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Research Article

Observer-Based Robust H_∞ Control for Switched Stochastic Systems with Time-Varying Delay

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This paper investigates the problem of observer-based robust H_∞ control for a class of switched stochastic systems with time-varying delay. Based on the average dwell time method, an exponential stability criterion for switched stochastic delay systems is proposed. Then, H_∞ performance analysis and observer-based robust H_∞ controller design for the underlying systems are developed. Finally, a numerical example is presented to illustrate the effectiveness of the proposed approach.

1. Introduction

Switched systems are a kind of hybrid dynamical systems composed of a set of continuous-time subsystems or discrete-time subsystems and a switching law that orchestrates the switching between them. Switched systems have attracted increasing attention during the past decades because of their wide applications in real-world systems, such as robot control systems [1], networked control systems [2, 3]. Many useful results on stability analysis and control synthesis for such systems have been reported in [4–8]. For example, H_∞ control of switched linear discrete-time systems with polytopic uncertainties was investigated in [8].

It is well known that the time delay phenomenon is frequently encountered in engineering and social systems, and the existence of which may cause instability or undesirable system performance. Therefore, many research efforts have been devoted to the study of switched time delay systems [9–15]. On the other hand, stochastic systems have attracted considerable attention during the past several decades. Early results can be found in [16], and the H_∞ control problem of stochastic systems with time delay was investigated in [17, 18]. The study on H_2/H_∞ control of stochastic system was developed in [19]. Stability analysis on stochastic system with multiple delays was proposed in [20]. Moreover, some results on switched stochastic systems with and without time delay have been obtained (see [21–25] and the references cited therein).

In many real-world systems, state feedback control will fail to guarantee the stabilization because the states of the systems are not all measurable [26]. One of the key approaches to solve the problem is to reconstruct the states of the systems and realize the required feedback control. Hence, the observer-based control has been an interesting topic in control theory. Some results on observer-based control for stochastic delay systems or Markovian jump systems have been presented in [27–29]. However, to the best of our knowledge, the problem of observer-based robust H_∞ control for switched stochastic systems with time delay has not been fully studied, which motivates the present study.

In this paper, we aim to design an observer-based robust H_∞ controller for switched stochastic systems with time delay such that the closed-loop system is mean-square exponentially stable with H_∞ performance. The major contributions of the work can be summarized as follows: (1) a new Lyapunov-Krasovskii functional candidate is introduced to derive the exponential stability of switched stochastic systems with time delay, and the free-weighting matrix method is employed to reduce the conservatism; (2) an observer-based robust H_∞ controller design scheme for the underlying systems is proposed.

The remainder of the paper is organized as follows. In Section 2, problem statement and some useful lemmas are given. In Section 3, the main results are presented. In Section 4, a numerical example is given to illustrate the effectiveness of the proposed approach. Finally, concluding remarks are provided in Section 5.

Notation. In this paper, the superscript "T" denotes the transpose, and the symmetric term in a matrix is denoted by $*$. The notation $X > Y$ $(X \geq Y)$ means that $X - Y$ is positive definite (positive semidefinite, respectively). R^n denotes the n-dimensional Euclidean space. $\|x(t)\|$ denotes the Euclidean norm. $|a|$ denotes the absolute value of a. $L_2[t_0, \infty)$ is the space of square integrable functions on $[t_0, \infty)$, and t_0 is the initial time. $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ denote the maximum and minimum eigenvalues of P, respectively. A^+ denotes the Moore-Penrose pseudoinverse of A. I is the identity matrix. $\operatorname{diag}\{a_i\}$ denotes a diagonal matrix with the diagonal elements $a_i, i = 1, 2, \ldots, n$.

2. Problem Formulation and Preliminaries

Consider the following switched stochastic system with time delay:

$$\begin{aligned} dx(t) = {} & \left[\widehat{A}_{\sigma(t)} x(t) + \widehat{A}_{\tau\sigma(t)} x(t - \tau(t)) \right. \\ & \left. + B_{\sigma(t)} u(t) + G_{\sigma(t)} v(t) \right] dt \qquad \text{(1a)} \\ & + \widehat{D}_{\sigma(t)} x(t)\, dw(t), \end{aligned}$$

$$y(t) = C_{\sigma(t)} x(t), \qquad \text{(1b)}$$

$$z(t) = J_{\sigma(t)} x(t), \qquad \text{(1c)}$$

$$x(t) = \varphi(t), \quad t \in [t_0 - \tau, t_0], \qquad \text{(1d)}$$

where $x(t) \in R^n$ is the state vector, $\varphi(t) \in R^n$ is the initial state function, $u(t) \in R^l$ is the control input, $v(t) \in R^p$ is the disturbance input which is assumed to belong to $L_2[t_0, \infty]$, $y(t) \in R^r$ is the measurable output, $z(t) \in R^q$ is the controlled output, and $w(t)$ is a one-dimensional zero-mean Wiener process on a probability space $(\Omega \ F \ P)$ and satisfies

$$E\{dw(t)\} = 0, \qquad E\{dw^2(t)\} = dt, \qquad (2)$$

where Ω is the sample space, F is σ-algebras of subsets of the sample space, P is the probability measure on F, and $E\{\cdot\}$ is the expectation operator. $\tau(t)$ is the time delay satisfying

$$0 \leq \tau(t) \leq \tau, \qquad \dot{\tau}(t) \leq \tau_d < 1, \qquad (3)$$

where τ and τ_d are known constants.

The function $\sigma(t) : [t_0, \infty] \to \underline{N} = \{1, 2, \ldots, N\}$ is a switching signal which is deterministic, piecewise constant, and right continuous. The switching sequence can be described as $\sigma : \{(t_0, \sigma(t_0)), (t_1, \sigma(t_1)), \ldots, (t_k, \sigma(t_k))\}, \sigma(t_k) \in \underline{N}$, where t_0 is the initial instant and t_k denotes the kth switching instant. Moreover, $\sigma(t) = i$ means that the ith subsystem is activated. For all $i \in \underline{N}$, B_i, C_i, J_i, and G_i are known real-value matrices with appropriate dimensions, $\widehat{A}_i$, $\widehat{A}_{\tau i}$, and $\widehat{D}_i$ are uncertain real matrices with appropriate dimensions and can be written as

$$\left[\widehat{A}_i \ \ \widehat{A}_{\tau i} \ \ \widehat{D}_i\right] = \left[A_i + \Delta A_i \ \ A_{\tau i} + \Delta A_{\tau i} \ \ D_i + \Delta D_i\right], \quad (4)$$

where $\left[\Delta A_i \ \ \Delta A_{\tau i} \ \ \Delta D_i\right] = H_i F_i(t) \left[E_{1i} \ \ E_{2i} \ \ E_{3i}\right]$, A_i, $A_{\tau i}$, D_i, H_i, E_{1i}, E_{2i}, and E_{3i} are known real-value matrices with appropriate dimensions, and $F_i(t)$ is an unknown time-varying matrix that satisfies

$$F_i^T(t) F_i(t) \leq I. \qquad (5)$$

The state feedback controller is designed as $u(t) = K_{\sigma(t)} x(t)$. In actual operation, however, the states of the systems are not all measurable. The following switched system is constructed to estimate the state of system (1a), (1b), (1c), and (1d):

$$\begin{aligned} d\widehat{x}(t) = {} & \left[A_{\sigma(t)} \widehat{x}(t) + A_{\tau\sigma(t)} \widehat{x}(t - \tau(t)) \right. \\ & \left. + B_{\sigma(t)} u(t) + L_{\sigma(t)} \left(y(t) - \widehat{y}(t)\right)\right] dt, \end{aligned} \qquad \text{(6a)}$$

$$\widehat{y}(t) = C_{\sigma(t)} \widehat{x}(t), \qquad \text{(6b)}$$

$$\widehat{x}(t) = \phi(t), \quad t \in [t_0 - \tau, t_0], \qquad \text{(6c)}$$

where $\widehat{x}(t) \in R^n$ is the estimation of $x(t)$, $\widehat{y}(t) \in R^r$ is the observer output, and $\phi(t) \in R^n$ is the initial observer state function. The real state feedback controller becomes $u(t) = K_{\sigma(t)} \widehat{x}(t)$. L_i and K_i are the observer gains and controller gains to be determined, respectively.

Remark 1. It is noted that the observer-based H_∞ control for stochastic systems or Markovian jump systems was considered in [27–29]. However, the results in the aforementioned papers cannot be directly applied to the switched stochastic system considered in the paper. This motivates our study. Also, the proposed observer in (6a), (6b), and (6c) is a switching observer, which is different from the existing ones given in [27–29].

From systems (1a), (1b), (1c), and (1d) and (6a), (6b), and (6c), we can obtain the following augmented closed-loop system:

$$\begin{aligned} d\xi(t) = {} & \left[\overline{A}_{\sigma(t)} \xi(t) + \overline{A}_{\tau\sigma(t)} \xi(t - \tau(t)) \right. \\ & \left. + \overline{G}_{\sigma(t)} v(t)\right] dt + \overline{D}_{\sigma(t)} \xi(t)\, dw(t), \end{aligned} \qquad \text{(7a)}$$

$$z(t) = \overline{J}_{\sigma(t)} \xi(t), \qquad \text{(7b)}$$

$$\xi(t) = \left[\varphi^T(t) \ \ \varphi^T(t) - \phi^T(t)\right]^T, \quad t \in [t_0 - \tau, t_0], \qquad \text{(7c)}$$

where $\xi(t) = \left[x^T(t) \ \ e^T(t)\right]^T$, $e(t) = x(t) - \widehat{x}(t)$ denotes the state estimated error.

For $\sigma(t)=i$, the parameters of system (7a), (7b), and (7c) are given as follows:

$$\overline{A}_i=\widetilde{A}_i+\Delta\widetilde{A}_i,\qquad \widetilde{A}_i=\begin{bmatrix}A_i+B_iK_i & -B_iK_i\\ 0 & A_i-L_iC_i\end{bmatrix},$$
$$\Delta\widetilde{A}_i=\widetilde{H}_i\widetilde{F}_i\widetilde{E}_{1i},$$
$$\overline{A}_{\tau i}=\widetilde{A}_{\tau i}+\Delta\widetilde{A}_{\tau i},\qquad \widetilde{A}_{\tau i}=\begin{bmatrix}A_{\tau i} & 0\\ 0 & A_{\tau i}\end{bmatrix},$$
$$\Delta\widetilde{A}_{\tau i}=\widetilde{H}_i\widetilde{F}_i\widetilde{E}_{2i},$$
$$\overline{D}_i=\widetilde{D}_i+\Delta\widetilde{D}_i,\qquad \widetilde{D}_i=\begin{bmatrix}D_i & 0\\ D_i & 0\end{bmatrix},\tag{8}$$
$$\Delta\widetilde{D}_i=\widetilde{H}_i\widetilde{F}_i\widetilde{E}_{3i},$$
$$\overline{G}_i=\widetilde{G}_i=\begin{bmatrix}G_i^T & G_i^T\end{bmatrix}^T,\qquad \overline{J}_i=\widetilde{J}_i=\begin{bmatrix}J_i & 0\end{bmatrix},$$
$$\widetilde{H}_i=\begin{bmatrix}H_i & 0\\ 0 & H_i\end{bmatrix},\qquad \widetilde{F}_i=\begin{bmatrix}F_i & 0\\ 0 & F_i\end{bmatrix},$$
$$\widetilde{E}_{gi}=\begin{bmatrix}E_{gi} & 0\\ E_{gi} & 0\end{bmatrix},\qquad g=1,2,3.$$

Assumption 2. B_i is full row rank, for all $i\in\underline{N}$.

Definition 3. System (1a), (1b), (1c), and (1d) with $v(t)=0$ is said to be mean-square exponentially stable under the switching signal $\sigma(t)$ if there exist scalars $\kappa>0$ and $\alpha>0$, such that the solution $x(t)$ of the system satisfies

$$E\left\{\|x(t)\|^2\right\}\le\kappa e^{-\alpha(t-t_0)}\sup_{-\tau\le\theta\le0}E\left\{\|x(t_0+\theta)\|^2\right\},\quad\forall t\ge t_0.\tag{9}$$

Definition 4 (see [24]). For any $T_2>T_1\ge t_0$, let $N_\sigma(T_1,T_2)$ denote the switching number of $\sigma(t)$ on an interval $[T_1,T_2)$. If

$$N_\sigma(T_1,T_2)\le N_0+\frac{T_2-T_1}{T_\alpha}\tag{10}$$

holds for given constants $N_0\ge0$ and $T_\alpha>0$, then the constant T_α is called the average dwell time. As commonly used in the literature, some chooses $N_0=0$.

Definition 5 (see [30]). For any $\lambda>0$ and $\gamma>0$, system (7a), (7b), and (7c) is said to be mean-square exponentially stable with a prescribed weighted H_∞ performance level γ if the following conditions are satisfied:

(1) when $v(t)=0$, system (7a), (7b), and (7c) is mean-square exponentially stable;

(2) under the zero initial condition, the output $z(t)$ satisfies

$$E\left\{\int_{t_0}^{\infty}e^{-\lambda(s-t_0)}z^T(s)z(s)\,ds\right\}\le\gamma^2\int_{t_0}^{\infty}v^T(s)v(s)\,ds,\quad\forall v(t)\in L_2[t_0,\infty).\tag{11}$$

Lemma 6 (see [31]). *For any positive symmetric constant matrix $M\in R^{n\times n}$ and a scalar $r>0$, if there exists a vector functiong $:[0,r]\to R^n$ such that integrations in the following are well defined, then the following inequality holds*

$$r\int_0^r g^T(s)Mg(s)\,ds\ge\left[\int_0^r g(s)\,ds\right]^T M\left[\int_0^r g(s)\,ds\right].\tag{12}$$

Lemma 7 (see [32]). *Let U, V, W, and X be constant matrices of appropriate dimensions with X satisfying $X=X^T$, then for all $V^TV\le I$, $X+UVW+W^TV^TU^T<0$ if and only if there exists a scalar $\varepsilon>0$ such that $X+\varepsilon UU^T+\varepsilon^{-1}W^TW<0$.*

The objective of this paper is to design an observer-based robust H_∞ controller for switched stochastic delay system (1a), (1b), (1c), and (1d) such that the augmented closed-loop system (7a), (7b), and (7c) is mean-square exponentially stable with a prescribed weighted H_∞ performance level γ.

3. Main Results

3.1. Stability Analysis. In this subsection, in order to obtain the main results, we first focus on the problem of stability analysis for the following switched stochastic systems with time delay

$$dx(t)=\left[A_{\sigma(t)}x(t)+A_{\tau\sigma(t)}x(t-\tau(t))\right]dt+D_{\sigma(t)}x(t)\,dw(t),\tag{13a}$$
$$x(t)=\varphi(t),\quad t\in[t_0-\tau,t_0].\tag{13b}$$

Theorem 8. *For a given scalar $\alpha>0$, if there exist symmetric positive definite matrices P_i, Q_i, and R_i and any matrices S_i such that*

$$\begin{bmatrix}\sum_{11}^{i} & P_iA_{\tau i} & S_i^T & S_i^T & D_i^TP_i\\ * & -(1-\tau_d)Q_i & -S_i^T & -S_i^T & 0\\ * & * & -S_i-S_i^T & -S_i^T & 0\\ * & * & * & -\tau^{-1}R_i & 0\\ * & * & * & * & -P_i\end{bmatrix}<0,\quad\forall i\in\underline{N}\tag{14}$$

then system (13a) *and* (13b) *is mean-square exponentially stable under arbitrary switching signal with the average dwell time*

$$T_\alpha>T_\alpha^*=\tau+\frac{\ln(\chi\mu)}{\lambda},\tag{15}$$

where μ, χ, and λ satisfy

$$P_i\le\mu P_j,\quad Q_i\le\mu Q_j,\quad R_i\le\mu R_j,$$
$$Q_i\le\beta_iP_i,\quad R_i\le\beta_iP_i,\quad\forall i,j\in\underline{N},\tag{16}$$

$$\lambda + \beta_i (1+\tau)\left(e^{\lambda\tau} - 1\right) \le \alpha,$$
$$\chi = \max_{i\in \underline{N}} \chi_i, \quad \chi_i = 1 + \tau\beta_i (1+\tau)\left(e^{\lambda\tau} - 1\right), \tag{17}$$
$$\sum_{11}^{i} = A_i^T P_i + P_i A_i + Q_i + \tau R_i + \alpha P_i.$$

Proof. Let $Y(t) = A_{\sigma(t)} x(t) + A_{\tau\sigma(t)} x(t-\tau(t))$, then (13a) can be described as

$$dx(t) = Y(t)\,dt + D_{\sigma(t)} x(t)\,dw(t). \tag{18}$$

Choose the following Lyapunov functional candidate for the ith subsystem

$$V_i(t, x(t)) = V_{1,i}(t, x(t)) + V_{2,i}(t, x(t)) + V_{3,i}(t, x(t)), \tag{19}$$

where

$$V_{1,i}(t, x(t)) = x^T(t) P_i x(t),$$
$$V_{2,i}(t, x(t)) = \int_{t-\tau(t)}^{t} x^T(s) Q_i x(s)\,ds, \tag{20}$$
$$V_{3,i}(t, x(t)) = \int_0^{\tau}\int_{t-\theta}^{t} x^T(s) R_i x(s)\,ds\,d\theta.$$

For the sake of simplicity, $V_i(t, x(t))$ is written as $V_i(t)$ in this paper. According to Itô's formula, along the trajectory of the ith subsystem, we have

$$dV_i(t) = \sum_{g=1}^{3} dV_{g,i}, \tag{21}$$

where

$$dV_{1,i}(t) = \mathscr{L}V_{1,i}(t)\,dt + 2x^T(t) P_i D_i x(t)\,dw(t),$$
$$\mathscr{L}V_{1,i}(t) = 2x^T(t) P_i\left[A_i x(t) + A_{\tau i} x(t-\tau(t))\right] + x^T(t) D_i^T P_i D_i x(t),$$
$$\begin{aligned} dV_{2,i}(t) &= \left[x^T(t) Q_i x(t) - (1-\dot{\tau}(t)) x^T(t-\tau(t)) \times Q_i x(t-\tau(t))\right] dt \\ &\le \left[x^T(t) Q_i x(t) - (1-\tau_d) x^T(t-\tau(t)) \times Q_i x(t-\tau(t))\right] dt, \end{aligned} \tag{22}$$
$$dV_{3,i}(t) = \left[\tau x^T(t) R_i x(t) - \int_{t-\tau}^{t} x^T(s) R_i x(s)\,ds\right] dt.$$

According to Lemma 6, we have

$$\begin{aligned} dV_{3,i}(t) &\le \left\{\tau x^T(t) R_i x(t) - \tau^{-1}\left[\int_{t-\tau(t)}^{t} x(s)\,ds\right]^T \times R_i\left[\int_{t-\tau(t)}^{t} x(s)\,ds\right]\right\} dt. \end{aligned} \tag{23}$$

Integrating both sides of (18) from $t-\tau(t)$ to t, we have

$$x(t) - x(t-\tau(t)) = \int_{t-\tau(t)}^{t} Y(s)\,ds + \int_{t-\tau(t)}^{t} D_{\sigma(s)} x(s)\,dw(s). \tag{24}$$

Thus,

$$2\left[\int_{t-\tau(t)}^{t} x(s)\,ds + \int_{t-\tau(t)}^{t} Y(s)\,ds\right]^T S_i \eta(t)\,dt = 0, \tag{25}$$

where $\eta(t) = x(t) - x(t-\tau(t)) - \int_{t-\tau(t)}^{t} Y(s) ds - \int_{t-\tau(t)}^{t} D_{\sigma(s)} x(s) dw(s)$.

Combining (21)–(25) leads to

$$dV_i(t) \le \mathscr{L}V_i(t)\,dt + W_i(t), \tag{26}$$

where

$$\mathscr{L}V_i(t) = \varsigma^T(t)\,\Theta_i\,\varsigma(t),$$
$$W_i(t) = 2x^T(t) P_i D_i x(t)\,dw(t) - 2\left[\int_{t-\tau(t)}^{t} x(s)\,ds + \int_{t-\tau(t)}^{t} Y(s)\,ds\right]^T \times S_i\left[\int_{t-\tau(t)}^{t} D_{\sigma(s)} x(s)\,dw(s)\right] dt,$$
$$\varsigma(t) = \left[x^T(t) \quad x^T(t-\tau(t)) \quad \int_{t-\tau(t)}^{t} Y^T(s)\,ds \quad \int_{t-\tau(t)}^{t} x^T(s)\,ds\right]^T,$$
$$\Theta_i = \begin{bmatrix} \Theta_{11}^i & P_i A_{\tau i} & S_i^T & S_i^T \\ * & -(1-\tau_d) Q_i & -S_i^T & -S_i^T \\ * & * & -S_i - S_i^T & -S_i^T \\ * & * & * & -\tau^{-1} R_i \end{bmatrix},$$
$$\Theta_{11}^i = A_i^T P_i + P_i A_i + Q_i + D_i^T P_i D_i + \tau R_i. \tag{27}$$

By using the Schur complement, we obtain from (14) that

$$\mathscr{L}V_i(t) < -\alpha V_{1,i}(t) < 0. \tag{28}$$

According to (26), one obtains that

$$dV_i(t) \le \mathscr{L}V_i(t)\,dt + W_i(t) < -\alpha V_{1,i}(t)\,dt + W_i(t). \tag{29}$$

Then, taking mathematical expectation, we have

$$E\left\{\frac{dV_i(t)}{dt}\right\} \le E\left\{\mathscr{L}V_i(t)\right\} < -\alpha E\left\{V_{1,i}(t)\right\} < 0. \tag{30}$$

From (16) and (19), we obtain that

$$\begin{aligned} V_i(t) &\le V_{1,i}(t) + \int_{t-\tau}^{t} x^T(s) Q_i x(s)\,ds + \tau\int_{t-\tau}^{t} x^T(s) R_i x(s)\,ds \\ &\le V_{1,i}(t) + \beta_i(1+\tau)\int_{t-\tau}^{t} V_{1,i}(s)\,ds. \end{aligned} \tag{31}$$

Let $\sigma(t_k) = i$. Then, using Itô's formula, we can obtain that for $t \in [t_k, t_{k+1})$,

$$\begin{aligned} E\left\{e^{\lambda t}V_i(t)\right\} - E\left\{e^{\lambda t_k}V_i(t_k)\right\} &= E\left\{\int_{t_k}^{t} \mathscr{L}\left(e^{\lambda s}V_i(s)\right)ds\right\} \\ &\le E\left\{\int_{t_k}^{t} e^{\lambda s}\left[\lambda V_{1,i}(s) + \lambda\beta_i(1+\tau)\right.\right. \\ &\qquad \left.\left.\times \int_{s-\tau}^{s} V_{1,i}(\vartheta)\,d\vartheta - \alpha V_{1,i}(s)\right]ds\right\}. \end{aligned} \tag{32}$$

Notice that

$$\begin{aligned} \int_{t_k}^{t} e^{\lambda s}ds\int_{s-\tau}^{s} V_{1,i}(\vartheta)\,d\vartheta &= \int_{t_k-\tau}^{t} d\vartheta\int_{\vartheta}^{\vartheta+\tau} e^{\lambda s}V_{1,i}(\vartheta)\,ds \\ &= \frac{1}{\lambda}\left(e^{\lambda\tau}-1\right)\int_{t_k-\tau}^{t} e^{\lambda s}V_{1,i}(s)\,ds. \end{aligned} \tag{33}$$

Thus, it can be obtained that

$$\begin{aligned} &E\left\{e^{\lambda t}V_i(t)\right\} - E\left\{e^{\lambda t_k}V_i(t_k)\right\} \\ &\quad\le E\left\{\int_{t_k}^{t} e^{\lambda s}\left[\lambda + \beta_i(1+\tau)\left(e^{\lambda\tau}-1\right) - \alpha\right]V_{1,i}(s)\,ds\right\} \\ &\quad + E\left\{\Omega_i(t_k)\right\}, \end{aligned} \tag{34}$$

where

$$\begin{aligned} \Omega_i(t_k) &= \beta_i(1+\tau)\left(e^{\lambda\tau}-1\right)\int_{t_k-\tau}^{t_k} e^{\lambda s}V_{1,i}(s)\,ds \\ &\le \beta_i(1+\tau)\left(e^{\lambda\tau}-1\right)e^{\lambda t_k}\int_{t_k-\tau}^{t_k} V_{1,i}(s)\,ds \\ &\le \tau\beta_i(1+\tau)\left(e^{\lambda\tau}-1\right)e^{\lambda t_k}\sup_{-\tau\le\theta\le 0} V_{1,i}(t_k+\theta) \\ &\le \tau\beta_i(1+\tau)\left(e^{\lambda\tau}-1\right)e^{\lambda t_k}\sup_{-\tau\le\theta\le 0} V_i(t_k+\theta). \end{aligned} \tag{35}$$

Noticing that $E\{V(t)\} \le \sup_{-\tau\le\theta\le 0} E\{V(t+\theta)\}$, one gets

$$\begin{aligned} E\left\{V_i(t)\right\} &\le \chi_i e^{-\lambda(t-t_k)}\sup_{-\tau\le\theta\le 0} E\left\{V_i(t_k+\theta)\right\} \\ &\le \chi e^{-\lambda(t-t_k)}\sup_{-\tau\le\theta\le 0} E\left\{V_i(t_k+\theta)\right\}. \end{aligned} \tag{36}$$

For any $t \in [t_k, t_{k+1})$, from (16) and (36), it follows that

$$\begin{aligned} E\{V(t)\} &= E\{V_i(t)\} \\ &\le \chi e^{-\lambda(t-t_k)}\sup_{-\tau\le\theta\le 0} E\left\{V_{\sigma(t_k)}(t_k+\theta)\right\} \\ &\le \chi\mu e^{-\lambda(t-t_k)}\sup_{-\tau\le\theta\le 0} E\left\{V_{\sigma(t_k^-)}(t_k+\theta)\right\} \\ &\le \chi^2\mu e^{-\lambda(t-t_k)}e^{-\lambda(t_k+\theta-t_{k-1})} \\ &\quad\times \sup_{-\tau\le\theta\le 0} E\left\{V_{\sigma(t_{k-1})}(t_{k-1}+\theta)\right\} \\ &\le \chi^2\mu e^{\lambda\tau}e^{-\lambda(t-t_{k-1})}\sup_{-\tau\le\theta\le 0} E\left\{V_{\sigma(t_{k-1})}(t_{k-1}+\theta)\right\} \\ &\le \chi^3\left(\mu e^{\lambda\tau}\right)^2 e^{-\lambda(t-t_{k-2})}\sup_{-\tau\le\theta\le 0} E\left\{V_{\sigma(t_{k-2})}(t_{k-2}+\theta)\right\} \\ &\le \cdots \\ &\le \chi\left(\chi\mu e^{\lambda\tau}\right)^{N_\sigma(t_0,t)}e^{-\lambda(t-t_0)}\sup_{-\tau\le\theta\le 0} E\left\{V_{\sigma(t_0)}(t_0+\theta)\right\}. \end{aligned} \tag{37}$$

When (15) holds, noticing that $N_\sigma(t_0,t) \le (t-t_0)/T_\alpha$, one has

$$\begin{aligned} E\left\{\|x(t)\|^2\right\} &\le \frac{b}{a}\chi e^{-(\lambda-(\lambda\tau+\ln(\chi\mu))/T_\alpha)(t-t_0)} \\ &\quad\times \sup_{-\tau\le\theta\le 0} E\left\{\|x(t_0+\theta)\|^2\right\}, \end{aligned} \tag{38}$$

where $a = \min_{\forall i\in\underline{N}}\lambda_{\min}(P_i)$, $b = \max_{\forall i\in\underline{N}}\lambda_{\max}(P_i) + \tau\max_{\forall i\in\underline{N}}\lambda_{\max}(Q_i) + (\tau^2/2)\max_{\forall i\in\underline{N}}\lambda_{\max}(R_i)$.

The proof is completed. □

Remark 9. In the derivation of Theorem 8, a new Lyapunov-Krasovskii functional candidate is constructed for the stability analysis of switched stochastic systems with time delay, and it is different from the ones given in [9–15]. Also, the free-weighting matrix method is utilized to reduce the conservatism.

Remark 10. If $\mu = 1$ in (15), which leads to $P_i = P_j$, $Q_i = Q_j$, and $R_i = R_j$, for all $i, j \in \underline{N}$, then system (13a) and (13b) possesses a common Lyapunov function, and the switching signals can be arbitrary.

When $w(t) = 0$, system (13a) and (13b) becomes the following switched system with time delay:

$$\dot{x}(t) = A_{\sigma(t)}x(t) + A_{\tau\sigma(t)}x(t-\tau(t)), \tag{39a}$$

$$x(t) = \varphi(t), \quad t \in [t_0-\tau, t_0]. \tag{39b}$$

From Theorem 8, we can readily get the exponential stability criterion for switched system (39a) and (39b).

Corollary 11. *Consider system* (39a) *and* (39b), *for a given scalar* $\alpha > 0$, *if there exist symmetric positive definite matrices* P_i, Q_i, *and* R_i *and any matrices* S_i *such that*

$$\begin{bmatrix} \sum_{11}^{i} & P_i A_{\tau i} & S_i^T & S_i^T \\ * & -(1-\tau_d) Q_i & -S_i^T & -S_i^T \\ * & * & -S_i - S_i^T & -S_i^T \\ * & * & * & -\tau^{-1} R_i \end{bmatrix} < 0, \quad \forall i \in \underline{N} \tag{40}$$

then system (39a) *and* (39b) *is exponentially stable under arbitrary switching signal with the average dwell time scheme* (15).

3.2. H_∞ Performance Analysis. In the sequel, we will investigate the problem of H_∞ performance analysis for switched stochastic systems with time delay. Consider the following system:

$$dx(t) = \left[A_{\sigma(t)} x(t) + A_{\tau\sigma(t)} x(t-\tau(t)) + G_{\sigma(t)} v(t)\right] dt + D_{\sigma(t)} x(t)\, dw(t), \tag{41a}$$

$$z(t) = J_{\sigma(t)} x(t), \tag{41b}$$

$$x(t) = \varphi(t), \quad t \in [t_0 - \tau, t_0]. \tag{41c}$$

Theorem 12. *For a given scalar* $\alpha > 0$, *if there exist symmetric positive definite matrices* P_i, Q_i, *and* R_i *and any matrices* S_i *such that*

$$\begin{bmatrix} \prod_{11}^{i} & P_i A_{\tau i} & S_i^T & S_i^T & P_i G_i & D_i^T P_i & J_i^T \\ * & -(1-\tau_d) Q_i & -S_i^T & -S_i^T & 0 & 0 & 0 \\ * & * & -S_i - S_i^T & -S_i^T & 0 & 0 & 0 \\ * & * & * & -\tau^{-1} R_i & 0 & 0 & 0 \\ * & * & * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & * & * & -P_i & 0 \\ * & * & * & * & * & * & -I \end{bmatrix} < 0, \quad \forall i \in \underline{N} \tag{42}$$

then system (41a), (41b), *and* (41c) *is mean-square exponentially stable with a weighted prescribed H_∞ performance level γ under arbitrary switching signal with the average dwell time*

$$T_\alpha > T_\alpha^* = \tau + \frac{\ln(\chi\mu)}{\lambda}, \tag{43}$$

where μ, χ, *and* λ *satisfy*

$$\begin{gathered} P_i \le \mu P_j, \quad Q_i \le \mu Q_j, \quad R_i \le \mu R_j, \\ Q_i \le \beta_i P_i, \quad R_i \le \beta_i P_i, \quad \forall i, j \in \underline{N}, \end{gathered} \tag{44}$$

$$\begin{gathered} \lambda + \beta_i (1+\tau)\left(e^{\lambda\tau} - 1\right) \le \alpha, \\ \chi = \max_{i \in \underline{N}} \chi_i, \quad \chi_i = 1 + \tau\beta_i (1+\tau)\left(e^{\lambda\tau} - 1\right), \\ \prod_{11}^{i} = A_i^T P_i + P_i A_i + Q_i + \tau R_i + \alpha P_i. \end{gathered} \tag{45}$$

Proof. We can easily obtain that (14) is satisfied if (42) holds. Thus, system (41a), (41b), and (41c) with $v(t) = 0$ is mean-square exponentially stable.

When $v(t) \ne 0$, let

$$\Gamma(t) = z^T(t) z(t) - \gamma^2 v^T(t) v(t). \tag{46}$$

Choosing the same Lyapunov functional candidate as (19) and following the proof line of Theorem 8, we have

$$dV_i(t) \le \mathscr{L} V_i(t)\, dt + W_i(t), \tag{47}$$

where

$$\begin{aligned} W_i(t) = {} & 2x^T(t) P_i D_i x(t)\, dw(t) \\ & - 2\left[\int_{t-\tau(t)}^{t} x(s) ds + \int_{t-\tau(t)}^{t} Y(s) ds\right]^T \\ & \times S_i \left[\int_{t-\tau(t)}^{t} D_{\sigma(s)} x(s)\, dw(s)\right] dt, \end{aligned} \tag{48}$$

and $\mathscr{L} V_i(t)$ satisfies

$$\begin{gathered} \mathscr{L} V_i(t) + \Gamma(t) = \bar{\varsigma}^T(t) \overline{\Theta}_i \bar{\varsigma}(t), \\ \bar{\varsigma}(t) = \left[x^T(t) \;\; x^T(t-\tau(t)) \;\; \int_{t-\tau(t)}^{t} Y^T(s) ds \;\; \int_{t-\tau(t)}^{t} x^T(s) ds \;\; v^T(t)\right]^T, \\ \overline{\Theta}_i = \begin{bmatrix} \overline{\Theta}_{11}^i & P_i A_{\tau i} & S_i^T & S_i^T & P_i G_i \\ * & -(1-\tau_d) Q_i & -S_i^T & -S_i^T & 0 \\ * & * & -S_i - S_i^T & -S_i^T & 0 \\ * & * & * & -\tau^{-1} R_i & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix}, \\ \overline{\Theta}_{11}^i = A_i^T P_i + P_i A_i + Q_i + D_i^T P_i D_i + \tau R_i + J_i^T J_i. \end{gathered} \tag{49}$$

Using the Schur complement, from (42), we get

$$\mathscr{L} V_i(t) + \Gamma(t) < -\alpha V_{1,i}(t) < 0. \tag{50}$$

It follows that

$$E\{dV_i(t)\} \le E\{\mathscr{L} V_i(t)\, dt\}. \tag{51}$$

From (44), we obtain that

$$E\left\{V_{\sigma(t_k)}\left(t_k\right)\right\} \leq \mu E\left\{V_{\sigma(t_{k-1})}\left(t_k\right)\right\}. \tag{52}$$

For any $t \in [t_k, t_{k+1})$, using Itô's formula and taking the mathematical expectation, one has

$$\begin{aligned}
E\left\{V_{\sigma(t)}(t)\right\} &= E\left\{V_{\sigma(t_k)}\left(t_k\right)\right\} + \int_{t_k}^{t} E\left\{dV_{\sigma(t_k)}(s)\right\} \\
&\leq E\left\{V_{\sigma(t_k)}\left(t_k\right)\right\} + \int_{t_k}^{t} E\left\{\mathscr{L}V_{\sigma(t_k)}(s) + \Gamma(s)\right\} ds \\
&\quad - \int_{t_k}^{t} E\left\{\Gamma(s)\, ds\right\} \\
&< \mu E\left\{V_{\sigma(t_{k-1})}\left(t_k\right)\right\} - \int_{t_k}^{t} E\left\{\Gamma(s)\, ds\right\} \\
&< \mu^2 E\left\{V_{\sigma(t_{k-2})}\left(t_{k-1}\right)\right\} \\
&\quad - E\left\{\mu \int_{t_{k-1}}^{t_k} \Gamma(s)\, ds + \int_{t_k}^{t} \Gamma(s)\, ds\right\} \\
&< \cdots \\
&< \mu^{N_\sigma(t_0,t)} E\left\{V\left(t_0\right)\right\} \\
&\quad - E\left\{\mu^{N_\sigma(t_0,t)} \int_{t_0}^{t_1} \Gamma(s)\, ds \right. \\
&\qquad \left. + \mu^{N_\sigma(t_1,t)} \int_{t_1}^{t_2} \Gamma(s)\, ds + \cdots + \int_{t_k}^{t} \Gamma(s)\, ds\right\} \\
&= \mu^{N_\sigma(t_0,t)} E\left\{V\left(t_0\right)\right\} - E\left\{\int_{t_0}^{t} e^{N_\sigma(s,t)\ln\mu}\Gamma(s)\, ds\right\}.
\end{aligned} \tag{53}$$

Under the zero initial condition, we obtain that

$$E\left\{\int_{t_0}^{t} e^{N_\sigma(s,t)\ln\mu}\Gamma(s)\, ds\right\} < 0. \tag{54}$$

According to (46), one has

$$\begin{aligned}
&E\left\{\int_{t_0}^{t} e^{N_\sigma(s,t)\ln\mu} z^T(s) z(s)\right\} ds \\
&\quad < \gamma^2 \int_{t_0}^{t} e^{N_\sigma(s,t)\ln\mu} v^T(s) v(s)\, ds.
\end{aligned} \tag{55}$$

Multiplying both sides of (55) by $e^{-N_\sigma(t_0,t)\ln\mu}$ leads to

$$\begin{aligned}
&E\left\{\int_{t_0}^{t} e^{-N_\sigma(t_0,s)\ln\mu} z^T(s) z(s)\, ds\right\} \\
&\quad < \gamma^2 \int_{t_0}^{t} e^{-N_\sigma(t_0,s)\ln\mu} v^T(s) v(s)\, ds.
\end{aligned} \tag{56}$$

Noticing that $N_\sigma(t_0, s) \leq (s - t_0)/T_\alpha$ and $T_\alpha \geq \tau + \ln(\chi\mu)/\lambda \geq \ln\mu/\lambda$, one obtains that

$$E\left\{\int_{t_0}^{t} e^{-\lambda(s-t_0)} z^T(s) z(s)\, ds\right\} < \gamma^2 \int_{t_0}^{t} v^T(s) v(s)\, ds. \tag{57}$$

When $t \rightarrow \infty$, the following inequality is derived:

$$E\left\{\int_{t_0}^{\infty} e^{-\lambda(s-t_0)} z^T(s) z(s)\, ds\right\} < \gamma^2 \int_{t_0}^{\infty} v^T(s) v(s)\, ds. \tag{58}$$

The proof is completed. □

3.3. Observer-Based Robust H_∞ Stabilization. Now, we are in a position to design an observer-based robust H_∞ controller for system (1a), (1b), (1c), and (1d) such that the augmented closed-loop system (7a), (7b), and (7c) is mean-square exponentially stable with a prescribed weighted H_∞ performance level γ. Based on Theorem 12, a sufficient condition for the existence of such a controller is presented in the following theorem.

Theorem 13. *Consider system* (1a), (1b), (1c), *and* (1d), *for a given scalar* $\alpha > 0$, *if there exist scalars* $\varepsilon_i > 0$, *symmetric positive definite matrices* P_i, Q_i, *and* R_i, *and any matrices* $\widetilde{S}_i$, Y_i, *and* Z_i *such that, for all* $i \in \underline{N}$,

$$\begin{bmatrix}
\Omega_{11}^{i} & \overline{P}_i\widetilde{A}_{\tau i} & \widetilde{S}_i^T & \widetilde{S}_i^T & \overline{P}_i\widetilde{G}_i & \widetilde{D}_i^T\overline{P}_i & \widetilde{J}_i^T & \overline{P}_i\widetilde{H}_i & 0 & \varepsilon_i\widetilde{E}_{1i}^T & \varepsilon_i\widetilde{E}_{3i}^T \\
* & -\left(1-\tau_d\right)\overline{Q}_i & -\widetilde{S}_i^T & -\widetilde{S}_i^T & 0 & 0 & 0 & 0 & 0 & \varepsilon_i\widetilde{E}_{2i}^T & 0 \\
* & * & -\widetilde{S}_i - \widetilde{S}_i^T & -\widetilde{S}_i^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & -\tau^{-1}\overline{R}_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & -\gamma^2 I & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & -\overline{P}_i & 0 & 0 & \overline{P}_i\widetilde{H}_i & 0 & 0 \\
* & * & * & * & * & * & -I & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -\varepsilon_i I & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & -\varepsilon_i I & 0 & 0 \\
* & * & * & * & * & * & * & * & * & -\varepsilon_i I & 0 \\
* & * & * & * & * & * & * & * & * & * & -\varepsilon_i I
\end{bmatrix} < 0, \tag{59}$$

then there exists an observer-based controller such that system (7a), (7b), *and* (7c) *is mean-square exponentially stable with a prescribed weighted H_∞ performance level γ under arbitrary switching signal with the average dwell time*

$$T_\alpha > T_\alpha^* = \tau + \frac{\ln(\chi\mu)}{\lambda}, \tag{60}$$

where μ, χ, and λ satisfy

$$\begin{gathered} P_i \le \mu P_j, \quad Q_i \le \mu Q_j, \quad R_i \le \mu R_j, \\ Q_i \le \beta_i P_i, \quad R_i \le \beta_i P_i, \quad \forall i, j \in \underline{N}, \end{gathered} \tag{61}$$

$$\lambda + \beta_i (1+\tau)\left(e^{\lambda\tau} - 1\right) \le \alpha, \tag{62}$$

$$\begin{gathered} \chi = \max_{i \in \underline{N}} \chi_i, \quad \chi_i = 1 + \tau\beta_i (1+\tau)\left(e^{\lambda\tau} - 1\right), \\ \Omega_{11}^i = \Upsilon_i^T + \Upsilon_i + \overline{Q}_i + \tau\overline{R}_i + \alpha\overline{P}_i, \\ \Upsilon_i = \begin{bmatrix} P_i A_i + Z_i & -Z_i \\ 0 & P_i A_i - Y_i C_i \end{bmatrix}, \\ \overline{P}_i = \operatorname{diag}\{P_i, P_i\}, \quad \overline{Q}_i = \operatorname{diag}\{Q_i, Q_i\}, \\ \overline{R}_i = \operatorname{diag}\{R_i, R_i\}. \end{gathered} \tag{63}$$

Moreover, if the above conditions have a feasible solution, the controller gain matrices and the observer gain matrices can be obtained by $K_i = B_i^+ P_i^{-1} Z_i$ and $L_i = P_i^{-1} Y_i$.

Proof. According to Theorem 12, we get that system (7a), (7b), and (7c) is mean-square exponentially stable with a weighted H_∞ performance level γ if the following inequality is satisfied:

$$\begin{aligned} &\overline{\Psi}_i \\ &= \begin{bmatrix} \overline{\Psi}_{11}^i & \widetilde{P}_i \overline{A}_{\tau i} & \widetilde{S}_i^T & \widetilde{S}_i^T & \widetilde{P}_i \overline{G}_i & \overline{D}_i^T \widetilde{P}_i & \overline{J}_i^T \\ * & -(1-\tau_d)\widetilde{Q}_i & -\widetilde{S}_i^T & -\widetilde{S}_i^T & 0 & 0 & 0 \\ * & * & -\widetilde{S}_i - \widetilde{S}_i^T & -\widetilde{S}_i^T & 0 & 0 & 0 \\ * & * & * & -\tau^{-1} R_i & 0 & 0 & 0 \\ * & * & * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & * & * & -\widetilde{P}_i & 0 \\ * & * & * & * & * & * & -I \end{bmatrix} \\ &< 0, \end{aligned} \tag{64}$$

where $\overline{\Psi}_{11}^i = \widetilde{A}_i^T \widetilde{P}_i + \widetilde{P}_i \widetilde{A}_i + \widetilde{Q}_i + \tau\widetilde{R}_i + \alpha\widetilde{P}_i$, and $\widetilde{P}_i$, $\widetilde{Q}_i$, and $\widetilde{R}_i$ are symmetric positive definite matrices with appropriate dimensions.

Then, we have

$$\overline{\Psi}_i = \Psi_i + \Delta\Psi_i < 0, \tag{65}$$

where

$$\begin{aligned} &\Psi_i \\ &= \begin{bmatrix} \Psi_{11}^i & \widetilde{P}_i \widetilde{A}_{\tau i} & \widetilde{S}_i^T & \widetilde{S}_i^T & \widetilde{P}_i \widetilde{G}_i & \widetilde{D}_i^T \widetilde{P}_i & \widetilde{J}_i^T \\ * & -(1-\tau_d)\widetilde{Q}_i & -\widetilde{S}_i^T & -\widetilde{S}_i^T & 0 & 0 & 0 \\ * & * & -\widetilde{S}_i - \widetilde{S}_i^T & -\widetilde{S}_i^T & 0 & 0 & 0 \\ * & * & * & -\tau^{-1} R_i & 0 & 0 & 0 \\ * & * & * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & * & * & -\widetilde{P}_i & 0 \\ * & * & * & * & * & * & -I \end{bmatrix}, \\ &\Delta\Psi_i \\ &= \begin{bmatrix} \widetilde{E}_{1i}^T \widetilde{F}_i^T \widetilde{H}_i^T \widetilde{P}_i + \widetilde{P}_i \widetilde{H}_i \widetilde{F}_i \widetilde{E}_{1i} & \widetilde{P}_i \widetilde{H}_i \widetilde{F}_i \widetilde{E}_{2i} & 0 & 0 & 0 & \widetilde{E}_{3i}^T \widetilde{F}_i^T \widetilde{H}_i^T \widetilde{P}_i & 0 \\ \widetilde{E}_{2i}^T \widetilde{F}_i^T \widetilde{H}_i^T \widetilde{P}_i & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \widetilde{P}_i \widetilde{H}_i \widetilde{F}_i \widetilde{E}_{3i} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} \widetilde{P}_i \widetilde{H}_i & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \widetilde{P}_i \widetilde{H}_i \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \widetilde{F}_i & 0 \\ 0 & \widetilde{F}_i \end{bmatrix} \begin{bmatrix} \widetilde{E}_{1i}^T & \widetilde{E}_{3i}^T \\ \widetilde{E}_{2i}^T & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}^T \\ &\quad + \begin{bmatrix} \widetilde{E}_{1i}^T & \widetilde{E}_{3i}^T \\ \widetilde{E}_{2i}^T & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \widetilde{F}_i & 0 \\ 0 & \widetilde{F}_i \end{bmatrix}^T \begin{bmatrix} \widetilde{P}_i \widetilde{H}_i & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \widetilde{P}_i \widetilde{H}_i \\ 0 & 0 \end{bmatrix}^T, \\ &\Psi_{11}^i = A_i^T \widetilde{P}_i + \widetilde{P}_i A_i + \widetilde{Q}_i + \tau\widetilde{R}_i + \alpha\widetilde{P}_i. \end{aligned} \tag{66}$$

From Lemma 7, we have

$$\begin{aligned} \overline{\Psi}_i \le \Psi_i &+ \varepsilon_i^{-1} \begin{bmatrix} \widetilde{P}_i \widetilde{H}_i & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \widetilde{P}_i \widetilde{H}_i \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \widetilde{P}_i \widetilde{H}_i & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \widetilde{P}_i \widetilde{H}_i \\ 0 & 0 \end{bmatrix}^T \\ &+ \varepsilon_i \begin{bmatrix} \widetilde{E}_{1i}^T & \widetilde{E}_{3i}^T \\ \widetilde{E}_{2i}^T & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \widetilde{E}_{1i}^T & \widetilde{E}_{3i}^T \\ \widetilde{E}_{2i}^T & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}^T < 0. \end{aligned} \tag{67}$$

Choose $\widetilde{P}_i = \overline{P}_i = \text{diag}\{P_i, P_i\}$, $\widetilde{Q}_i = \overline{Q}_i = \text{diag}\{Q_i, Q_i\}$, and $\widetilde{R}_i = \overline{R}_i = \text{diag}\{R_i, R_i\}$, and let $Z_i = P_i B_i K_i$ and $Y_i = P_i L_i$. By using the Schur complement, we can obtain that (67) is equivalent to (59).

Thus, according to Theorem 12, we obtain from (59)–(62) that system (7a), (7b), and (7c) is mean-square exponentially stable with a weighted H_∞ performance level γ. Moreover, we can obtain the controller gain matrices $K_i = B_i^+ P_i^{-1} Z_i$ and the observer gain matrices $L_i = P_i^{-1} Y_i$.

The proof is completed. □

Remark 14. An observer-based H_∞ controller design scheme is proposed in the paper. Compared with the existing results presented in [27–29], a remark advantage of the work is that the proposed observer is mode-dependent, which means that each subsystem has its individual observer. Moreover, the proposed observer not only ensures the convergence of the estimated error of each subsystem, but also guarantees that the estimated error of the whole system converges to zero exponentially.

In Theorem 13, when we choose that $Q_i = R_i = P_i$, it is not difficult to get that $\beta_i = 1$ and $\chi = \chi_i = 1 + \tau(1+\tau)(e^{\lambda\tau} - 1)$, for all $i \in \underline{N}$. Then, the following corollary is derived.

Corollary 15. *Consider system* (1a), (1b), (1c), *and* (1d), *for a given scalar $\alpha > 0$, if there exist scalars $\varepsilon_i > 0$, symmetric positive definite matrices $P_i > 0$, and matrices $\widetilde{S}_i$, Y_i, and Z_i such that, for all $i \in \underline{N}$,*

$$\begin{bmatrix} \Xi_{11}^i & \overline{P}_i \widetilde{A}_{\tau i} & \widetilde{S}_i^T & \widetilde{S}_i^T & \overline{P}_i \widetilde{G}_i & \widetilde{D}_i^T \overline{P}_i & \widetilde{J}_i^T & \overline{P}_i \widetilde{H}_i & 0 & \varepsilon_i \widetilde{E}_{1i}^T & \varepsilon_i \widetilde{E}_{3i}^T \\ * & -(1-\tau_d)\overline{P}_i & -\widetilde{S}_i^T & -\widetilde{S}_i^T & 0 & 0 & 0 & 0 & 0 & \varepsilon_i \widetilde{E}_{2i}^T & 0 \\ * & * & -\widetilde{S}_i - \widetilde{S}_i^T & -\widetilde{S}_i^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\tau^{-1}\overline{P}_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\gamma^2 I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\overline{P}_i & 0 & 0 & \overline{P}_i \widetilde{H}_i & 0 & 0 \\ * & * & * & * & * & * & -I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\varepsilon_i I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -\varepsilon_i I & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -\varepsilon_i I & 0 \\ * & * & * & * & * & * & * & * & * & * & -\varepsilon_i I \end{bmatrix} < 0, \tag{68}$$

then there exists an observer-based robust H_∞ controller such that system (7a), (7b), *and* (7c) *is mean-square exponentially stable with a prescribed weighted H_∞ performance level γ under arbitrary switching signal with the average dwell time*

$$T_\alpha > T_\alpha^* = \tau + \frac{\ln(\chi\mu)}{\lambda}, \tag{69}$$

where μ, χ, and λ satisfy

$$\begin{gathered} \chi = 1 + \tau(1+\tau)\left(e^{\lambda\tau} - 1\right), \\ \lambda + (1+\tau)\left(e^{\lambda\tau} - 1\right) \le \alpha, \\ P_i \le \mu P_j, \quad \forall i, j \in \underline{N}, \\ \Xi_{11}^i = \Lambda_i^T + \Lambda_i + \overline{P}_i + \tau\overline{P}_i + \alpha\overline{P}_i, \\ \Lambda_i = \begin{bmatrix} P_i A_i + Z_i & -Z_i \\ 0 & P_i A_i - Y_i C_i \end{bmatrix}, \\ \overline{P}_i = \text{diag}\{P_i, P_i\}. \end{gathered} \tag{70}$$

Moreover, the controller gain matrices are $K_i = B_i^+ P_i^{-1} Z_i$, and the observer gain matrices are $L_i = P_i^{-1} Y_i$.

4. Numerical Example

Consider system (1a), (1b), (1c), and (1d) with the following parameters

$$A_1 = \begin{bmatrix} 3 & 1 \\ 0 & -3 \end{bmatrix}, \quad A_{\tau 1} = \begin{bmatrix} -0.1 & 0 \\ 0.1 & 0.1 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix},$$

$$J_1 = \begin{bmatrix} 3 & 0 \\ 1 & 4 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 0.1 & 0 \\ 0.1 & 0.6 \end{bmatrix},$$

$$G_1 = \begin{bmatrix} 1.5 & 0 \\ 0.1 & 0.4 \end{bmatrix}, \quad H_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0 \end{bmatrix},$$

$$E_{11} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0 \end{bmatrix}, \quad E_{21} = \begin{bmatrix} 0 & 0.1 \\ 0 & 0 \end{bmatrix},$$

$$E_{31} = \begin{bmatrix} 0 & 0 \\ 0.1 & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 4 & 0 \\ 0 & -2 \end{bmatrix}, \quad A_{\tau 2} = \begin{bmatrix} -0.1 & 0.1 \\ 0 & -0.2 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix},$$

$$J_2 = \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix}, \qquad D_2 = \begin{bmatrix} 0.1 & 0 \\ 0.3 & 0.5 \end{bmatrix},$$
$$G_2 = \begin{bmatrix} 0.5 & 0.2 \\ 0 & 0.6 \end{bmatrix}, \qquad H_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0.1 \end{bmatrix},$$
$$E_{12} = \begin{bmatrix} 0 & 0 \\ 0 & 0.1 \end{bmatrix}, \qquad E_{22} = \begin{bmatrix} 0 & 0 \\ 0.1 & 0 \end{bmatrix},$$
$$E_{32} = \begin{bmatrix} 0 & 0.1 \\ 0 & 0 \end{bmatrix},$$
$$F_1(t) = \begin{bmatrix} \sin t & 0 \\ 0 & \sin t \end{bmatrix}, \qquad F_2(t) = \begin{bmatrix} \cos t & 0 \\ 0 & \cos t \end{bmatrix}. \tag{71}$$

The disturbance input $v(t) = \begin{bmatrix} 15e^{-0.2t} & 12e^{-0.3t} \end{bmatrix}^T$, and $\tau(t) = 0.3 + 0.2\sin t$; by calculation, we can obtain that $\dot{\tau}(t) \leq \tau_d = 0.2$ and $\tau(t) \leq \tau = 0.5$. Choosing $\alpha = 1.4$, $\gamma = 1.0$ and solving the LMIs in Corollary 15, we have

$$\begin{gathered}
P_1 = \begin{bmatrix} 4.0319 & -0.7058 \\ -0.7058 & 13.0192 \end{bmatrix}, \\
Y_1 = \begin{bmatrix} 82.4579 & 19.7441 \\ -15.9926 & 25.9295 \end{bmatrix}, \\
Z_1 = \begin{bmatrix} -98.8224 & -4.9793 \\ -5.0165 & -48.5838 \end{bmatrix}, \\
P_2 = \begin{bmatrix} 4.1782 & -0.5687 \\ -0.5687 & 11.9156 \end{bmatrix}, \\
Y_2 = \begin{bmatrix} 91.2715 & -45.8464 \\ -48.7364 & 99.0875 \end{bmatrix}, \\
Z_2 = \begin{bmatrix} -105.7969 & -5.8641 \\ -5.1805 & -64.8133 \end{bmatrix}, \\
\tilde{S}_1 = \begin{bmatrix} 1.9716 & -0.2408 & -0.0809 & 0.0342 \\ -0.2301 & 5.2280 & 0.0555 & -0.4124 \\ -0.0822 & 0.0553 & 2.0559 & -0.3049 \\ 0.0286 & -0.4044 & -0.3017 & 6.0017 \end{bmatrix}, \\
\tilde{S}_2 = \begin{bmatrix} 1.5102 & -0.0781 & -0.0730 & 0.1131 \\ -0.0796 & 4.6523 & 0.0254 & -0.6518 \\ -0.0792 & 0.0119 & 2.1184 & -0.2329 \\ 0.1161 & -0.6486 & -0.2413 & 5.2970 \end{bmatrix}, \\
\varepsilon_1 = 54.6307, \qquad \varepsilon_2 = 51.0439.
\end{gathered} \tag{72}$$

Then, we can obtain the following observer gain matrices:

$$\begin{gathered}
L_1 = \begin{bmatrix} 20.4301 & 5.2958 \\ -0.1209 & 2.2787 \end{bmatrix}, \\
L_2 = \begin{bmatrix} 21.4270 & -9.9052 \\ -3.0675 & 7.8431 \end{bmatrix}
\end{gathered} \tag{73}$$

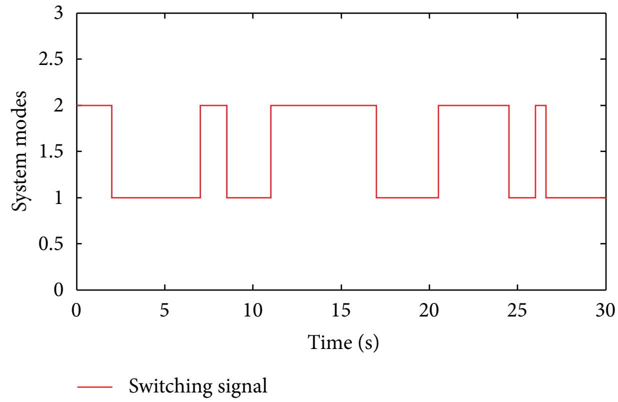

Figure 1: Switching signal.

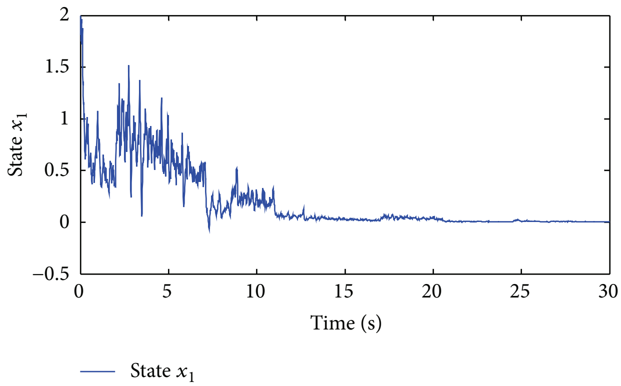

Figure 2: State x_1 of the closed-loop system.

and the controller gain matrices

$$\begin{gathered}
K_1 = \begin{bmatrix} 21.3520 & -5.7638 \\ -23.0825 & 1.9288 \end{bmatrix}, \\
K_2 = \begin{bmatrix} 23.8921 & -3.3845 \\ -49.4382 & 1.2267 \end{bmatrix}.
\end{gathered} \tag{74}$$

Moreover, we have $\mu = 4.3051$, $\lambda = 0.7$, $\chi = 1.3143$, and $T_\alpha^* = \tau + \ln(\chi\mu)/\lambda = 2.9759$. Taking $T_\alpha = 3 > T_\alpha^*$, and letting $x(t) = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$, $t \in [-0.5, 0)$, $x(0) = \begin{bmatrix} 2 & -2 \end{bmatrix}^T$, and $e(0) = \begin{bmatrix} 1 & 1 \end{bmatrix}^T$, and simulation results are shown in Figures 1–5.

Figure 1 shows the switching signal of the switched system with the average dwell time $T_\alpha = 3$. Figures 2 and 3 illustrate the state trajectories of the closed-loop system. The estimated errors are plotted in Figures 4 and 5, respectively. We can see from Figures 2–5 that the proposed observer can guarantee the convergence of the estimated error and the designed controller can guarantee the stability of the corresponding closed-loop system. This demonstrates the effectiveness of the proposed method.

In addition, some observer-based controller design approaches proposed in the existing literature [27–29] are only applicable to stochastic systems or Markovian jump

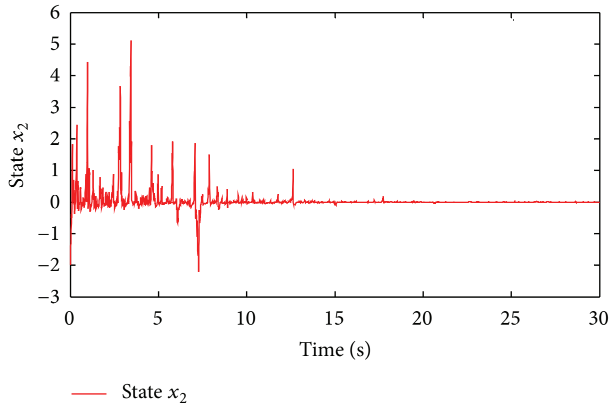

FIGURE 3: State x_2 of the closed-loop system.

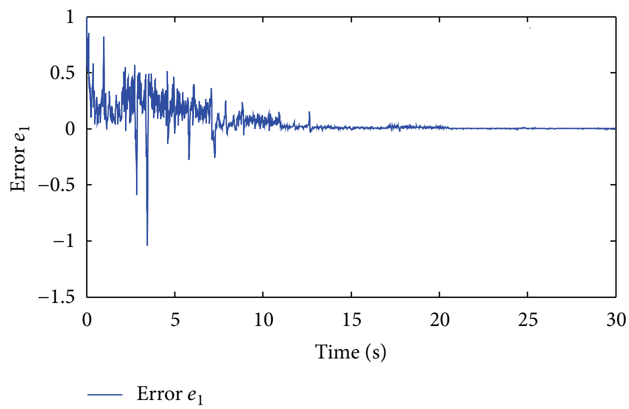

FIGURE 4: The estimated error e_1.

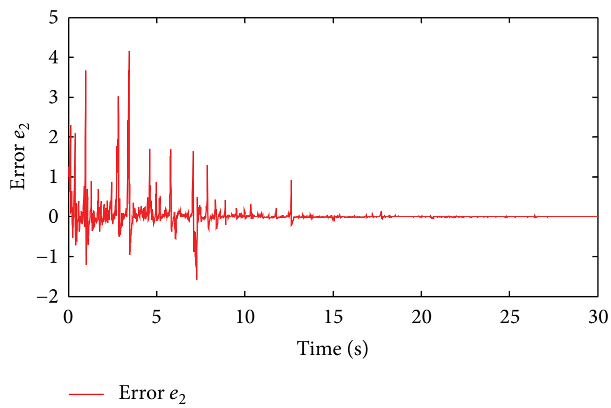

FIGURE 5: The estimated error e_2.

systems and they cannot be used to stabilize the system considered in this section, which also shows the advantage of the proposed method.

5. Conclusions

In this paper, the problem of observer-based robust H_∞ stabilization for stochastic switched systems with time delay has been investigated. By using the average dwell time method, sufficient conditions which guarantee the mean-square exponential stability of switched stochastic systems with time delay are derived. Then, H_∞ performance analysis and observer-based H_∞ control for such systems are developed. Finally, a numerical example is given to demonstrate the effectiveness of the proposed approach.

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References

[1] T. C. Lee and Z. P. Jiang, "Uniform asymptotic stability of nonlinear switched systems with an application to mobile robots," *IEEE Transactions on Automatic Control*, vol. 53, no. 5, pp. 1235–1252, 2008.

[2] W.-A. Zhang and L. Yu, "New approach to stabilisation of networked control systems with time-varying delays," *IET Control Theory & Applications*, vol. 2, no. 12, pp. 1094–1104, 2008.

[3] M. C. F. Donkers, W. P. M. H. Heemels, N. van de Wouw, and L. Hetel, "Stability analysis of networked control systems using a switched linear systems approach," *IEEE Transactions on Automatic Control*, vol. 56, no. 9, pp. 2101–2115, 2011.

[4] K. S. Narendra and J. Balakrishnan, "A common Lyapunov function for stable LTI systems with commuting A-matrices," *IEEE Transactions on Automatic Control*, vol. 39, no. 12, pp. 2469–2471, 1994.

[5] M. S. Branicky, "Multiple Lyapunov functions and other analysis tools for switched and hybrid systems," *IEEE Transactions on Automatic Control*, vol. 43, no. 4, pp. 475–482, 1998.

[6] J. P. Hespanha and A. S. Morse, "Stability of switched systems with average dwell-time," in *Proceedings of the 38th IEEE Conference on Decision and Control (CDC '99)*, pp. 2655–2660, December 1999.

[7] L. Zhang, P. Shi, C. Wang, and H. Gao, "Robust H_∞ filtering for switched linear discrete-time systems with polytopic uncertainties," *International Journal of Adaptive Control and Signal Processing*, vol. 20, no. 6, pp. 291–304, 2006.

[8] L. Zhang, P. Shi, E.-K. Boukas, and C. Wang, "H_∞ control of switched linear discrete-time systems with polytopic uncertainties," *Optimal Control Applications & Methods*, vol. 27, no. 5, pp. 273–291, 2006.

[9] L. Zhang, P. Shi, and E.-K. Boukas, "H_∞ output-feedback control for switched linear discrete-time systems with time-varying delays," *International Journal of Control*, vol. 80, no. 8, pp. 1354–1365, 2007.

[10] L. Zhang, P. Shi, E.-K. Boukas, and C. Wang, "Robust l_2–l_∞ filtering for switched linear discrete time-delay systems with polytopic uncertainties," *IET Control Theory & Applications*, vol. 1, no. 3, pp. 722–730, 2007.

[11] X.-M. Sun, J. Zhao, and D. J. Hill, "Stability and L_2-gain analysis for switched delay systems: a delay-dependent method," *Automatica*, vol. 42, no. 10, pp. 1769–1774, 2006.

[12] Y. G. Sun, L. Wang, and G. Xie, "Exponential stability of switched systems with interval time-varying delay," *IET Control Theory & Applications*, vol. 3, no. 8, pp. 1033–1040, 2009.

[13] R. Wang and J. Zhao, "Exponential stability analysis for discrete-time switched linear systems with time-delay," *International Journal of Innovative Computing, Information and Control*, vol. 3, no. 6 B, pp. 1557–1564, 2007.

[14] D. Xie and Y. Wu, "Stabilisation of time-delay switched systems with constrained switching signals and its applications in networked control systems," *IET Control Theory & Applications*, vol. 4, no. 10, pp. 2120–2128, 2010.

[15] Z. Xiang, Y.-N. Sun, and Q. Chen, "Robust reliable stabilization of uncertain switched neutral systems with delayed switching," *Applied Mathematics and Computation*, vol. 217, no. 23, pp. 9835–9844, 2011.

[16] J. Samuels, "On the mean square stability of random linear systems," *IRE Transactions on Circuit Theory*, vol. 6, no. 5, pp. 248–259, 1959.

[17] S. Xu and T. Chen, "Robust H_∞ control for uncertain stochastic systems with state delay," *IEEE Transactions on Automatic Control*, vol. 47, no. 12, pp. 2089–2094, 2002.

[18] S. Xu and T. Chen, "H_∞ output feedback control for uncertain stochastic systems with time-varying delays," *Automatica*, vol. 40, no. 12, pp. 2091–2098, 2004.

[19] B.-S. Chen and W. Zhang, "Stochastic H_2/H_∞ control with state-dependent noise," *IEEE Transactions on Automatic Control*, vol. 49, no. 1, pp. 45–57, 2004.

[20] W.-H. Chen, Z.-H. Guan, and X. Lu, "Delay-dependent exponential stability of uncertain stochastic systems with multiple delays: an LMI approach," *Systems & Control Letters*, vol. 54, no. 6, pp. 547–555, 2005.

[21] W. Feng, J. Tian, and P. Zhao, "Stability analysis of switched stochastic systems," *Automatica*, vol. 47, no. 1, pp. 148–157, 2011.

[22] Z. Xiang, R. Wang, and Q. Chen, "Robust reliable stabilization of stochastic switched nonlinear systems under asynchronous switching," *Applied Mathematics and Computation*, vol. 217, no. 19, pp. 7725–7736, 2011.

[23] Z. Xiang, C. Qiao, and R. Wang, "Robust H_∞ reliable control of uncertain stochastic switched non-linear systems," *International Journal of Advance Mechatronic Systems*, vol. 3, no. 2, pp. 98–108, 2011.

[24] J. Liu, X. Liu, and W.-C. Xie, "Exponential stability of switched stochastic delay systems with non-linear uncertainties," *International Journal of Systems Science*, vol. 40, no. 6, pp. 637–648, 2009.

[25] Y. Chen and W. X. Zheng, "On stability of switched time-delay systems subject to nonlinear stochastic perturbations," in *Proceedings of the 49th IEEE Conference on Decision and Control (CDC '10)*, pp. 2644–2649, December 2010.

[26] Z. Xiang, R. Wang, and B. Jiang, "Nonfragile observer for discrete-time switched nonlinear systems with time delay," *Circuits, Systems, and Signal Processing*, vol. 30, no. 1, pp. 73–87, 2011.

[27] A. H. Abolmasoumi and H. R. Momeni, "Robust observer-based H_∞ control of a Markovian jump system with different delay and system modes," *International Journal of Control, Automation and Systems*, vol. 9, no. 4, pp. 768–776, 2011.

[28] L. Li, Y. Jia, J. Du, and F. Yu, "Observer-based L_2–L_∞ control for a class of stochastic systems with time-varying delay," in *Proceedings of the 49th IEEE Conference on Decision and Control (CDC '10)*, pp. 1023–1028, December 2010.

[29] G. Chen, Y. Shen, and S. Zhu, "Non-fragile observer-based H_∞ control for neutral stochastic hybrid systems with time-varying delay," *Neural Computing and Applications*, vol. 20, no. 8, pp. 1149–1158, 2011.

[30] G. Zhai, B. Hu, K. Yasuda, and A. N. Michel, "Disturbance attenuation properties of time-controlled switched systems," *Journal of the Franklin Institute*, vol. 338, no. 7, pp. 765–779, 2001.

[31] K. Gu, "A further refinement of discretized Lyapunov functional method for the stability of time-delay systems," *International Journal of Control*, vol. 74, no. 10, pp. 967–976, 2001.

[32] L. Xie, "Output feedback H_∞ control of systems with parameter uncertainty," *International Journal of Control*, vol. 63, no. 4, pp. 741–750, 1996.