\
&\quad + n\nu(1+\beta)\,\overline{\alpha}_r \\
&\quad \times \int_{-\tau}^{0} e^{\lambda(s+\tau)} \int_\Omega \left|v^T(s,x)\right| |v(s,x)|\,dx\,ds \\
&\quad + 2\int_0^t e^{\lambda s} \sum_{i=1}^{n} \int_\Omega p_{ri} v_i(s,x) \sum_{j=1}^{n} \sigma_{ij}(s)\,dw_j(s)\,dx \\
&\quad = \int_\Omega \sum_{i=1}^{n} p_{ri} v_i^2(0,x)\,dx \\
&\quad - \int_0^t e^{\lambda s} \int_\Omega \zeta^T(s,x)\,\widehat{\Theta}_r \zeta(s,x)\,dx\,ds \\
&\quad + n\nu(1+\beta)\,\overline{\alpha}_r \int_{-\tau}^{0} e^{\lambda(s+\tau)} \int_\Omega \left|v^T(s,x)\right| |v(s,x)|\,dx\,ds \\
&\quad + 2\int_0^t e^{\lambda s} \sum_{i=1}^{n} \int_\Omega p_{ri} v_i(s,x) \sum_{j=1}^{n} \sigma_{ij}(s)\,dw_j(s)\,dx
\end{aligned} \tag{50}$$

which together with (43) implies

$$\begin{aligned}
&\mathscr{V}(t, v(t), r) \\
&\quad\leqslant \max_{r\in S}\overline{\alpha}_r \int_\Omega \sum_{i=1}^n v_i^2(0,x)\,dx \\
&\qquad + nve^{\lambda\tau}(1+\beta)\max_{r\in S}\overline{\alpha}_r \\
&\qquad \times \int_{-\tau}^0 e^{\lambda s}\int_\Omega v^T(s,x)\,v(s,x)\,dx\,ds \\
&\qquad + 2\max_{r\in S}\overline{\alpha}_r \int_0^t e^{\lambda s}\sum_{i=1}^n \int_\Omega v_i(s,x)\sum_{j=1}^n \sigma_{ij}(s)\,dw_j(s)\,dx.
\end{aligned} \tag{51}$$

□

Remark 19. The methods employed in (38)–(50) are different from ones in the proof of [4, Theorem 3] so that our efficient LMI criterion can be constructed. In large numerical calculations, LMI-based criterion in Theorem 18 is more effective than the complicated condition (8) in [4, Theorem 3]. To some extent, Theorem 18 is more effective than [58, Theorem 3.1] to some extent if fuzzy system (6) is simplified to system (8) without Markovian jumping (see, e.g., Example 38).

It is obvious that the right-hand side of (51) is a nonnegative semi-martingale. And hence the semi-martingale convergence theorem derives

$$\limsup_{t\to\infty}\mathscr{V}(t, v(t), r) < \infty, \quad \mathbb{P}-\text{a.s.} \tag{52}$$

Note that

$$\begin{aligned}
e^{\lambda t}\min_{r\in S}\left(\min_i p_{ri}\right)\int_\Omega v^T v\,dx &\leqslant e^{\lambda t}\int_\Omega \sum_{i=1}^n p_{ri}v_i^2(t,x)\,dx \\
&= \mathscr{V}(t, v(t), r).
\end{aligned} \tag{53}$$

Then we can conclude from (52)

$$\limsup_{t\to\infty}\frac{1}{t}\log\left(\|v\|_2^2\right) \leqslant -\lambda, \quad \mathbb{P}-a.s. \tag{54}$$

So we can see it from Definition 5 that the null solution of stochastic fuzzy system (6) is almost sure exponentially stable.

Corollary 20. *The null solution of stochastic fuzzy system* (1) *is almost sure exponentially stable if there exist positive scalars λ, $\overline{\alpha}$, β and positive definite diagonal matrices P such that*

$$\begin{gathered}
\widehat{\Theta} \triangleq -\begin{pmatrix} \widehat{\mathscr{A}} & P\overline{A}\left(\left|\widehat{D}\right| + \left|\check{D}\right|\right)G \\ * & -\overline{\alpha}\beta\boldsymbol{\mathscr{V}} \end{pmatrix} > 0, \\
P < \overline{\alpha}I,
\end{gathered} \tag{55}$$

where

$$\begin{aligned}
\widehat{\mathscr{A}} = {}& \lambda P - 2PB + P\overline{A}\left(\left|\widehat{C}\right| + \left|\check{C}\right|\right)F \\
&+ F\left(\left|\widehat{C}^T\right| + \left|\check{C}^T\right|\right)\overline{A}P + \overline{\alpha}\boldsymbol{\mathscr{U}} \\
&+ nve^{\lambda\tau}\overline{\alpha}(1+\beta)I.
\end{aligned} \tag{56}$$

Remark 21. It seems from Remark 19 that Corollary 20 is obviously more effective than [4, Theorem 3] and [58, Theorem 3.1], which may shown by numerical examples below.

Corollary 22. *Assume $p > 1$. The null solution of Markovian jumping stochastic system* (8) *is almost sure exponentially stable if there exist positive scalars λ, $\overline{\alpha}_r$ $(r \in S)$, β and positive definite matrices $P_r = \operatorname{diag}(p_{r1}, p_{r2}, \ldots, p_{rn})$, $(r \in S)$ such that*

$$\begin{gathered}
\breve{\Theta}_r \triangleq -\begin{pmatrix} \mathscr{B}_r & P_r\overline{A}\left|D_r\right|G \\ * & -\overline{\alpha}_r\beta\boldsymbol{\mathscr{V}} \end{pmatrix} > 0, \quad r \in S, \\
P_r < \overline{\alpha}_r I, \quad r \in S,
\end{gathered} \tag{57}$$

where

$$\begin{aligned}
\mathscr{B}_r = {}& \lambda P_r - 2P_rB + P_r\overline{A}\left|C_r\right|F + F\left|C_r^T\right|\overline{A}P_r \\
&+ \overline{\alpha}_r\boldsymbol{\mathscr{U}} + nve^{\lambda\tau}\overline{\alpha}_r(1+\beta)I + \sum_{j\in S}\pi_{rj}P_j.
\end{aligned} \tag{58}$$

Corollary 23. *Assumed $p = 2$. The null solution of Markovian jumping stochastic system* (8) *is almost sure exponentially stable if there exist positive scalars λ, $\overline{\alpha}_r$, $\underline{\alpha}_r$ $(r \in S)$, β and positive definite matrices $P_r = \operatorname{diag}(p_{r1}, p_{r2}, \ldots, p_{rn})$, $(r \in S)$ such that*

$$\begin{gathered}
\overline{\Theta}_r \triangleq -\begin{pmatrix} \mathscr{B}_r - 2\lambda_1\mathscr{D}\underline{\alpha}_r I & P_r\overline{A}\left|D_r\right|G \\ * & -\overline{\alpha}_r\beta\boldsymbol{\mathscr{V}} \end{pmatrix} > 0, \quad r \in S, \\
P_r < \overline{\alpha}_r I, \quad r \in S, \\
P_r > \underline{\alpha}_r I, \quad r \in S,
\end{gathered} \tag{59}$$

where $\mathscr{B}_r$ satisfies (58).

Remark 24. As pointed out in Remark 19, Corollary 23 is obviously more effective than [58, Theorem 3.1] if Markovian jumping stochastic system (8) is simplified to a stochastic system without Markovian juamping.

4. The Robust Exponential Stability Criteria

Theorem 25. *Assume that $p > 1$. In addition, there exist a sequence of positive scalars $\overline{\alpha}_r$ $(r \in S)$ and positive definite diagonal matrices $P_r = \operatorname{diag}(p_{r1}, p_{r2}, \ldots, p_{rn})$ $(r \in S)$ and $Q = \operatorname{diag}(q_1, q_2, \ldots, q_n)$ such that for all $r \in S$,*

$$\begin{pmatrix} \mathbb{A}_r & P_r\overline{A}\left(\left|\widehat{D}_r\right|+\left|\check{D}_r\right|\right)G & P_r\overline{A}\left|E_{1r}\right| & F\left(\left|\mathcal{N}_{1r}^T\right|+\left|\mathcal{N}_{2r}^T\right|\right) & P_r\overline{A}\left|E_{2r}\right| & 0 \\ * & -e^{-\lambda\tau}Q+\overline{\alpha}_r\mathcal{V} & 0 & 0 & 0 & G\left(\left|\mathcal{N}_{1r}^T\right|+\left|\mathcal{N}_{2r}^T\right|\right) \\ * & * & -I & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 \\ * & * & & & -I & 0 \\ * & * & * & * & * & -I \end{pmatrix} < 0, \tag{60}$$

$$P_r < \overline{\alpha}_r I, \quad r \in S, \tag{61}$$

where matrices $\widehat{C}_r = (\widehat{c}_{ij}^{(r)})_{n\times n}$, $\widehat{D}_r = (\widehat{d}_{ij}^{(r)})_{n\times n}$, $\check{C}_r = (\check{c}_{ij}^{(r)})_{n\times n}$, $\check{D}_r = (\check{d}_{ij}^{(r)})_{n\times n}$, *and*

$$\begin{aligned} \mathbb{A}_r = {} & \lambda P_r - 2P_rB + P_r\overline{A}\left(\left|\widehat{C}_r\right|+\left|\check{C}_r\right|\right)F \\ & + F\left(\left|\widehat{C}_r^T\right|+\left|\check{C}_r^T\right|\right)\overline{A}P_r + \overline{\alpha}_r\mathcal{U} \\ & + Q + \sum_{j\in S}\pi_{rj}P_j , \end{aligned} \tag{62}$$

then the null solution of Markovian jumping stochastic fuzzy system (9) *is stochastically exponentially robust stable in the mean square.*

Proof. Similarly as the proof of Theorem 11, we consider the same Lyapunov-Krasovskii functional

$$\begin{aligned} V(t,v(t),r) = {} & e^{\lambda t}\int_\Omega \sum_{i=1}^n p_{ri}v_i^2(t,x)\,dx \\ & + \int_\Omega\int_{t-\tau}^t e^{\lambda s}\sum_{i=1}^n q_i v_i^2(s,x)\,ds\,dx, \quad \forall r \in S, \end{aligned} \tag{63}$$

where $v(t,x) = (v_1(t,x), v_2(t,x), \dots, v_n(t,x))^T$ is a solution for stochastic fuzzy system (9). Then it follows by the proof of Theorem 11 that

$$\begin{aligned} & \mathcal{L}V(t,v(t),r) \\ & \leqslant e^{\lambda t}\Bigg\{\lambda\int_\Omega\left|v^T\right|\left(\lambda P_r + \sum_{j\in S}\pi_{rj}P_j\right)|v|\,dx \\ & \quad - 2\int_\Omega\left|v^T\right|P_rB|v|\,dx \\ & \quad + 2\int_\Omega\Big[\left|v^T\right|P_r\overline{A}\left(\left|\widehat{C}_r(t)\right|+\left|\check{C}_r(t)\right|\right) \\ & \qquad \times F|v| + \left|v^T\right|P_r\overline{A} \\ & \qquad \times\left(\left|\widehat{D}_r(t)\right|+\left|\check{D}_r(t)\right|\right) \\ & \qquad \times G\left|v(t-\tau,x)\right|\Big]\,dx \\ & \quad + \int_\Omega\overline{\alpha}_r\Big(\left|v^T\right|\mathcal{U}|v| + \left|v^T(t-\tau,x)\right| \\ & \qquad \times\mathcal{V}\left|v(t-\tau,x)\right|\Big)\,dx \\ & \quad + \int_\Omega\Big(\left|v^T\right|Q|v| - e^{-\lambda\tau}\left|v^T(t-\tau,x)\right| \\ & \qquad \times Q\left|v(t-\tau,x)\right|\Big)\,dx\Bigg\}. \end{aligned} \tag{29*}$$

Denote $\zeta^T(t,x) = (|v^T(t,x)|, |v^T(t-\tau,x)|)$, and $\mathbb{B}_r(t) = \begin{pmatrix} \mathbb{A}_r(t) & P_r\overline{A}(|\widehat{D}_r(t)|+|\check{D}_r(t)|)G \\ * & -e^{-\lambda\tau}Q+\overline{\alpha}_r\mathcal{V} \end{pmatrix}$, where $\mathbb{A}_r(t) = \lambda P_r - 2P_rB + P_r\overline{A}(|\widehat{C}_r(t)| + |\check{C}_r(t)|)F + F(|\widehat{C}_r^T(t)| + |\check{C}_r^T(t)|)\overline{A}P_r + \overline{\alpha}_r\mathcal{U} + Q + \sum_{j\in S}\pi_{rj}P_j$. It is obvious that

$$\begin{aligned} & \zeta^T(t,x)\,\mathbb{B}_r(t)\,\zeta(t,x) \\ & \quad \leqslant \zeta^T(t,x)\Bigg[\begin{pmatrix} \mathbb{A}_r & P_r\overline{A}\left(\left|\widehat{D}_r\right|+\left|\check{D}_r\right|\right)G \\ * & -e^{-\lambda\tau}Q+\overline{\alpha}_r\mathcal{V} \end{pmatrix} \\ & \qquad + \begin{pmatrix} P_r\overline{A}\left(\left|\Delta\widehat{C}_r(t)\right|+\left|\Delta\check{C}_r(t)\right|\right)F + F\left(\left|\Delta\widehat{C}_r^T(t)\right|+\left|\Delta\check{C}_r^T(t)\right|\right)\overline{A}P_r & P_r\overline{A}\left(\left|\Delta\widehat{D}_r(t)\right|+\left|\Delta\check{D}_r(t)\right|\right)G \\ * & 0 \end{pmatrix}\Bigg]\zeta(t,x). \end{aligned} \tag{64}$$

Then we can conclude it by (29^*), (11), Lemma 10, and applying Schur Complement ([66]) to (60) that

$$\mathscr{L}V(t,v(t),r) \leqslant -\int_{\Omega} \zeta^T(t,x)\,\mathbb{B}_r(t)\,\zeta(t,x)\,dx \leqslant 0, \quad r \in S. \tag{65}$$

Thereby, we can complete the rest of the proof by the same methods employed in (32)–(34). □

Similarly, we can derive the following Theorem by (11), Lemma 10, Schur Complement Theorem and the proof of Theorem 18.

Theorem 26. *Assume $p > 1$. The null solution of Markovian jumping stochastic fuzzy system* (9) *is the almost sure robust exponential stability if there exist positive scalars λ, $\overline{\alpha}_r$ $(r \in S)$, β and positive definite matrices $P_r = \operatorname{diag}(p_{r1}, p_{r2}, \ldots, p_{rn})$, $(r \in S)$ such that for all $r \in S$,*

$$\begin{pmatrix} \widehat{\mathbb{A}}_r & P_r\overline{A}\left(\left|\widehat{D}_r\right|+\left|\check{D}_r\right|\right)G & P_r\overline{A}\left|E_{1r}\right| & F\left(\left|\mathcal{N}_{1r}^T\right|+\left|\mathcal{N}_{2r}^T\right|\right) & P_r\overline{A}\left|E_{2r}\right| & 0 \\ * & \overline{\alpha}_r\beta\mathscr{V} & 0 & 0 & 0 & G\left(\left|\mathcal{N}_{1r}^T\right|+\left|\mathcal{N}_{2r}^T\right|\right) \\ * & * & -I & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 \\ * & * & & & -I & 0 \\ * & * & * & * & * & -I \end{pmatrix} < 0, \tag{66}$$

$$P_r < \overline{\alpha}_r I, \quad r \in S,$$

where

$$\begin{aligned} \widehat{\mathbb{A}}_r = {} & \lambda P_r - 2P_rB + P_r\left(\left|\widehat{C}_r\right|+\left|\check{C}_r\right|\right)F \\ & + F\left(\left|\widehat{C}_r^T\right|+\left|\check{C}_r^T\right|\right)\overline{A}P_r + \overline{\alpha}_r\mathscr{U} \\ & + nve^{\lambda\tau}\overline{\alpha}_r(1+\beta)I + \sum_{j\in S}\pi_{rj}P_j. \end{aligned} \tag{67}$$

Remark 27. Although the stability of Laplace diffusion stochastic neural networks are studied by previous literature. However, it is the first attempt that the robust stability criteria about the nonlinear p-Laplace diffusion stochastic fuzzy neural networks with Markovian jumping are obtianed, and the first time that the exponential stability criteria of p-Laplace diffusion stochastic fuzzy neural networks with Markovian jumping are provided. It is also the first attempt to synthesize the variational methods in $W^{1,p}(\Omega)$ (see, e.g., Lemma 6, Remark 7), *Itô* formula, Dynkin formula, the semi-martingale convergence theorem, Schur Complement Theorem, and LMI technique to set up the LMI-based (robust) exponential stability or almost sure exponential (robust) stability criteria.

5. Comparisons and Numerical Examples

Example 28. Consider the following stochastic fuzzy CGNNs:

$$\begin{aligned} & dv_1(t,x) \\ & = \Bigg\{\sum_{k=1}^{2}\frac{\partial}{\partial x_k}\left(\mathscr{D}_{1k}(t,x,v)\left|\nabla v_1(t,x)\right|^{p-2}\frac{\partial v_1}{\partial x_k}\right) \\ & \quad - a_1(v_1(t,x))\Bigg[b_1(v_1(t,x)) \\ & \quad - \bigwedge_{j=1}^{2}\widehat{c}_{1j}(r(t))\,f_j\left(v_j(t,x)\right) \\ & \quad - \bigvee_{j=1}^{2}\check{c}_{1j}(r(t))\,f_j\left(v_j(t,x)\right) \\ & \quad - \bigwedge_{j=1}^{2}\widehat{d}_{1j}(r(t)) \times g_j\left(v_j(t-\tau,x)\right) \\ & \quad - \bigvee_{j=1}^{2}\check{d}_{1j}(r(t)) \times g_j\left(v_j(t-\tau,x)\right)\Bigg]\Bigg\}dt \\ & \quad + \sum_{j=1}^{2}\sigma_{1j}\left(v_j(t,x), v_j(t-\tau,x)\right)dw_j(t), \\ & \qquad \forall t \geqslant t_0, \quad x \in \Omega, \end{aligned}$$

$$\begin{aligned} & dv_2(t,x) \\ & = \Bigg\{\sum_{k=1}^{2}\frac{\partial}{\partial x_k}\left(\mathscr{D}_{2k}(t,x,v)\left|\nabla v_2(t,x)\right|^{p-2}\frac{\partial v_2}{\partial x_k}\right) \end{aligned}$$

$$
\begin{aligned}
&- a_2(v_2(t,x))\Bigg[b_2(v_2(t,x)) \\
&\qquad - \bigwedge_{j=1}^{2} \hat{c}_{2j}(r(t)) f_j(v_j(t,x)) \\
&\qquad - \bigvee_{j=1}^{2} \check{c}_{2j}(r(t)) f_j(v_j(t,x)) \\
&\qquad - \bigwedge_{j=1}^{2} \hat{d}_{2j}(r(t)) \\
&\qquad\qquad \times g_j(v_j(t-\tau,x)) \\
&\qquad - \bigvee_{j=1}^{2} \check{d}_{2j}(r(t)) \\
&\qquad\qquad \times g_j(v_j(t-\tau,x)) \Bigg]\Bigg\} dt \\
&+ \sum_{j=1}^{2} \sigma_{2j}(v_j(t,x), v_j(t-\tau,x)) \, dw_j(t), \\
&\qquad\qquad \forall t \geqslant t_0,\ x \in \Omega, \\
&v(\theta,x) = \phi(\theta,x), \qquad (\theta,x) \in [-\tau,0]\times\Omega
\end{aligned} \tag{68}
$$

under Dirichlet boundary condition, where the initial value function

$$
\begin{aligned}
\phi(s,x) &= \begin{pmatrix} 0.25\left(1-\cos\left(5\pi x^2\right)\right)\cos^{189}\left(x^2-0.25\right)e^{-100s} \\ 0.2\sin^2\left(4\pi x^2\right)\cos^{201}\left(x^2-0.55\right)e^{-100s} \end{pmatrix}, \\
&\qquad -\tau \le s \le 0,
\end{aligned} \tag{69}
$$

$p = 2.011$, $v = (v_1(t,x), v_2(t,x))^T \in R^2$, $x = (x_1,x_2)^T \in \Omega = \{(x_1,x_2)^T \in R^2 : |x_j| < \sqrt{2}, j = 1,2\}$, $a_1(v_1) = 0.11 + 0.01\sin^2(v_1^2)$, $a_2(v_2) = 0.12 + 0.01\cos^2(v_2^2)$, $b_1(v_1) = 0.12v_1 + 2v_1\sin^2(v_1^2)$, $b_2(v_2) = 0.125v_2 + v_2\cos^2(v_2^2)$, $f(v) = g(v) = (0.16v_1, 0.166v_2 + 0.001v_2\sin^2(v_2^2))^T$, and

$$
\begin{gathered}
\overline{A} = \begin{pmatrix} 0.12 & 0 \\ 0 & 0.13 \end{pmatrix}, \qquad B = \begin{pmatrix} 0.0132 & 0 \\ 0 & 0.0150 \end{pmatrix}, \\
\mathscr{D}(t,x,v) = \begin{pmatrix} 0.003 & 0.005 \\ 0.004 & 0.006 \end{pmatrix}, \qquad F = \begin{pmatrix} 0.16 & 0 \\ 0 & 0.167 \end{pmatrix} = G, \\
\mathscr{U} = \begin{pmatrix} 0.0003 & 0 \\ 0 & 0.0003 \end{pmatrix} = \mathscr{V},
\end{gathered} \tag{70}
$$

$$
\begin{gathered}
\widehat{C}_1 = \begin{pmatrix} 0.11 & -0.003 \\ -0.003 & 0.12 \end{pmatrix} = \widehat{D}_1, \\
\check{C}_1 = \begin{pmatrix} 0.16 & -0.003 \\ -0.003 & 0.18 \end{pmatrix} = \check{D}_1, \\
\widehat{C}_2 = \begin{pmatrix} 0.13 & -0.003 \\ -0.003 & 0.15 \end{pmatrix} = \widehat{D}_2, \\
\check{C}_2 = \begin{pmatrix} 0.17 & -0.003 \\ -0.003 & 0.19 \end{pmatrix} = \check{D}_2, \\
\widehat{C}_3 = \begin{pmatrix} 0.12 & -0.003 \\ -0.003 & 0.13 \end{pmatrix} = \widehat{D}_3, \\
\check{C}_3 = \begin{pmatrix} 0.175 & -0.003 \\ -0.003 & 0.196 \end{pmatrix} = \check{D}_3.
\end{gathered} \tag{71}
$$

The transition rates matrices are considered as

$$
\begin{aligned}
\prod &= \begin{pmatrix} \pi_{11} & \pi_{12} & \pi_{13} \\ \pi_{21} & \pi_{22} & \pi_{23} \\ \pi_{31} & \pi_{32} & \pi_{33} \end{pmatrix} \\
&= \begin{pmatrix} -0.6 & 0.4 & 0.2 \\ 0.2 & -0.7 & 0.5 \\ 0.5 & 0.3 & -0.8 \end{pmatrix}.
\end{aligned} \tag{72}
$$

Fix $\lambda = 0.001$. Let $\tau = 50.78$, then we use matlab LMI toolbox to solve LMIs (21) and (22), and obtain $t\min = -2.5977 * 10^{-5} < 0$, $\overline{\alpha}_1 = 7025.2$, $\overline{\alpha}_2 = 7051.3$, $\overline{\alpha}_3 = 7038.5$, and

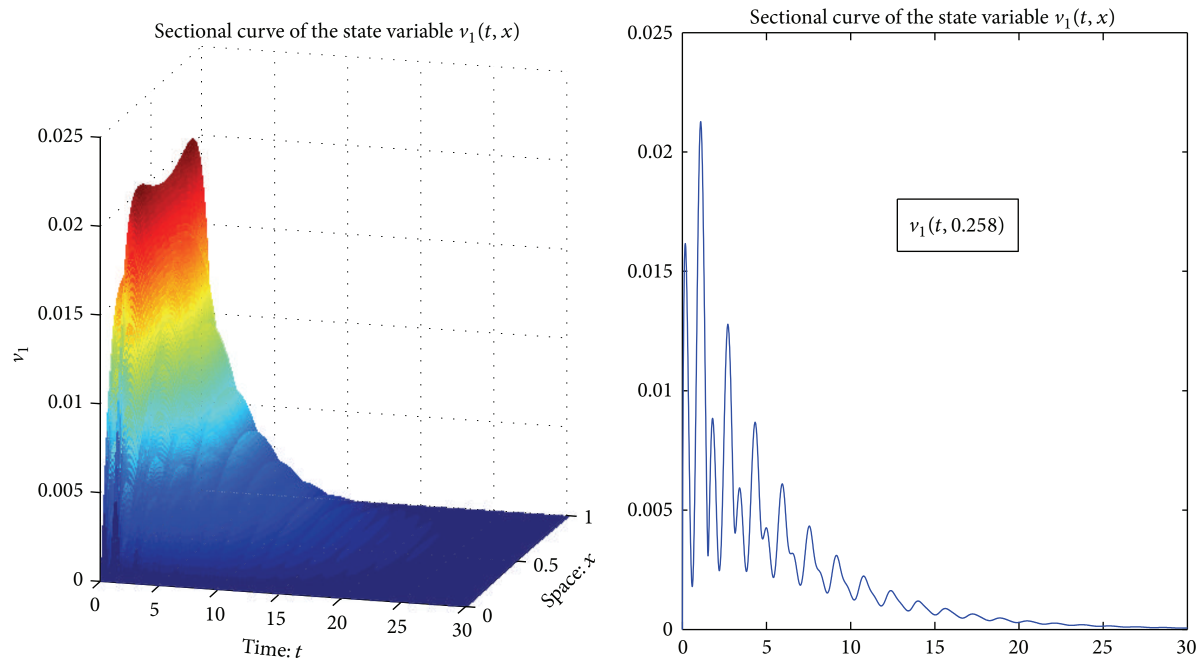
$$
\begin{gathered}
P_1 = \begin{pmatrix} 7023.4 & 0 \\ 0 & 7025.1 \end{pmatrix}, \qquad P_2 = \begin{pmatrix} 7041.5 & 0 \\ 0 & 7051.1 \end{pmatrix}, \\
P_3 = \begin{pmatrix} 7035.1 & 0 \\ 0 & 7038.3 \end{pmatrix}, \\
Q = \begin{pmatrix} 46.4745 & 0 \\ 0 & 52.8358 \end{pmatrix}.
\end{gathered} \tag{73}
$$

Then by Theorem 11 we know that the null solution of Markovian jumping stochastic fuzzy system (68) is stochastically exponentially stable in the mean square with the allowable upper bounds of time delays $\tau = 50.78$ (see, Figures 1 and 2).

Remark 29. (1) Thanks to some novel techniques employed in this paper (Remark 12), LMI-based criterion of Theorem 11 is more effective and feasible in consideration of its significant improvement in the allowable upper bounds of time delays.

(2) Because the stability of the stochastic fuzzy CGNN with p-Laplace diffusion is never studied by any previous literature, below we have to compare the corollaries of Theorem 11 with some existing results to demonstrate the advantages of the proposed method (see, e.g., Example 30).

(3) Finding a solution x to the LMI system $A(x) < B(x)$ is called the feasibility problem. In matlab LMI toolbox, the feasibility problem is solved by the so-called feasp solver. And



(a) (b)

FIGURE 1: The state variable $v_1(t,x)$.

the feasp solver always judges the feasibility of the feasibility problem by solving the following convex optimization problem:

$$\begin{aligned} &\min t \\ \text{s.t.}\quad & A(x) - B(x)\, t \leqslant I. \end{aligned} \tag{74}$$

The global optimum value of the convex optimization problem is always denoted by $t \min$, which is the first datum of the output data of the feasp solver. Particularly, the system is feasible if $t\min < 0$, and infeasible if $t \min > 0$.

Example 30. Consider the following fuzzy system:

$$\begin{aligned} dx_1(t) &= \Bigg\{ -a_1(x_1(t)) \Bigg[b_1(x_1(t)) - \bigwedge_{j=1}^{2} \widehat{c}_{1j} f_j(x_j(t)) - \bigvee_{j=1}^{2} \check{c}_{1j} f_j(x_j(t)) - \bigwedge_{j=1}^{2} \widehat{d}_{1j} g_j(x_j(t-\tau)) - \bigvee_{j=1}^{2} \check{d}_{1j} g_j(x_j(t-\tau)) \Bigg] \Bigg\} dt \\ &\quad + \sum_{j=1}^{2} \sigma_{1j}(x_j(t), x_j(t-\tau))\, dw_j(t), \\ dx_2(t) &= \Bigg\{ -a_2(x_2(t)) \Bigg[b_2(x_2(t)) - \bigwedge_{j=1}^{2} \widehat{c}_{2j} f_j(x_j(t)) - \bigvee_{j=1}^{2} \check{c}_{2j} f_j(x_j(t)) - \bigwedge_{j=1}^{2} \widehat{d}_{2j} g_j(x_j(t-\tau)) - \bigvee_{j=1}^{2} \check{d}_{2j} g_j(x_j(t-\tau)) \Bigg] \Bigg\} dt \\ &\quad + \sum_{j=1}^{2} \sigma_{2j}(x_j(t), x_j(t-\tau))\, dw_j(t), \\ x_i(t) &= \phi_i(t), \quad -\tau \leqslant t \leqslant 0,\ i = 1,2 \end{aligned} \tag{75}$$

with all the parameters mentioned in Example 28. In addition, denote

$$\begin{aligned} \widehat{C} &= \frac{1}{3}\sum_{j=1}^{3} \widehat{C}_j = \begin{pmatrix} 0.1200 & -0.0030 \\ -0.0030 & 0.1333 \end{pmatrix} = \widehat{D}, \\ \check{C} &= \frac{1}{3}\sum_{j=1}^{3} \check{C}_j = \begin{pmatrix} 0.1683 & -0.0030 \\ -0.0030 & 0.1887 \end{pmatrix} = \check{D}. \end{aligned} \tag{76}$$

Let $\tau = 45.37$, then we solve LMIs (35), and get $t\min = -1.6257 * 10^{-4} < 0$, $\overline{\alpha} = 194020$, and

$$P = \begin{pmatrix} 193970 & 0 \\ 0 & 194020 \end{pmatrix}, \quad Q = \begin{pmatrix} 1278.1 & 0 \\ 0 & 1453.3 \end{pmatrix}. \tag{77}$$

Then by Corollary 13, the null solution of stochastic fuzzy system (75) is stochastically exponentially stable in the mean square with the allowable upper bounds of time delays $\tau = 45.37$.

Remark 31. With all the above parameters in Example 30, we solve the inequalities condition (2) in [4, Theorem 1], and obtain $t\min = 1.8569 * 10^{-11} > 0$ which implies infeasible (see, Remark 29 (3)). However, the inequalities condition (2) in [4, Theorem 1] is only sufficient, not necessary for the stability. In Example 30, we can conclude from LMI-base criterion of Corollary 13 that the null solution of stochastic fuzzy system (75) is stochastically exponentially stable in the mean square. Hence, as pointed out in Remarks 12 and 14, Corollary 13 is more feasible and effective than [4, Theorem 1].

Example 32. Consider the Markovian jumping system (8) with all the parameters mentioned in Example 28. In addition, denote

$$C_1 = \frac{1}{2}\left(\widehat{C}_1 + \check{C}_1\right) = \begin{pmatrix} 0.1350 & -0.0030 \\ -0.0030 & 0.1500 \end{pmatrix} = D_1,$$
$$C_2 = \frac{1}{2}\left(\widehat{C}_2 + \check{C}_2\right) = \begin{pmatrix} 0.1500 & -0.0030 \\ -0.0030 & 0.1700 \end{pmatrix} = D_2, \tag{78}$$
$$C_3 = \frac{1}{2}\left(\widehat{C}_3 + \check{C}_3\right) = \begin{pmatrix} 0.1475 & -0.0030 \\ -0.0030 & 0.1630 \end{pmatrix} = D_3.$$

Let $\tau = 108.9$. Then one can solve LMIs (37), and obtain $t\min = -0.0168 < 0$, $\overline{\alpha}_1 = 51.4646$, $\overline{\alpha}_2 = 51.0590$, $\overline{\alpha}_3 = 50.4225$, and

$$P_1 = \begin{pmatrix} 31.2880 & 0 \\ 0 & 31.8027 \end{pmatrix}, \quad P_2 = \begin{pmatrix} 31.3729 & 0 \\ 0 & 31.8989 \end{pmatrix}, \quad P_3 = \begin{pmatrix} 31.3785 & 0 \\ 0 & 31.8989 \end{pmatrix}, \quad Q = \begin{pmatrix} 0.3659 & 0 \\ 0 & 0.4103 \end{pmatrix}. \tag{79}$$

And hence Corollary 15 derives that the null solution of Markovian jumping stochastic system (8) is stochastically exponentially stable in the mean square with the allowable upper bounds of time delays $\tau = 108.9$.

If $p = 2$, we can solve LMIs (42) with $\tau = 109.88$, and get $t\min = -0.0348$, $\underline{\alpha}_1 = 44.8100$, $\underline{\alpha}_2 = 44.4562$, $\underline{\alpha}_3 = 43.6935$, $\overline{\alpha}_1 = 77.9603$, $\overline{\alpha}_2 = 77.5548$, $\overline{\alpha}_3 = 76.8509$, and

$$P_1 = \begin{pmatrix} 60.6629 & 0 \\ 0 & 60.7682 \end{pmatrix}, \quad P_2 = \begin{pmatrix} 60.8995 & 0 \\ 0 & 61.0238 \end{pmatrix}, \quad P_3 = \begin{pmatrix} 60.9916 & 0 \\ 0 & 61.1055 \end{pmatrix}, \quad Q = \begin{pmatrix} 2.1773 & 0 \\ 0 & 2.2520 \end{pmatrix}, \tag{80}$$

where $\lambda_1 = 9.8696$ for $\Omega = \{(x_1, x_2)^T \in R^2 : |x_j| < \sqrt{2}, j = 1, 2\}$ (see, e.g. [58, Remark 2.5]). Then Corollary 16 yields that the null solution of Markovian jumping stochastic system (8) is stochastically exponentially stable in the mean square with the allowable upper bounds of time delays $\tau = 109.88$.

Remark 33. Example 32 illustrates that LMI-based criteria of Corollaries 15 and 16 are more effective and feasible than some existing results (see, e.g., [58, Theorem 3.2]) due to the significant improvement in the allowable upper bounds of time delays.

There are some interesting comparisons among Examples 28, 30, and 32 as follows.

From Table 1, we know that the ambiguity of the fuzzy system affect the analysis and judgement on the stability. The maximum allowable upper bounds decrease when the fuzzy factors occur. In addition, both the randomicity of Markovian jumping and nonlinear p-Laplace diffusion exercised a malign influence on judging the stability.

Remark 34. Table 1 also illustrates that the diffusion item plays an active role in the LMI-based criterion of Corollary 16.

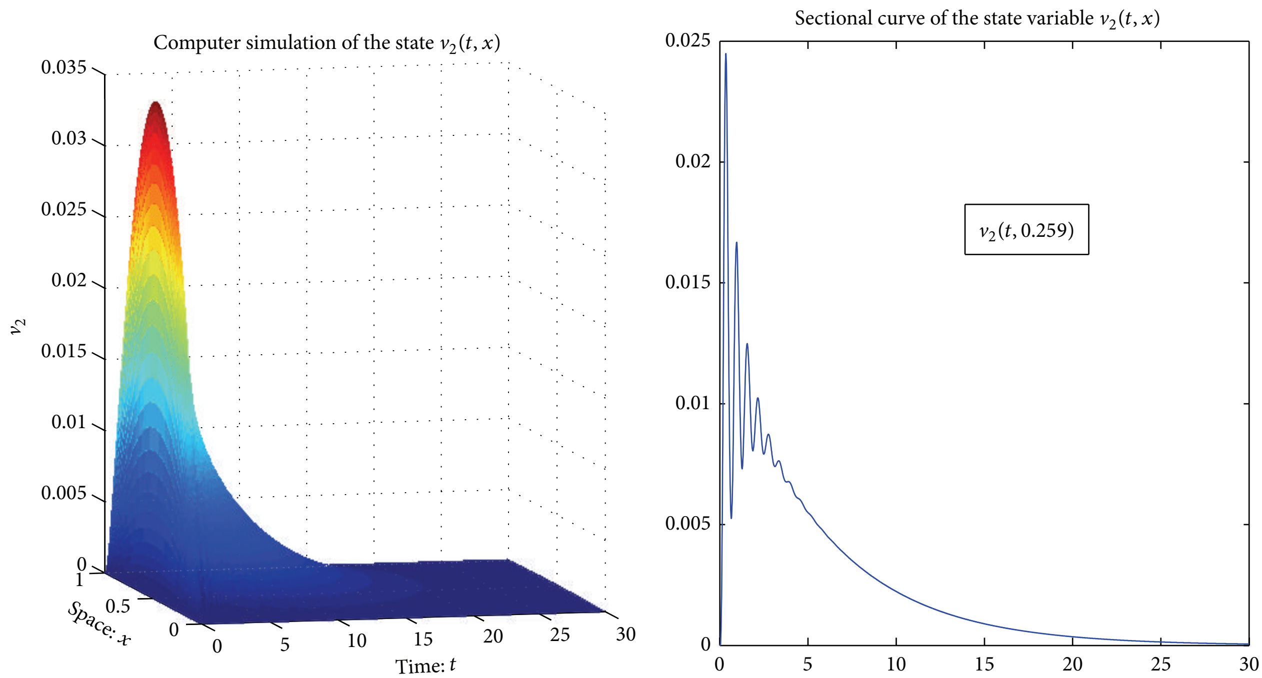
Example 35. Consider (68) with the parameters (71), (72), and

$$\underline{A} = \begin{pmatrix} 0.32 & 0 \\ 0 & 0.31 \end{pmatrix}, \quad \overline{A} = \begin{pmatrix} 0.38 & 0 \\ 0 & 0.36 \end{pmatrix}, \quad B = \begin{pmatrix} 6.433 & 0 \\ 0 & 6.61 \end{pmatrix}, \quad F = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.1 \end{pmatrix} = G, \quad \mathscr{U} = \begin{pmatrix} 0.0003 & 0 \\ 0 & 0.0003 \end{pmatrix} = \mathscr{V}, \quad \mathscr{D}(t, x, v) = \begin{pmatrix} 0.003 & 0.005 \\ 0.004 & 0.006 \end{pmatrix}. \tag{70*}$$

Assume, in addition, $n = 2$. Fix $\lambda = 0.0001$ and $\beta = 0.0001$. Let $\tau = 27.15$, then we solve LMIs (43), and obtain

Table 1: Allowable upper bound of τ for various cases.

	Theorem 11	Corollary 13	Corollary 15	Corollary 16
Value of p	2.011		2.011	2
Markovian jumping	Yes	No	Yes	Yes
Fuzzy	Yes	Yes	No	No
Time delays τ	50.78	45.37	108.9	109.88



(a) (b)

Figure 2: The state variable $v_2(t, x)$.

$t\min = -3.6996 * 10^{-9} < 0$, $\bar{\alpha}_1 = 49955$, $\bar{\alpha}_2 = 92134$, $\bar{\alpha}_3 = 68884$, and

$$P_1 = \begin{pmatrix} 24.0279 & 0 \\ 0 & 23.8713 \end{pmatrix}, \quad P_2 = \begin{pmatrix} 37.8443 & 0 \\ 0 & 39.0478 \end{pmatrix}, \quad P_3 = \begin{pmatrix} 30.8866 & 0 \\ 0 & 30.2698 \end{pmatrix}. \tag{82}$$

Hence, we can conclude from Theorem 18 that the null solution of Markovian jumping stochastic fuzzy system (6) is almost sure exponentially stable with the allowable upper bounds of time delays $\tau = 27.15$.

Example 36. Consider stochastic fuzzy system (75) with all the parameters in Example 35. In addition, denote

$$\widehat{C} = \frac{1}{3}\sum_{j=1}^{3}\widehat{C}_j = \begin{pmatrix} 0.1200 & -0.0030 \\ -0.0030 & 0.1333 \end{pmatrix} = \widehat{D}, \quad \check{C} = \frac{1}{3}\sum_{j=1}^{3}\check{C}_j = \begin{pmatrix} 0.1683 & 0.0030 \\ 0.0030 & 0.1887 \end{pmatrix} = \check{D}. \tag{76*}$$

Let $\tau = 88.15$, then one can solve LMIs (55), and obtain $t\min = -7.5392 * 10^{-7} < 0$, $\alpha = 22255$ and

$$P = \begin{pmatrix} 12.3530 & 0 \\ 0 & 10.6073 \end{pmatrix}. \tag{84}$$

Then by Corollary 20, the null solution of stochastic fuzzy system (75) is almost sure exponentially stable with the allowable upper bounds of time delays $\tau = 88.15$.

Remark 37. With all the above data in Example 36, we solve the inequalities condition (8) in [4, Theorem 3], and obtain $t\min = 8.9843 * 10^{-12} > 0$ which implies infeasible. However, the inequalities condition (8) in [4, Theorem 3] is only sufficient, not necessary for the stability. In Example 36, we can conclude from LMI-base criterion of Corollary 20 that the null solution of stochastic fuzzy system (1) is almost sure exponentially stable. Hence, as pointed out in Remarks 19 and 21, Corollary 20 is more feasible and effective than [4, Theorem 3].

Table 2: Allowable upper bound of τ for various cases.

	Theorem 18	Corollary 20	Corollary 22	Corollary 23
Value of p	2.011		2.011	2
Markovian jumping	Yes	No	Yes	Yes
Fuzzy	Yes	Yes	No	No
Time delays τ	27.15	88.15	98.85	99.89

Example 38. Consider Markovian jumpimg stochastic system (8) with all the parameters in Example 35. In addition, denote

$$C_1 = \frac{1}{2}\left(\widehat{C}_1 + \check{C}_1\right) = \begin{pmatrix} 0.1350 & -0.0030 \\ -0.0030 & 0.1500 \end{pmatrix} = D_1,$$
$$C_2 = \frac{1}{2}\left(\widehat{C}_2 + \check{C}_2\right) = \begin{pmatrix} 0.1500 & -0.0030 \\ -0.0030 & 0.1700 \end{pmatrix} = D_2, \quad (78^*)$$
$$C_3 = \frac{1}{2}\left(\widehat{C}_3 + \check{C}_3\right) = \begin{pmatrix} 0.1475 & -0.0030 \\ -0.0030 & 0.1630 \end{pmatrix} = D_3.$$

Let $\tau = 98.85$, then we solve LMIs (43), and obtain $t\min = -7.9520 * 10^{-7} < 0$, $\overline{\alpha}_1 = 131.8414$, $\overline{\alpha}_2 = 130.0644$, $\overline{\alpha}_3 = 131.3207$, and

$$P_1 = \begin{pmatrix} 0.4182 & 0 \\ 0 & 0.3723 \end{pmatrix},$$
$$P_2 = \begin{pmatrix} 0.3328 & 0 \\ 0 & 0.2830 \end{pmatrix}, \quad (86)$$
$$P_3 = \begin{pmatrix} 0.3525 & 0 \\ 0 & 0.3155 \end{pmatrix}.$$

Hence, we can conclude from Corollary 22 that the null solution of Markovian jumping stochastic system (8) is almost sure exponentially stable with the allowable upper bounds of time delays $\tau = 98.85$.

If $p = 2$, then one can solve LMIs (59) with $\tau = 99.89$, and get $t\min = -3.5692 * 10^{-7} < 0$, $\underline{\alpha}_1 = 0.8469$, $\underline{\alpha}_2 = 0.6080$, $\underline{\alpha}_3 = 0.6509$, $\overline{\alpha}_1 = 452.9764$, $\overline{\alpha}_2 = 457.1149$, $\overline{\alpha}_3 = 459.5402$, and

$$P_1 = \begin{pmatrix} 1.6566 & 0 \\ 0 & 1.4943 \end{pmatrix},$$
$$P_2 = \begin{pmatrix} 1.3220 & 0 \\ 0 & 1.1359 \end{pmatrix}, \quad (87)$$
$$P_3 = \begin{pmatrix} 1.3732 & 0 \\ 0 & 1.2426 \end{pmatrix},$$

where $\lambda_1 = 9.8696$ for $\Omega = \{(x_1, x_2)^T \in R^2 : |x_j| < \sqrt{2}, j = 1, 2\}$.

Hence, Corollary 23 yields that the null solution of Markovian jumping stochastic system (8) is almost sure exponentially stable with the allowable upper bounds of time delays $\tau = 99.89$.

Remark 39. Example 38 illustrates that LMI-based criteria of Corollaries 22 and 23 are more effective and feasible than some existing results (see, e.g., [58, Theorem 3.1]) due to the significant improvement in the allowable upper bounds of time delays.

There are some interesting comparisons among Examples 35, 36, and 38 as follows.

From Table 2, we know that the ambiguity of the fuzzy system affect the analysis and judgement on the stability. The maximum allowable upper bounds decrease when the fuzzy factors occur. In addition, both the randomicity of Markovian jumping and nonlinear p-Laplace diffusion exercised a malign influence on judging the stability.

Remark 40. Table 2 also illustrates that the diffusion item plays an active role in the LMI-based criterion of Corollary 23.

Remark 41. From Examples 28, 30, 32, 35, 36, and 38 we learn that owing to some novel techniques employed in this paper (see, Remarks 12 and 19), LMI-based criteria of Theorems 11 and 18, and their corollaries are more effective and feasible than recent related results (Remarks 12, 14, 17, 19, 21, and 24), and improve significantly the allowable upper bounds of time delays (Remarks 29, 31, 33, 34, 37, and 39).

6. Conclusions

In this paper, the stability for delayed nonlinear reaction-diffusion Markovian jumping stochastic fuzzy Cohen-Grossberg neural networks is investigated. The fuzzy factors and the nonlinear p-Laplace diffusion bring a great difficulty in setting up the LMI-based criteria for the stability. By way of some variational methods in $W^{1,p}(\Omega)$, *Itô* formula, Dynkin formula, the semi-martingale convergence theorem, Schur Complement Theorem and LMI technique, the LMI-based criteria on the (robust) exponential stability and almost sure exponential (robust) stability are finally obtained, the feasibility of which can efficiently be computed and confirmed by computer matlab LMI toolbox. As the stability of the nonlinear p-Laplace diffusion fuzzy CGNNs has never been studied before, we compare some corollaries of our main results with existing results in numerical examples. In Examples 30 and 36, Corollaries 13 and 20 judge what existing criteria cannot do. Numerical examples and simulations illustrate the effectiveness and less conservatism of the proposed method due to the significant improvement in the allowable upper bounds of time delays (see, Remarks 29, 31, 33 and Remarks 34, 37, 39). Tables 1 and 2 show that fuzzy factors and stochastic factors give some difficulties to judge the stability, for the allowable upper bounds of time delays decrease when fuzzy factors and stochastic factors occur. In addition, when $p = 2$, the diffusion item plays a positive role. So for the future work, the p-Laplace diffusion item ($p > 1$ and $p \neq 2$) should play its role in stability criteria, which still remains open and challenging.

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