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Research Article

Forced Oscillation of Second-Order Half-Linear Dynamic Equations on Time Scales

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We will establish a new interval oscillation criterion for second-order half-linear dynamic equation $(r(t)[x^{\Delta}(t)]^{\alpha})^{\Delta} + p(t)x^{\alpha}(\sigma(t)) = f(t)$ on a time scale $\mathbb{T}$ which is unbounded, which is a extension of the oscillation result for second order linear dynamic equation established by Erbe et al. (2008). As an application, we obtain a sufficient condition of oscillation of the second-order half-linear differential equation $([x'(t)]^{\alpha})' + c\sin t x^{\alpha}(t) = \cos t$, where $\alpha = p/q$, p, q are odd positive integers.

1. Introduction

The theory of time scales, which has recently received a lot of attention, was introduced by Hilger in his Ph.D. thesis [1] in order to unify continuous and discrete analysis. Not only can this theory of so-called "dynamic equations" unify the theories of differential equations and of difference equations, but also it is able to extend these classical cases to cases "in between," for example, to so-called q-difference equations. A time scale $\mathbb{T}$ is an arbitrary closed subset of the reals, and the cases when this time scale is equal to the reals or to the integers represent the classical theories of differential and of difference equations. Many other interesting time scales exist, and they give rise to plenty of applications, among them the study of population dynamic models (see [2]). A book on the subject of time scale by Bohner and Peterson [2] summarizes and organizes much of the time scale calculus (see also [3]). For the notions used below, we refer to [2] and to the next section, where we recall some of the main tools used in the subsequent sections of this paper.

In the last years, there has been much research activity concerning the oscillation and nonoscillation of solutions of some dynamic equations on time scales, and we refer the reader to the paper [4–6]. Following this trend, in this paper we will provide some sufficient conditions for oscillation of second-order half-linear dynamic equation.

Consider the second-order half-linear dynamic equation

$$\left(r(t)\left[x^{\Delta}(t)\right]^{\alpha}\right)^{\Delta}+p(t)x^{\alpha}(\sigma(t))=f(t) \tag{1.1}$$

on a time scale $\mathbb{T}$ which is unbounded above, $r(t)$, $p(t)$, and $f(t)$ are rd-continuous functions. α is a quotient of odd positive integer. When $\alpha=1$, (1.1) is the second-order linear dynamic equation

$$\left(r(t)\left(x^{\Delta}(t)\right)\right)^{\Delta}+p(t)x(\sigma(t))=f(t). \tag{1.2}$$

In [7], by using the Riccati substitution the authors established a interval oscillation criterion, that is, a criterion given by the behavior of $p(t)$ and $q(t)$ on a sequence of subintervals of $[a,\infty)_{\mathbb{T}}$. In this paper, we extend the result of [7] to the second-order half-linear dynamic (1.1). As a application, we prove the equation

$$\left(\left[x'(t)\right]^{\alpha}\right)'+c\sin t x^{\alpha}(t)=\cos t \tag{1.3}$$

is oscillatory, if $c\geq c_0$, where $\alpha=p/q$, p,q are odd positive integers and c_0 is defined in Example 3.1.

For completeness, (see [2, 3] for elementary results for the time scale calculus), we recall some basic results for dynamic equations and the calculus on time scales. Let $\mathbb{T}$ be a time scale (i.e., a closed nonempty subset of $\mathbb{R}$) with $\sup\mathbb{T}=\infty$. The forward jump operator is defined by

$$\sigma(t)=\inf\{s\in\mathbb{T}:s>t\} \tag{1.4}$$

and the backward jump operator is defined by

$$\rho(t)=\sup\{s\in\mathbb{T}:s<t\}, \tag{1.5}$$

where $\sup\emptyset=\inf\mathbb{T}$, where $\emptyset$ denotes the empty set. If $\sigma(t)>t$, we say t is right-scattered, while if $\rho(t)<t$, we say t is left-scattered. If $\sigma(t)=t$, we say t is right-dense, while if $\rho(t)=t$ and $t\neq\inf\mathbb{T}$, we say t is left-dense. Given a time scale interval $[c,d]_{\mathbb{T}}:=\{t\in\mathbb{T}:c\leq t\leq d\}$ in $\mathbb{T}$ the notation $[c,d]_{\mathbb{T}}^{\kappa}$ denotes the interval $[c,d]_{\mathbb{T}}$ in case $\rho(d)=d$ and denotes the interval $[c,d)_{\mathbb{T}}$ in case $\rho(d)<d$. The graininess function μ for a time scale $\mathbb{T}$ is defined by $\mu(t)=\sigma(t)-t$, and for any function $f:\mathbb{T}\to\mathbb{R}$ the notation $f^{\sigma}(t)$ denotes $f(\sigma(t))$. We say that $x:\mathbb{T}\to\mathbb{R}$ is differentiable at $t\in\mathbb{T}$ provided that

$$x^{\Delta}(t):=\lim_{s\to t}\frac{x(t)-x(s)}{t-s} \tag{1.6}$$

exists, when $\sigma(t) = t$ (here by $s \to t$ it is understood that s approaches t in the time scale) and when x is continuous at t and $\sigma(t) > t$,

$$x^{\Delta}(t) := \frac{x(\sigma(t)) - x(t)}{\mu(t)}. \tag{1.7}$$

Note that if $\mathbb{T} = \mathbb{R}$, then the delta derivative is just the standard derivative, and when $\mathbb{T} = \mathbb{Z}$ the delta derivative is just the forward difference operator. Hence, our results contain the discrete and continuous cases as special cases and generalize these results to arbitrary time scales (e.g., the time scale $q^{\mathbb{N}_0} := \{1, q, q^2, \ldots\}$ which is very important in quantum theory [8]).

2. Main Theorem

Theorem 2.1. *Assume that given any $T \in [a, \infty)_{\mathbb{T}}$, there exists points $T \le s_1 < t_1 \le s_2 < t_2$ such that*

$$f(t) \begin{cases} \le 0, & t \in [s_1, t_1]_{\mathbb{T}}, \\ \ge 0, & t \in [s_2, t_2]_{\mathbb{T}}. \end{cases} \tag{2.1}$$

Further assume that there exists a function $u \in C^1_{rd}$ such that for $i = 1, 2$, one has

$$Q_i[u] := \int_{s_i}^{t_i} \left[r(t) \left(u^{\Delta}(t) \right)^{\alpha+1} - p(t) (u^{\sigma}(t))^{\alpha+1} \right] \Delta t \tag{2.2}$$

satisfies $Q_i[u] \le 0$, and $u(t) \not\equiv 0$ on $[s_i, t_i]_{\mathbb{T}}$, with $u(s_i) = 0 = u(t_i)$. Then the dynamic (1.1) is oscillatory on $[a, \infty)_{\mathbb{T}}$.

Remark 2.2. When $\alpha = 1$, the above theorem becomes [7, Theorem 2.1].

Proof. Assume that (1.1) is nonoscillatory. Then there is a solution $x(t)$ of (1.1) and a $T \in [a, \infty)_{\mathbb{T}}$ such that $x(t)$ is of one sign on $[T, \infty)_{\mathbb{T}}$. We consider the case $x(t) > 0$ on $[T, \infty)_{\mathbb{T}}$. Make the substitution

$$\omega(t) = r(t) \left[\frac{x^{\Delta}(t)}{x(t)} \right]^{\alpha} u^{\alpha+1}(t), \quad t \in [\mathbb{T}, \infty)_{\mathbb{T}}. \tag{2.3}$$

Then, for $t \in [s_1, t_1]_{\mathbb{T}}$ (note that $f(t) \le 0$ on $[s_1, t_1]_{\mathbb{T}}$),

$$\begin{aligned}
\omega^{\Delta}(t) &= \left[r(t)\left(\frac{x^{\Delta}(t)}{x(t)}\right)^{\alpha}\right]^{\Delta} u^{\alpha+1}(\sigma(t)) + r(t)\left[\frac{x^{\Delta}(t)}{x(t)}\right]^{\alpha}\left(u^{\alpha+1}(t)\right)^{\Delta} \\
&= -p(t)u^{\alpha+1}(\sigma(t)) + r(t)\left(u^{\Delta}(t)\right)^{\alpha+1} + \frac{f(t)}{x^{\alpha}(\sigma(t))}u^{\alpha+1}(\sigma(t)) \\
&\quad - r(t)\left[\left(u^{\Delta}(t)\right)^{\alpha+1} - \left(\frac{x^{\Delta}(t)}{x(t)}\right)^{\alpha}\left(u^{\alpha+1}(t)\right)^{\Delta} + \frac{\left(x^{\Delta}(t)\right)^{\alpha}(x^{\alpha}(t))^{\Delta}}{x^{\alpha}(t)x^{\alpha}(\sigma(t))}u^{\alpha+1}(\sigma(t))\right] \\
&\le -p(t)u^{\alpha+1}(\sigma(t)) + r(t)\left(u^{\Delta}(t)\right)^{\alpha+1} \\
&\quad - r(t)\left[\left(u^{\Delta}(t)\right)^{\alpha+1} - \left(\frac{x^{\Delta}(t)}{x(t)}\right)^{\alpha}\left(u^{\alpha+1}(t)\right)^{\Delta} + \frac{\left(x^{\Delta}(t)\right)^{\alpha}(x^{\alpha}(t))^{\Delta}}{x^{\alpha}(t)x^{\alpha}(\sigma(t))}u^{\alpha+1}(\sigma(t))\right].
\end{aligned} \tag{2.4}$$

If we define

$$F(t) := r(t)\left[\left(u^{\Delta}(t)\right)^{\alpha+1} - \left(\frac{x^{\Delta}(t)}{x(t)}\right)^{\alpha}\left(u^{\alpha+1}(t)\right)^{\Delta} + \frac{\left(x^{\Delta}(t)\right)^{\alpha}(x^{\alpha}(t))^{\Delta}}{x^{\alpha}(t)x^{\alpha}(\sigma(t))}u^{\alpha+1}(\sigma(t))\right], \tag{2.5}$$

then we have

$$\omega^{\Delta}(t) \le -p(t)u^{\alpha+1}(\sigma(t)) + r(t)\left(u^{\Delta}(t)\right)^{\alpha+1} - F(t), \quad t \in [s_1, t_1]_{\mathbb{T}}. \tag{2.6}$$

(i) Suppose that $t \in [s_1, t_1]_{\mathbb{T}}$ is right-dense. Then,

$$\left(u^{\alpha+1}(t)\right)^{\Delta} = (\alpha+1)u^{\alpha}(t)u^{\Delta}(t), \tag{2.7}$$

so we have

$$F(t) = (\alpha+1)r(t)\left[\frac{\left(u^{\Delta}(t)\right)^{\alpha+1}}{\alpha+1} - u^{\Delta}(t)\left[\frac{x^{\Delta}(t)u(t)}{x(t)}\right]^{\alpha} + \frac{\left[\left(x^{\Delta}(t)u(t)/x(t)\right)^{\alpha}\right]^{(\alpha+1)/\alpha}}{(\alpha+1)/\alpha}\right]. \tag{2.8}$$

We use Young's inequality [9], which says that

$$\frac{|A|^{p}}{p} - AB + \frac{|B|^{q}}{q} \ge 0, \quad p > 1,\ q > 1,\ \frac{1}{p} + \frac{1}{q} = 1, \tag{2.9}$$

with equality if and only if $B = A^{\alpha}, \alpha := p/q$.

So if we let

$$A = u^{\Delta}(t), \qquad B = \left[\frac{x^{\Delta}(t)u(t)}{x(t)}\right]^{\alpha}, \qquad p = \alpha + 1, \qquad q = \frac{\alpha+1}{\alpha}, \tag{2.10}$$

then we have $F(t) \geq 0, t \in [s_1, t_1]_{\mathbb{T}}$, and

$$F(t) = 0, \quad \text{iff } \frac{x^{\Delta}(t)u(t)}{x(t)} = u^{\Delta}(t). \tag{2.11}$$

(ii) Suppose next that $t \in [s_1, t_1)_{\mathbb{T}}$ is right-scattered and $u(t) \neq 0$. Then,

$$\begin{aligned} x^{\Delta}(t) &= \frac{x(\sigma(t)) - x(t)}{\mu(t)}, \qquad (x^{\alpha}(t))^{\Delta} = \frac{x^{\alpha}(\sigma(t)) - x^{\alpha}(t)}{\mu(t)}, \\ u^{\Delta}(t) &= \frac{u(\sigma(t)) - u(t)}{\mu(t)}, \qquad \left(u^{\alpha+1}(t)\right)^{\Delta} = \frac{u^{\alpha+1}(\sigma(t)) - u^{\alpha+1}(t)}{\mu(t)}. \end{aligned} \tag{2.12}$$

Let us put $a := u(\sigma(t))/u(t)$, $b := x(\sigma(t))/x(t)$. Then we have

$$F(t) = \frac{r(t)u^{\alpha+1}(t)}{\mu^{\alpha+1}(t)} f(a,b), \tag{2.13}$$

where $f(a,b) := (1-a^{-1})^{\alpha+1} - (b-1)^{\alpha}(1-a^{-(\alpha+1)}) + (b-1)^{\alpha}(1-b^{-\alpha})$.

Note that $f(b,b) = 0$ and

$$\frac{\partial f}{\partial a}(a,b) = \frac{(\alpha+1)a^{-2}}{a^{\alpha}}\left[(a-1)^{\alpha} - (b-1)^{\alpha}\right]. \tag{2.14}$$

It follows that if $a > b$, then $\partial f/\partial a > 0$, and so $f(a,b) > 0$. Likewise, if $a < b$, then $\partial f/\partial a < 0$, and so $f(a,b) > 0$.

In other words, $f(a,b) \geq 0$ and

$$f(a,b) = 0 \Longleftrightarrow a = b \Longleftrightarrow \frac{u(\sigma(t))}{u(t)} = \frac{x(\sigma(t))}{x(t)} \Longleftrightarrow \frac{x^{\Delta}(t)}{x(t)} = \frac{u^{\Delta}(t)}{u(t)}. \tag{2.15}$$

(iii) Suppose next that $t \in [s_1, t_1)_{\mathbb{T}}$ is right-scattered but $u(t) = 0$. It is easy to get that

$$F(t) = \frac{r(t)u^{\alpha+1}(\sigma(t))}{\mu^{\alpha+1}(t)}\left[1 - \left(1 - \frac{x(t)}{x(\sigma(t))}\right)^{\alpha}\right]. \tag{2.16}$$

So $F(t) \geq 0$ and

$$F(t) = 0 \Longleftrightarrow u(\sigma(t)) = 0 \Longleftrightarrow \frac{x^{\Delta}(t)u(t)}{x(t)} = u^{\Delta}(t). \tag{2.17}$$

From (i), (ii), and (iii), we get that $F(t) \geq 0, t \in [s_1, t_1)_{\mathbb{T}}$ and

$$F(t) = 0, \quad \text{iff } \frac{x^{\Delta}(t)u(t)}{x(t)} = u^{\Delta}(t). \tag{2.18}$$

Integrating (2.6) from s_1 to t_1 (using $u(s_1) = u(t_1) = 0$), we get that

$$0 \leq \int_{s_1}^{t_1} \left[r(t)\left(u^{\Delta}(t)\right)^{\alpha+1} - p(t)u^{\alpha+1}(\sigma(t)) \right] \Delta t - \int_{s_1}^{t_1} F(t)\Delta t. \tag{2.19}$$

Since $Q_1[u] \leq 0$, we obtain that

$$\int_{s_1}^{t_1} F(t)\Delta t \leq 0. \tag{2.20}$$

From (2.18) and $F(t) \geq 0, t \in [s_1, t_1]_{\mathbb{T}}$, we get that $F(t) \equiv 0, t \in [s_1, t_1)_{\mathbb{T}}$. That is,

$$\frac{x^{\Delta}(t)u(t)}{x(t)} = u^{\Delta}(t). \tag{2.21}$$

So,

$$\left(\frac{u(t)}{x(t)}\right)^{\Delta} = 0. \tag{2.22}$$

Hence $u(t)/x(t) = C$. From $u(s_1) = 0$, we get that $u(t) \equiv 0, t \in [s_1, t_1]_{\mathbb{T}}$, which is a contradiction. □

3. Example

Example 3.1. Consider the second-order half-linear differential equation

$$\left(\left[x'(t)\right]^{\alpha}\right)' + c\sin t x^{\alpha}(t) = \cos t, \tag{3.1}$$

where $\alpha = p/q$, p, q are odd positive integers. Let

$$u(t) = \begin{cases} \sin^q 2t, & 2k\pi + \dfrac{\pi}{2} \leq t \leq 2k\pi + \pi, \\ -\sin^q 2t, & 2(k+1)\pi \leq t \leq 2(k+1)\pi + \dfrac{\pi}{2}, \end{cases} \quad k \in \mathbb{N}_0, \tag{3.2}$$

and $[s_1, t_1] = [2k\pi + \pi/2, 2k\pi + \pi]$. We have

$$\begin{aligned}
Q_1[u] &:= \int_{2k\pi+\pi/2}^{2k\pi+\pi} \left[\left(u'(t)\right)^{(p+q)/q} - c \sin t u^{(p+q)/q}(t) \right] dt \\
&= \int_{\pi/2}^{\pi} \left\{ \left[2q \sin^{q-1} 2t \cos 2t \right]^{(p+q)/q} - c \sin t \sin^{p+q} 2t \right\} dt \\
&= \int_{0}^{\pi/2} \left\{ \left[2q \sin^{q-1} 2s \cos 2s \right]^{(p+q)/q} - c \cos s \sin^{p+q} 2s \right\} ds \\
&= \frac{1}{2} \int_0^{\pi} \left[2q \sin^{q-1} t \cos t \right]^{(p+q)/q} dt - 2^{p+q} c \int_0^{\pi/2} \sin^{p+q} t \cos^{p+q+1} t \, dt \\
&= \frac{1}{2} \int_0^{\pi/2} \left[2q \sin^{q-1} t \cos t \right]^{(p+q)/q} dt + \frac{1}{2} \int_0^{\pi/2} \left[2q \cos^{q-1} t \sin t \right]^{(p+q)/q} dt \\
&\quad - 2^{p+q} c \frac{\Gamma((p+q+1)/2)\Gamma((p+q+2)/2)}{2\Gamma(p+q+3/2)} \\
&= \frac{1}{2} (2q)^{(p+q)/q} \frac{\Gamma((p+q)/2 - p/2q)\Gamma(1+p/2q)}{\Gamma((p+q)/2+1)} \\
&\quad - 2^{p+q} c \frac{\Gamma((p+q+1)/2)\Gamma((p+q+2)/2)}{2\Gamma(p+q+3/2)}.
\end{aligned} \tag{3.3}$$

Noticing that $p+q$ is even number and

$$\Gamma\left(\frac{p+q}{2} + \frac{p}{2q}\right) = \left(\frac{p+q}{2} - 1 + \frac{p}{2q}\right)\left(\frac{p+q}{2} - 2 + \frac{p}{2q}\right) \cdots \left(1 + \frac{p}{2q}\right)\Gamma\left(1 + \frac{p}{2q}\right), \tag{3.4}$$

using the following formula (see [10])

$$\Gamma(n+z)\Gamma(n-z) = \frac{\pi z}{\sin \pi z}[(n-1)!]^2 \prod_{k=1}^{n-1}\left(1 - \frac{z^2}{k^2}\right), \quad \Gamma\left(n + \frac{1}{2}\right) = \frac{(2n-1)!!}{2^n}\sqrt{\pi}, \tag{3.5}$$

we get that

$$
\begin{aligned}
Q_1[u] = {} & \frac{1}{2}(2q)^{(p+q)/q}\frac{\Gamma((p+q)/2-p/2q)\Gamma((p+q)/2+p/2q)}{((p+q)/2-1+p/2q)((p+q)/2-2+p/2q)\cdots(1+p/2q)((p+q)/2)!} \\
& - 2^{p+q}c\frac{((p+q-1)!!/2^{(p+q)/2})\sqrt{\pi}\cdot((p+q)/2)!}{((2(p+q+1)-1)!!/2^{p+q})\sqrt{\pi}} \\
= {} & \frac{1}{2}(2q)^{(p+q)/q}\frac{((p\pi/2q)/\sin(p\pi/2q))\left[((p+q)/2-1)!\right]^2\prod_{k=1}^{(p+q)/2-1}\left[1-p^2/4k^2q^2\right]}{((p+q)/2-1+p/2q)((p+q)/2-2+p/2q)\cdots(1+p/2q)((p+q)/2)!} \\
& - 2^{3(p+q)/2}c\frac{(p+q-1)!!}{(2(p+q)+1)!!}\left(\frac{p+q}{2}\right)!
\end{aligned}
\tag{3.6}
$$

It is easy to see that $Q_2[u]=Q_1[u]$, so from (3.6), we obtain that when

$$
c \geq \frac{p\pi(2q)^{\mathfrak{A}/q}2^{-3\mathfrak{A}/2}(2\mathfrak{A}+1)!![(\mathfrak{A}/2-1)!]^2\prod_{k=1}^{\mathfrak{A}/2-1}\left[1-p^2/4k^2q^2\right]}{4q\sin(p\pi/2q)(p+q-1)!!(\mathfrak{A}/2-1+p/2q)(\mathfrak{A}/2-2+p/2q)\cdots(1+p/2q)[(\mathfrak{A}/2)!]^2},
\tag{3.7}
$$

where $\mathfrak{A}$ denotes $(p+q)$. , equation (3.1) is oscillatory.

In particular, take $p=1, q=3$. From (3.7), we get that when

$$
c \geq \frac{525\pi\sqrt[3]{6}}{512},
\tag{3.8}
$$

the second-order half-linear equation

$$
\left(\left[x'(t)\right]^{1/3}\right)' + c\sin t x^{1/3}(t) = \cos t
\tag{3.9}
$$

is oscillatory.

Example 3.2. Consider the second-order half-linear difference equation

$$
\Delta\left([\Delta x(n)]^{\alpha}\right) + p(n)x^{\alpha}(n+1) = n^{\beta}f(n),
\tag{3.10}
$$

where $p(n)=c(-1)^n$, α is a quotient of odd positive integers, $\beta\in\mathbb{R}$,

$$
f(n) = \begin{cases} 0, & n = 4k \text{ or } 4k+2, \\ 1, & n = 4k+1, \\ -1, & n = 4k+3, \end{cases} \qquad k\in\mathbb{N}_0,
\tag{3.11}
$$

Let $u(n) = f(n), t \in N_0$, and note that

$$f(n) = \begin{cases} \geq 0, & n = 4k,\ 4k+1,\ 4k+2k \in \mathbb{N}_0, \\ \leq 0, & n = 4k+2,\ 4k+3,\ 4k+4 \in \mathbb{N}_0. \end{cases} \tag{3.12}$$

Furthermore, we have (note that $(-1)^{\alpha+1} = 1$)

$$\begin{aligned} Q_1[u] &:= \int_{2k}^{2k+2} \left[\left(u^{\Delta}(n) \right)^{\alpha+1} - p(n) u^{\alpha+1}(n+1) \right] \Delta n \\ &= [u(2k+1) - u(2k)]^{\alpha+1} - p(2k) u^{\alpha+1}(2k+1) \\ &\quad + [u(2k+2) - u(2k+1)]^{\alpha+1} - p(2k+1) u^{\alpha+1}(2k+2) \\ &= 2 - c. \end{aligned} \tag{3.13}$$

Therefore, if $c \geq 2$, then (3.10) is oscillatory.

Example 3.3. Consider the second-order half-linear q-difference equation

$$\left(\left[x^{\Delta}(t) \right]^{\alpha} \right)^{\Delta} + p(t) x^{\alpha}(qt) = t^{\beta} f(t), \tag{3.14}$$

where $p(t) = c(-1)^n$, $t = q^n \in \mathbb{T} = q^{\mathbb{N}_0}, q > 1, c > 0, \beta \in R$. α is a quotient of odd positive integers.

$$f(t) = \begin{cases} 0, & t = q^n,\ n = 4k \text{ or } 4k+2, \\ 1, & t = q^n,\ n = 4k+1, \\ -1, & t = q^n,\ n = 4k+3, \end{cases} \qquad k \in \mathbb{N}_0, \tag{3.15}$$

Let $u(t) = f(t), t = q^n \in \mathbb{T}$. We have

$$\begin{aligned} Q_1[u] &:= \int_{q^{4k}}^{q^{4k+2}} \left[\left(u^{\Delta}(t) \right)^{\alpha+1} - p(t) u^{\alpha+1}(qt) \right] \Delta t \\ &= \left\{ \left[\frac{u(q^{4k+1}) - u(q^{4k})}{q^{4k}(q-1)} \right]^{\alpha+1} - c u^{\alpha+1}\left(q^{4k+1} \right) \right\} q^{4k}(q-1) \\ &\quad + \left\{ \left[\frac{u(q^{4k+2}) - u(q^{4k+1})}{q^{4k+1}(q-1)} \right]^{\alpha+1} + c u^{\alpha+1}\left(q^{4k+2} \right) \right\} q^{4k+1}(q-1) \\ &= \left[\frac{1}{q^{4k(\alpha+1)}(q-1)^{\alpha+1}} - c + \frac{q}{q^{(4k+1)(\alpha+1)}(q-1)^{\alpha+1}} \right] q^{4k}(q-1). \end{aligned} \tag{3.16}$$

Similarly, we have

$$Q_2[u] := \int_{q^{4k+2}}^{q^{4k+4}} \left[\left(u^{\Delta}(t)\right)^{\alpha+1} - p(t) u^{\alpha+1}(qt) \right] \Delta t$$
$$= \left[\frac{1}{q^{(4k+2)(\alpha+1)}(q-1)^{\alpha+1}} - c + \frac{q}{q^{(4k+3)(\alpha+1)}(q-1)^{\alpha+1}} \right] q^{4k+2}(q-1). \tag{3.17}$$

Therefore, if $c > 0$, then $Q_1[u] \le 0, Q_2[u] \le 0$, for large k. So (3.14) is oscillatory.

Many other interesting examples can be similarly given.

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