Hindawi Publishing Corporation
Journal of Applied Mathematics
Volume 2013, Article ID 295493, 6 pages
http://dx.doi.org/10.1155/2013/295493

Research Article

Adaptive Waveform Design for Multiple Radar Tasks Based on Constant Modulus Constraint

Wang Bin, Yang Wenfang, and Wang Jinkuan

Northeastern University at Qinhuangdao, Qinhuangdao, Hebei 066004, China

Correspondence should be addressed to Wang Bin; wangbinneu@163.com

Received 29 May 2013; Accepted 12 July 2013

Academic Editor: Neal N. Xiong

Cognitive radar is an intelligent system, and it can adaptively transmit waveforms to the complex environment. The intelligent radar system should be able to provide different trade-offs among a variety of performance objectives. In this paper, we investigate the mutual information (MI) in signal-dependent interference and channel noise. We propose a waveform design method which can efficiently synthesize waveforms and provide a trade-off between estimation performance and detection performance. After obtaining a local optimal waveform, we apply the technique of generating a constant modulus signal with the given Fourier transform magnitude to the waveform. Finally we obtain a waveform that has constant modulus property.

1. Introduction

Cognitive radar (CR) is a new concept of radar system proposed by Haykin in 2006 [1, 2]. In CR, the radar continuously learns about the environment through experience gained from interactions of the receiver with the environment, the transmitter adjusts its illumination of the environment in an intelligent manner and the whole radar system constitutes a closed-loop dynamic system. Therefore, adaptive waveform design is important to the performance of radar system. Recently, advances in flexible waveform generators and high-speed signal processing hardware have made it possible for transmitted waveforms to vary with the complex environment.

Many researches focused on waveform design for different tasks, for example, target detection, estimation, tracking, and recognition. An early attempt to the problem of matched waveform design for detecting a known target in additive Gaussian noise was addressed via the SNR criterion in [3]. From the frequency domain approach, the SNR-based optimal matched waveform for a known target in signal-dependent interference was derived in [4]. Information theory is also an important tool for waveform design. Bell [5] firstly proposed the method of maximizing the mutual information between the received signal and target impulse response to optimize the waveform, and many articles also used mutual information as the optimal criterion for waveform design [6, 7]. Because a more flexible design framework is required, CR should be able to provide different trade-offs among a variety of performance objectives. Haykin et al. [8] proposed a waveform design method that efficiently synthesizes waveforms which provide a trade-off between estimation performance for a Gaussian ensemble of targets and detection performance for a specific target in channel-noise-only environment.

In this paper, we will consider a situation when the signal-dependent interference is not negligible, and provide an optimal trade-off between the detection and estimation criteria. Thus we seek to maximize the mutual information between a random target impulse response and the received radar waveform, subjected to a lower bound on the SINR for the target and energy constraints. We assume that the target hypotheses are statistically characterized by known power spectral density (PSD) as in [9]. Therefore, the actual target realization is an unknown sample function generated from the PSD of the true target class.

One consideration in forming practical radar waveforms is the constant modulus constraint, which permits efficient use of the front-end power amplifier [10]. With proper manipulation of the waveforms in the temporal domain, it should be possible to design constant modulus waveforms that approximate MI-based waveform spectrum with some loss of optimality. Pillai et al. give us a technique of generating a constant modulus signal with the given Fourier transform magnitude in [11]; thus we can use this method to get a waveform that has constant modulus property.

This paper is organized in the following manner. Section 2 describes the target model for waveform design in signal-dependent interference. Section 3 explains how to generate the constant modulus waveform from a given Fourier transform magnitude. Section 4 shows the derivation of the mutual information between the random target impulse response and the received radar waveform in signal-dependent interference and waveform design technique for target detection and estimation in signal-dependent interference. Section 5 shows some simulation results. The whole paper is summarized in Section 6.

2. Signal Model

The block diagram in Figure 1 represents the signal model of a target ensemble in ground clutter being considered. Let $x(t)$ be a finite-energy waveform with duration T. Let $g(t)$ be a zero-mean extended target with energy spectral variance $\sigma_G^2(f)$. Let T_g be the time duration where most of the target impulse's energy resides. It is necessary to have $T_g > T$ to capture the target impulse response's energy. The clutter $c(t)$ is a zero-mean complex Gaussian random process with power spectral density (PSD) $\sigma_C^2(f)$, and $n(t)$ is the zero-mean receiver noise process with one-sided PSD $P_n(f)$. In addition, $n(t)$ is assumed to be statistically independent of the transmitted waveform $x(t)$, the target impulse response $g(t)$, and the clutter $c(t)$.

The waveform received at the receiver is filtered by the ideal lowpass filter $B(f)$, passing only frequencies in the band ω. This is just a statement of the fact that we assume that the transmitted signal has no significant energy outside the frequency interval $w = [-W, W]$. Since $z(t)$ and $d(t)$ are the response of a linear time-invariant system to the transmitted signal, they do not have significant energy outside the frequency interval $w = [-W, W]$. Hence we will not consider frequencies outside this interval.

Let $y(t)$ be the received signal given by

$$y(t) = z(t) + d(t) + n(t). \tag{1}$$

$z(t)$ and $d(t)$ are defined by

$$\begin{aligned} z(t) &= x(t) * g(t), \\ d(t) &= x(t) * c(t), \end{aligned} \tag{2}$$

where $*$ denotes the convolution operator.

3. Constant Envelope Signals with Given Fourier Transform Magnitude

Pillai et al. give us a technique of generating a constant modulus signal with the given Fourier transform magnitude in [11]. It is summarized as follows.

Let C_M denote the set of functions $\{g(t)\}$ that have the prescribed Fourier transform magnitude $M(\omega)$ over a prescribed frequency set Ω. The operator P_M will assign every arbitrary function $x(t)$ a "nearest neighbor" $P_M x(t)$ that belongs to C_M such that there exists no other element $g \in C_M$ for which $\|x - g\| < \|x - P_M x\|$ is satisfied.

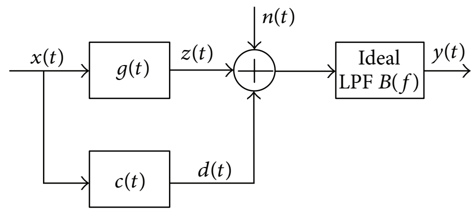

Figure 1: Signal model of a target ensemble in signal-dependent interference.

Given an arbitrary function $x(t)$, its corresponding Fourier transform is $X(\omega) = |X(\omega)|e^{j\Omega(\omega)}$ and the magnitude projection of $x(t)$ is defined as

$$P_M x(t) \longleftrightarrow \begin{cases} M(\omega) e^{j\Omega(\omega)}, & \omega \in \Omega \\ X(\omega), & \omega \in \Omega'. \end{cases} \tag{3}$$

For a constant envelope signal $x(t)$, it can be expressed as

$$x(t) = Ae^{j\theta(t)}, \tag{4}$$

where A is a suitable positive constant that can be used to maintain a prescribed energy level for $x(t)$.

Interestingly, constant envelope signals also share properties similar to the Fourier transform magnitude situation. Notice that if C_A denotes the set of functions $\{g(t)\}$ which have constant envelope level A, the operator P_A will assign every arbitrary function $x(t)$ a nearest neighbor $P_A x(t)$ that belongs to C_A such that no other element $g \in C_A$ satisfies $\|x - g\| < \|x - P_A x\|$.

Given an arbitrary signal $x(t) = a(t)e^{j\theta(t)}$, the projection procedure is

$$P_A x(t) = \begin{cases} Ae^{j\theta(t)}, & t \in T \\ x(t), & \text{otherwise}. \end{cases} \tag{5}$$

The magnitude and amplitude projection are combined according to

$$x_{k+1} = P_A P_M x_k, \tag{6}$$

where x_k is the kth iterative function. After a number of magnitude and amplitude projections, the function x_k satisfies the constant modulus property exactly while approximately maintaining the prescribed Fourier transform magnitude.

4. Waveform Design Based on Constant Modulus Constraint

We note that $x(t)$ is a deterministic waveform. It is explicitly denoted in $I(y(t); g(t) \mid x(t))$ because the mutual information is a function of $x(t)$, and we are interested in finding those functions $x(t)$ that maximize $I(y(t); g(t) \mid x(t))$ under constrains on their energy and bandwidth.

Bell proposed the derivation of the mutual information in the channel-noise-only case and derived the information-based waveform solution. Here we provide the derivation of the mutual information in the presence of signal-dependent clutter.

Here we have a channel (as shown in Figure 2) with input Z (a zero-mean Gaussian random variable with variance σ_Z^2), clutter D (a zero-mean Gaussian random variable with variance σ_D^2), and additive zero-mean Gaussian noise N with variance σ_N^2. The mutual information $I(Y;Z)$ between Y and Z is

$$I(Y;Z) = H(Y) - H(Y \mid Z). \quad (7)$$

The differential entropies $H(Y)$ and $H(Y \mid Z)$ are

$$\begin{aligned} H(Y) &= \frac{1}{2}\ln 2\pi\sigma_Y^2 = \frac{1}{2}\ln 2\pi\left(\sigma_Z^2 + \sigma_N^2 + \sigma_D^2\right), \\ H(Y \mid Z) &= \frac{1}{2}\ln 2\pi\left(\sigma_N^2 + \sigma_D^2\right). \end{aligned} \quad (8)$$

Thus the mutual information is given by the expression

$$I(Y;Z) = H(Y) - H(Y \mid Z) = \frac{1}{2}\ln\left(1 + \frac{\sigma_Z^2}{\sigma_N^2 + \sigma_D^2}\right). \quad (9)$$

Consider again the signal model of Figure 1. Assume that $\hat{z}_k(t)$, $\hat{y}_k(t)$, $\hat{d}_k(t)$, and $\hat{n}_k(t)$ are the sample signal in the frequency band $F_k = [f_k, f_k + \Delta f]$ and the sampling rate is $2\Delta f$. The samples $\hat{z}_k(t)$ are independent, identically distributed random variables with zero mean and variance σ_Z^2. Note that the total energy of $\hat{z}_k(t)$ is

$$\varepsilon_Z(F_k) = 2\Delta f\left|X(f_k)\right|^2\sigma_G^2(f_k). \quad (10)$$

The factor 2 in the previous formula is due to the fact that $X(f_k)$ is the two-sided spectrum of $x(t)$ and that we are carrying out our calculations using only positive frequencies. In the time interval T_y, the total samples statistically independent are $2\Delta f T_y$. So the variance of each sample is

$$\sigma_Z^2 = \frac{\varepsilon_Z(F_k)}{2\Delta f T_y} = \frac{2\Delta f\left|X(f_k)\right|^2\sigma_G^2(f_k)}{2\Delta f T_y} = \frac{\left|X(f_k)\right|^2\sigma_G^2(f_k)}{T_y}. \quad (11)$$

The clutter process has the total energy on the interval T_y given by

$$\varepsilon_D(F_k) = 2\Delta f\left|X(f_k)\right|^2\sigma_c^2(f_k)\,T_y. \quad (12)$$

The variance of each sample is

$$\begin{aligned} \sigma_D^2 &= \frac{\varepsilon_D(F_k)}{2\Delta f T_y} = \frac{2\Delta f\left|X(f_k)\right|^2\sigma_C^2(f_k)\,T_y}{2\Delta f T_y} \\ &= \left|X(f_k)\right|^2\sigma_C^2(f_k). \end{aligned} \quad (13)$$

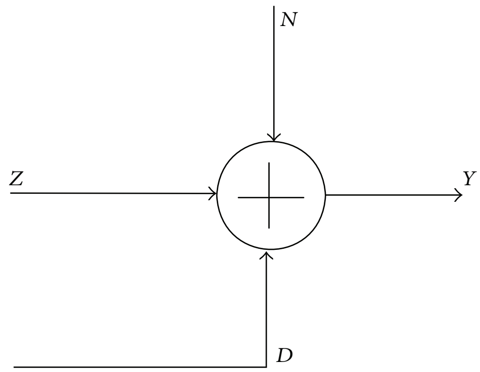

Figure 2: Channel model in the presence of clutter and additive Gaussian noise.

The noise process has the total energy on the interval T_y given by

$$\varepsilon_N(F_k) = \Delta f P_n(f)\,T_y. \quad (14)$$

Hence, the variance σ_N^2 of each sample is

$$\sigma_N^2 = \frac{\varepsilon_N(F_k)}{2\Delta f T_y} = \frac{\Delta f P_n(f)\,T_y}{2\Delta f T_y} = \frac{P_n(f)}{2}. \quad (15)$$

The mutual information between each sample Z_m of $\hat{z}_k(t)$ and the corresponding sample Y_m of $\hat{y}_k(t)$ is

$$I(Y_m; Z_m) = \frac{1}{2}\ln\left[1 + \frac{2\left|X(f_k)\right|^2\sigma_G^2(f_k)}{T_y\left\{P_n(f_k) + 2\left|X(f_k)\right|^2\sigma_C^2(f_k)\right\}}\right]. \quad (16)$$

Now there are $2\Delta f T_y$ statistically independent sample values for both $\hat{z}_k(t)$ and $\hat{y}_k(t)$ in the observation interval T_y. Thus,

$$\begin{aligned} &I(\hat{y}_k(t); \hat{z}_k(t) \mid x(t)) \\ &\quad = 2\Delta f T_y I(Y_m; Z_m) \\ &\quad = \Delta f T_y \ln\left[1 + \frac{2\left|X(f_k)\right|^2\sigma_G^2(f_k)}{T_y\left\{P_n(f_k) + 2\left|X(f_k)\right|^2\sigma_C^2(f_k)\right\}}\right]. \end{aligned} \quad (17)$$

If we now consider the frequency interval $\omega = [0, W]$, partition it into a large number of disjoint intervals of bandwidth Δf; then let the number of intervals increase as $\Delta f \to 0$, in the limit we obtain an integral for the mutual information $I(y(t); z(t) \mid x(t))$, where we assume the $x(t)$, $y(t)$, and $z(t)$ are confined to the frequency interval ω. Hence the mutual information $I(y(t); z(t) \mid x(t))$ is

$$\begin{aligned} &I(y(t); z(t) \mid x(t)) \\ &\quad = T_y \int_W \ln\left(1 + \frac{2\left|X(f)\right|^2\sigma_G^2(f)}{T_y\left\{P_n(f) + 2\left|X(f)\right|^2\sigma_C^2(f)\right\}}\right) df, \end{aligned} \quad (18)$$

as

$$I(y(t); g(t) \mid x(t)) = I(y(t); z(t) \mid x(t)). \quad (19)$$

Thus the mutual information between the random target impulse response and the received radar waveform is

$$I(y(t); g(t) \mid x(t)) = T_y \int_W \ln\left(1 + \frac{2|X(f)|^2 \sigma_G^2(f)}{T_y \{P_n(f) + 2|X(f)|^2 \sigma_C^2(f)\}}\right) df. \quad (20)$$

For the MI waveform derivation, we treat the receiver filter as an ideal lowpass filter with approximate time duration $T_B \leq T$ and $T_B \leq T_g$. Therefore T_B can be effectively ignored, and the receive filter simply becomes an explicit statement that the radar system is band limited. Therefore, T_y is

$$T_y = T + T_g. \quad (21)$$

The mutual information between the random target impulse response and the received radar waveform is shown in formula (20). The output SINR is defined to be the ratio of the average power of the signal component to the average power of the noise and interference component [12]. Thus, SINR is expressed as

$$\mathrm{SINR} = \int_W \frac{2|X(f)|^2 \sigma_G^2(f)}{P_n(f) + 2|X(f)|^2 \sigma_C^2(f)} df. \quad (22)$$

We can assume a lower bound SINR_0 on the SINR for the target

$$\int_W \frac{2|X(f)|^2 \sigma_G^2(f)}{P_n(f) + 2|X(f)|^2 \sigma_C^2(f)} df \geq \mathrm{SINR}_0. \quad (23)$$

The energy constraint in the band $\omega = [0, W]$ is expressed as

$$\int_W |X(f)|^2 df \leq E_x. \quad (24)$$

With these constraints in mind, we can now formulate the arbitrary waveform design problem as the following constrained optimization problem:

$$\begin{aligned} \max \quad & T_y \int_W \ln\left(1 + \frac{2|X(f)|^2 \sigma_G^2(f)}{T_y \{2|X(f)|^2 \sigma_C^2(f) + P_n(f)\}}\right) df \\ \text{s.t.} \quad & \int_W \frac{2|X(f)|^2 \sigma_G^2(f)}{P_n(f) + 2|X(f)|^2 \sigma_C^2(f)} df \geq \mathrm{SINR}_0 \\ & \int_W |X(f)|^2 df - E_x \leq 0. \end{aligned} \quad (25)$$

The previous constrained problem can be formulated as a convex optimization problem by introducing the autocorrelation sequence of the transmitted signal. Then an interior-point method can be used to carry out the optimization task.

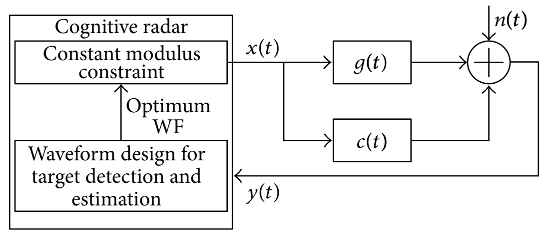

Figure 3: Closed-loop radar system.

Although the solution to this formulation is local optimal, this process is complicated. We need to solve the nonlinear constrained maximization problem.

After applying the technique of generating a constant modulus signal with the given Fourier transform magnitude to the above obtained waveform, we can get a waveform under multiple constraints.

Figure 3 represents the closed-loop radar system in signal-dependent interference proposed for target detection and estimation. In this figure, the CR signal processing involved is best described by a block labeled "COGNITIVE RADAR". CR is an intelligent system. Through sensing the environment, CR transmits the waveform suited to the working conditions. The radar returns and environment factors help to construct the new waveform that achieves a trade-off between the mutual information and the SINR for the target, that is, an optimal trade-off between the detection and estimation criteria. Then it reconstructs a signal with constant envelope property in the time domain according to its Fourier transform magnitude. The signal satisfies the constant modulus property exactly while approximately maintaining the prescribed Fourier transform magnitude. Then the waveform is transmitted to the environment. It forms a feed-back loop, and the cycle goes on and on.

5. Simulation

We consider an arbitrary target spectrum and clutter spectrum shown in Figure 4. The total energy is 1. The noise variance is 0.1. The lower bound SINR_0 is -8 dB. The number of sample points is 128. Sampling frequency is 2. Modulus value is 0.25.

Figure 5 is energy spectrum of unconstrained waveform. It shows that the optimized radar waveform only selects the dominant frequency components of the target spectrum. However, it does not distribute energy among different modes of the target. Investigating the reason, there are approximately two: one is the spectrum amplitudes scale in order to compensate for the clutter spectrum, and the other is to balance the detection performance. Hence it provides an optimal trade-off between the detection and estimation criteria.

Figure 6 is energy spectrum of constant modulus constrained waveform. It shows that the constant modulus constraint spreads the waveform energy into additional frequency bands, but the four peak amplitudes are maintained. The energy spectrum of constant modulus constrained waveform in Figure 6 is similar to the energy spectrum of unconstrained waveform in Figure 5. Thus it guarantees the performance of the nonconstant modulus optimized waveform.

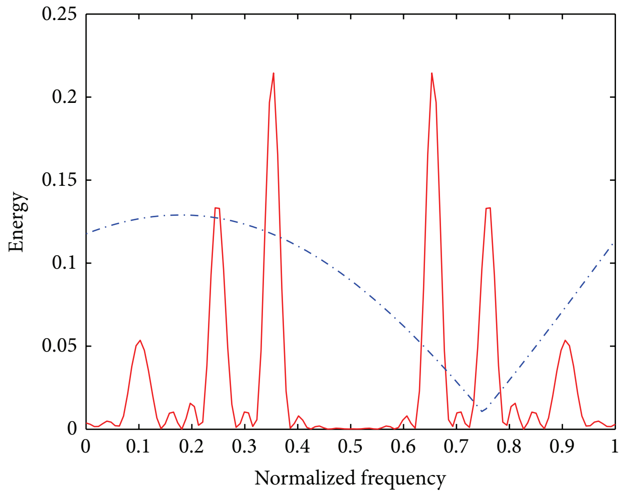

FIGURE 4: Target spectrum and clutter spectrum.

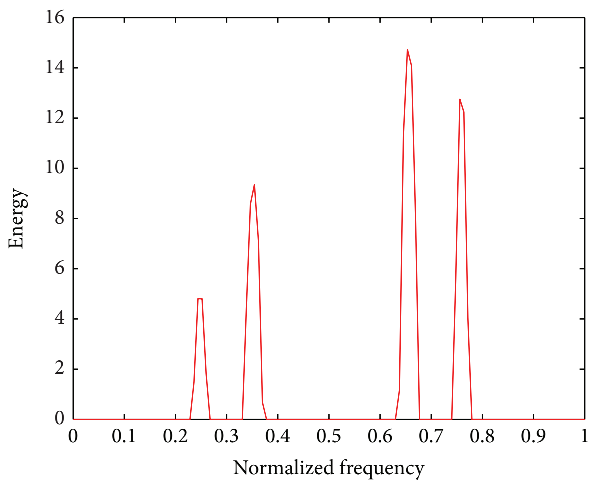

FIGURE 5: Energy spectrum of unconstrained waveform.

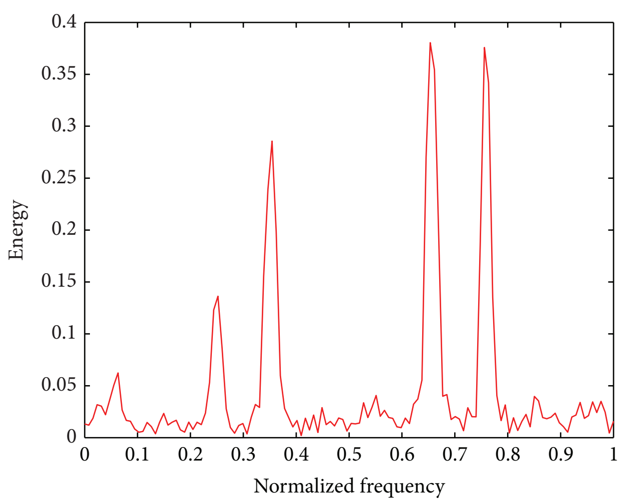

FIGURE 6: Energy spectrum of constant modulus constrained waveform.

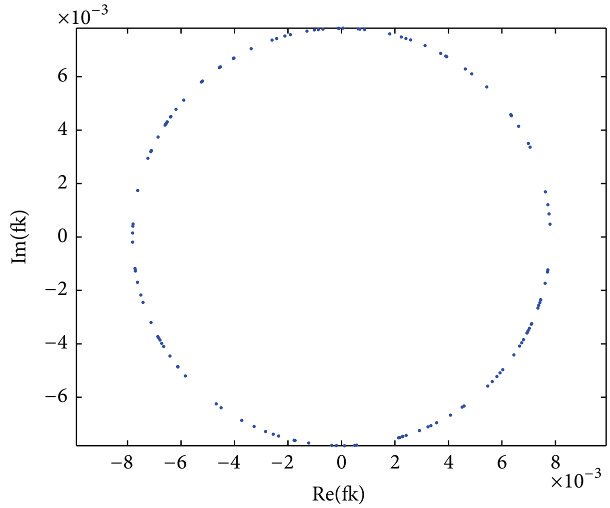

FIGURE 7: Complex constellation of constant modulus constrained waveform.

Figure 7 shows the time domain representation of the signal in the complex domain with real and imaginary parts of each instant plotted as x-axis and y-axis. The figure shows that after applying the constant modulus constraint, the temporal waveform has constant amplitude. Thus the transmitted waveform has no longer high peak amplitude in time domain and can effectively through DAC and PA of transmitter.

6. Conclusions

In this paper, we investigate the mutual information between the target impulse response and received radar waveform in signal-dependent interference and channel noise. Then we discuss the problem of radar waveform design under multiple constraints. Here we consider a situation when the signal-dependent interference is not negligible. An optimal trade-off between the detection and estimation criteria is provided. After applying the technique of generating a constant modulus signal with the given Fourier transform magnitude to the optimal waveform, a waveform that has constant modulus property is obtained. Simulation results have a significant meaning in the waveform design in cognitive radar. They show that the energy spectrum of constant modulus constrained waveform is similar to the energy spectrum of unconstrained waveform. Hence the performance of the non-constant modulus optimized waveform is guaranteed. The waveform can also be applied to a CR performing target identification.

Acknowledgments

This work was supported by the Fundamental Research Funds for the Central Universities (no. N110323005), the Natural Science Foundation of Hebei Province (no. F2013501075), and the Doctoral Scientific Research Foundation of Liaoning Province (no. 20131030).

References

[1] S. Haykin, “Cognitive radar: a way of the future,” *IEEE Signal Processing Magazine*, vol. 23, no. 1, pp. 30–40, 2006.

[2] S. Haykin, “Cognition is the key to the next generation of radar systems,” in *Proceedings of the Digital Signal Processing Workshop and 5th IEEE Signal Processing Education Workshop (DSP/SPE ’09)*, pp. 463–467, IEEE Press, January 2009.

[3] S. U. Pillai, H. S. Oh, D. C. Youla, and J. R. Guerci, “Optimum transmit-receiver design in the presence of signal-dependent interference and channel noise,” *IEEE Transactions on Information Theory*, vol. 46, no. 2, pp. 577–584, 2000.

[4] R. A. Romero, J. Bae, and N. A. Goodman, “Theory and application of SNR and mutual information matched illumination waveforms,” *IEEE Transactions on Aerospace and Electronic Systems*, vol. 47, no. 2, pp. 912–927, 2011.

[5] M. R. Bell, “Information theory and radar waveform design,” *IEEE Transactions on Information Theory*, vol. 39, no. 5, pp. 1578–1597, 1993.

[6] A. Leshem, O. Naparstek, and A. Nehorai, “Information theoretic adaptive radar waveform design for multiple extended targets,” *IEEE Journal on Selected Topics in Signal Processing*, vol. 1, no. 1, pp. 42–55, 2007.

[7] Y. Nijsure, Y. Chen, P. Rapajic, C. Yuen, Y. H. Chew, and T. F. Qin, “Information-theoretic algorithm for waveform optimization within ultra wideband cognitive radar network,” in *Proceedings of the IEEE International Conference on Ultra-Wideband (ICUWB ’10)*, pp. 1–4, IEEE Press, September 2010.

[8] S. Haykin, Y. B. Xue, and T. N. Davidson, “Optimal waveform design for cognitive radar,” in *Proceedings of the 42nd Asilomar Conference on Signals*, pp. 3–7, Systems and Computers, Pacific Grove, Calif, USA, October 2008.

[9] H. B. Jun and N. A. Goodman, “Adaptive waveforms for target class discrimination,” in *Proceedings of the International Waveform Diversity and Design Conference*, pp. 395–399, June 2007.

[10] L. K. Patton and B. D. Rigling, “Modulus constraints in adaptive radar waveform design,” in *Proceedings of the IEEE Radar Conference (RADAR ’08)*, pp. 1–6, IEEE Press, Rome, Italy, May 2008.

[11] S. U. Pillai, Y. L. Ke, and H. Beyer, “Reconstruction of constant envelope signals with given Fourier transform magnitude,” in *Proceedings of the IEEE Radar Conference (RADAR ’09)*, pp. 1–4, IEEE Press, Pasadena, Calif, USA, May 2009.

[12] R. A. Romero and N. A. Goodman, “Waveform design in signal-dependent interference and application to target recognition with multiple transmissions,” *IET Radar, Sonar and Navigation*, vol. 3, no. 4, pp. 328–340, 2009.