CARL STÖRMER IN MEMORIAM

BY

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On August 13, 1957, CARL STÖRMER died, nearly 83 years old. He was born in the town of Skien in southern Norway on September 3, 1874, as the only child of a pharmacist, G. L. Störmer and his wife née Mülertz.

When he was barely 18 years old he published his first mathematical work, *Summation of some trigonometric series,* in which he proves the formula

1 – 583801. *Acta mathematica.* 100. Imprimé le 25 septembre 1958.

$$\frac{\varphi_1 \cdot \varphi_2 \cdots \varphi_n}{2} = \frac{\sin \varphi_1}{1} \cdot \frac{\sin \varphi_2}{1} \cdots \frac{\sin \varphi_n}{1} - \frac{\sin 2\varphi_1}{2} \cdot \frac{\sin 2\varphi_2}{2} \cdots \frac{\sin 2\varphi_n}{2} + \\ + \frac{\sin 3\varphi_1}{3} \cdot \frac{\sin 3\varphi_2}{3} \cdots \frac{\sin 3\varphi_n}{3} - \cdots$$

when $$|\varphi_1| + |\varphi_2| + \cdots + |\varphi_n| < \pi.$$

Truly an amazing accomplishment by a young man still in his teens! Störmer discussed the same problem again in 1895 in *Acta Mathematica* in an article entitled «Sur une généralisation de la formule $\frac{\varphi}{2} = \frac{\sin \varphi}{1} - \frac{\sin 2\varphi}{2} + \frac{\sin 3\varphi}{3} - \cdots$».

Amusing evidence that the writer was still of tender years comes in a letter which he wrote to Professor Phragmén in October 1896 and which reeds in translation: "I send you enclosed the biographical material which you request. Since I have not yet passed any university exam and am at present enrolled as a student at the university, I cannot suggest any title—unless you could use 'Étudiant de Mathématique à l'Université à Christiania'. I leave it to you to decide whether to include this information or not." (Harald Cramér has drawn my attention to this letter.)

His next works were inspired by Machin's formula for π. He gives a complete solution to the problem of finding the integers m, n, x, y and k which will give

$$m \text{ arc tg } \frac{1}{x} + n \text{ arc tg } \frac{1}{y} = k \cdot \frac{\pi}{4}.$$

He finds four sets of solutions, Machin's own among them. In 1896 he enlarges the problem to include

$$m \text{ arc tg } \frac{1}{x} + n \text{ arc tg } \frac{1}{y} + r \text{ arc tg } \frac{1}{z} = k \cdot \frac{\pi}{4}.$$

He finds 102 sets of solutions for integer m, n, r, k, x, y and z when k is not zero. He discusses the problem whether there are more than 102 solutions and whether the solutions may, in fact, be innumerable. He mentions that Gauss had discussed the same problem in a similar manner without publishing his work.

His treatment of these problems led Störmer to the study of Pell's equation. In 1897 he published his *Quelques théorèmes sur l'équation de Pell* $x^2 - Dy^2 = \pm 1$ *et leurs applications*. Of his many important theorems I shall only mention one: For the equation

$$1 + x^2 = 2y^n$$

to be satisfied by integers x, y and n larger than 1, n must be a power of 2.

In 1902 he published in *Acta Mathematica* "Quelques propriétés arithmétiques des intégrales elliptiques et leurs applications à la théorie des fonctions entières transcendantes", in which he emphasizes that the mathematician is often slowed down by difficulties of a purely arithmetic nature which may at first seem to be rather remote from the problem. In this work as well as in an article in *Bulletin de la Société Mathématique de France*, Störmer gives a far-reaching generalisation of the study of transcendence of Liouville, Hermite and Borel.

In 1903 he was given an opportunity to watch the experiments carried out by the physicist Kristian Birkeland to find an explanation of Aurora Borealis—and from that moment the Aurora took all his interest. He watched Birkeland sending cathode rays through an evacuated glass container against a magnetic sphere representing the globe. It struck Störmer that he was facing a fascinating mathematical problem: to find the paths of the electric particles moving in the magnetic field surrounding a magnetic sphere.

Störmer attacked the problem in the following way: With a magnetic dipole placed in a three-dimensional coordinate system with direction along the z-axis and with the middle point at the origin, the task will be to study the paths of a negatively charged corpuscle. If the length of the arc s is chosen as independent variable, one gets a simultaneous system of three second-order differential equations to determine the coordinates x, y, z of the corpuscle as functions of s. By introducing polar coordinates for the projection of the point in the xy-plane ($x = R \cos \varphi$, $y = R \sin \varphi$), Störmer deduces the following equation:

$$\frac{d}{ds}\left(R^2 \frac{d\varphi}{ds}\right) = \frac{d}{ds}\left(\frac{R^2}{r^3}\right).$$

Here r equals the distance from the origin to the corpuscle. This equation can be integrated and we thus get

$$R^2 \frac{d\varphi}{ds} = 2\gamma + \frac{R^2}{r^3},$$

where γ is a constant of integration. Let the angle between the tangent of the path in the point (x, y, z) and the plane P which contains the z-axis and the point be θ. We thus get

$$R \frac{d\varphi}{ds} = \sin \theta = \frac{2\gamma}{R} + \frac{R}{r^3}.$$

This will indicate the areas of space not containing any of the curves, since the condition $|\sin \theta| \leqslant 1$ must be fulfilled. Thus Störmer has obtained a valuable discussion of the curves. He also gets the strikingly simple result

$$\varrho = r^3,$$

where ϱ is the radius of curvature of the curve.

In order to carry out an integration of the differential equations, it was necessary to use numerical methods, and a staff of assistants carried out the mass of calculations. In this connection, Störmer developed a method of approach which has had considerable importance. He presented "Störmer's method" in a lecture given at the international congress in Strassbourg in 1921, and the method has been adopted in text books in a number of countries. The method is worked out for the integration of a simultaneous system of second order differential equations, and the Russian mathematician Kryloff has modified it to make it applicable to a system of the first order. In 1930 and 1931 the German physicist Brüche published the results of some experiments with cathode rays, and the similarity between them and the curves calculated by Störmer was striking.

Störmer describes all this work in a fascinating chapter of his book *From the depths of space to the heart of the atom*. The great popularity of his book is illustrated by the fact that it was translated from Norwegian into Swedish, German, French, Dutch and Italian. It is an exciting book which clearly demonstrates the wide range of interests of the author in all the various fields of natural sciences. In this respect I do not think that he had many equals.

When Störmer had completed his university education in 1898, he studied mathematics in Paris and in 1899 was appointed a research fellow in mathematics at the University of Christiania. In this position he was among the editors of a special Niels Henrik Abel Commemorative, published on the occasion of the 100th anniversary of the birth of Abel. The very valuable collection of Abel documents contained in this book is his achievement.

He was 29 years old when he was appointed professor of pure mathematics at the University of Christiania in 1903. This was a chair with great traditions—his predecessors being C. A. Bjerknes, Ole Jacob Broch, Holmboe and Rasmusen, a few names only, but they take us right back to the classic age of Abel. Störmer was fresh from his studies in Paris and his lectures were an inspiration to us young students. They excelled in French elegance combined with clarity and simplicity. The mathematical seminar led by Störmer at that time was of great importance to those of us who were particularly interested in mathematics, and Störmer followed our work as scientists in the years to come with great interest.

Through more than forty years Störmer in an outstanding manner directed the work of those studying for the minor degree at the university. Among the major degree subjects which he took up during this period were Sophus Lie's transformation groups, the gamma function and the elliptic functions. The mimeographed edition of the last two subjects clearly illustrates his deep insight into these theories.

Störmer was an active participant in the Scandinavian congresses of mathematicians. He took part in each of the twelve congresses held during his lifetime, and he was president of the International Congress of Mathematicians held in Oslo in 1936.

Some pure mathematicians may have noticed with concern that his burning interest in and ceaseless work with Aurora kept him from using to their full extent those mathematical gifts of which his first published works had given such convincing evidence. I do not think, however, that there is any reason to regret his choice of a subject somewhat outside the scope of his professorate. A scientist must always be free to follow his inspiration, and Störmer himself gives a convincing demonstration that such freedom of choice may bring the greatest results. He also had the good fortune to receive countless expressions of recognition for his great accomplishments both at home and abroad. One such sign of recognition which possibly pleased him the most was the request made by Sir Edward Appleton that he gather all his studies of Aurora in one volume. The result was *The Polar Aurora*, published in Oxford in 1955. During his last years Störmer enjoyed working on this book—the preface of which concludes with the following words: "My work has given me infinite pleasure and satisfaction .... This fascinating phenomenon, the Aurora, guards its secrets well and it may be far in the future before they are completely yielded up to man."

The mathematical works of Carl Störmer

Summation af nogle trigonometriske rekker. 21 pp. *Chra. Videnskabsselskabs forhandlinger.* No. 17 (1892).

Om en generalisation af integralet $\int_0^\infty \frac{\sin a x}{x} d x = \frac{\pi}{2}$. 11 pp. *Chra. Videnskabsselskabs forhandlinger.* No. 4 (1895).

Sur une généralisation de la formule $\frac{\varphi}{2} = \frac{\sin \varphi}{1} - \frac{\sin 2\varphi}{2} + \frac{\sin 3\varphi}{3} - \cdots$. *Acta Math.*, 19 (1895), 341–350.

Solution complète en nombres entiers m, n, x, y, k de l'équation $m \operatorname{arc\,tg} \frac{1}{x} + n \operatorname{arc\,tg} \frac{1}{y} = \frac{k\pi}{4}$. 21 pp. *Videnskabsselskabets skrifter. Math.-naturv. Klasse.* No. 11 (1895).

Sur les solutions entières $x_1, x_2 \dots x_n, \varkappa_1, \varkappa_2 \dots \varkappa_n, k$ de l'équation

$$x_1 \text{ arc tang } \frac{1}{\varkappa_1} + x_2 \text{ arc tang } \frac{1}{\varkappa_2} + \cdots + x_n \text{ arc tang } \frac{1}{\varkappa_n} = k\frac{\pi}{4}.$$

C. R. Acad. Sci. Paris, 122 (1896), 175–177, 225–227.

Sur l'application de la théorie des nombres entiers complexes à la solution en nombres rationnels $x_1 x_2 \dots x_n \; c_1 c_2 \dots c_n, k$ de l'équation

$$c_1 \text{ arc tg } x_1 + c_2 \text{ arc tg } x_2 + \cdots c_n \text{ arc tg } x_n = k\frac{\pi}{4}.$$

96 pp. *Arch. Math. Naturvid.*, 19, No. 3 (1896).

Om en Egenskab ved Lösningerne af den Pellske Ligning $x^2 - Ay^2 = \pm 1$. *Nyt Tidsskrift f. Math. Kbh.* Aarg. 7, Afd. B. (1896) 49–52.

Quelques théorèmes sur l'équation de Pell $x^2 - Dy^2 = \pm 1$ et leurs applications. 48 pp. *Videnskabsselskabets Skrifter. I. Mat.-naturv. klasse.* No. 2 (1897).

Sur une équation indéterminée. *C. R. Acad. Sci. Paris*, 127 (1898), 752–754.

Solution complète en nombres entiers de l'équation $m \cdot \text{arc tang } \frac{1}{x} + n \cdot \text{arc tang } \frac{1}{y} = k\frac{\pi}{4}$. *Bull. Soc. Math. France*, 27 (1899), 160–170.

Sur les logarithmes des nombres algébriques. *C. R. Acad. Sci. Paris*, 130 (1900), 1603–1605.

Sur une propriété arithmétique des logarithmes des nombres algébriques. *Bull. Soc. Math. France*, 28 (1900), 146–157.

Une application d'un théoréme de Tchebycheff. 26 pp. *Arch. Math. Naturvid.*, 24, No. 5 (1901).

Documents concernant Abel. Eclaircissement sur les documents. 64 pp. (*Abel, N. H.*) *Memorial publié à l'occasion du centenaire de sa naissance.* 1902.

Remarque preliminaire sur l'équation indéterminée $x_1^2 - Ax_2^2 - 2Bx_2x_3 - Cx_3^2 + (AC - B^2)x_4^2 = \pm 4$. *Videnskabsselskabets Skrifter. I. Math.-naturv. klasse.* No. 8 (1902).

Nogle geometriske satser fra den moderne taltheori. 28 pp. *Chra. Videnskabsselskabs Forhandlinger.* No. 2 (1902).

Om nogle bestemte integraler. 9 pp. *Chra. Videnskabsselskabs Forhandlinger.* No. 6 (1902).

Quelques propriétés arithmétiques des intégrales elliptiques et leurs applications à la théorie des fonctions entières transcendantes. *Acta Math.*, 27 (1902), 185–208.

Sur quelques resultats obtenus dans la théorie des intégrales définies les plus générales à n dimensions contenant des paramètres. 25 pp. *Videnskabsselskabets Skrifter. I. Math. naturv. klasse.* No. 4 (1903).

Sur les integrales de Fourier–Cauchy. *C. R. Acad. Sci. Paris*, 137 (1903), 408–411.

Sur le mouvement d'un point matériel portant une charge d'électricité sous l'action d'un aimant élémentaire. 32 pp., 1 pl. fig. *Videnskabsselskabets Skrifter I. Math.-naturv.klasse.* No. 3 (1904).

Solution d'un problème curieux qu'on rencontre dans la théorie élémentaire des logarithmes. *Nyt Tidsskrift f. Math. Kbh.* Aarg. 19, Afd. B. (1908), 1–7.

Méthode d'intégration numérique des équations différentielles ordinaires. *Comptes rendus du Congres international des Mathématiciens, Strasbourg*, 22–30 sept. 1920. Toulouse (1921), 243–257.

En numerisk integrationsmetode for differentialligninger. (Foredrag holdt 4de mai 1921 i Norsk matematisk forening.) *Norsk Mat. Tidsskr.* 3 årg. (1921), 121–134.

Les phénomènes d'aurore boréale et les problèmes qui s'y rattachent. *5. skandinaviske matematikerkongres. Helsingf.* (1923), 57–76 [11 pl. fig.].

Indieren Ramanujan — et merkelig matematisk geni. *Norsk Mat. Tidsskr.* 16. årg. (1934), 1–13. Port.

Noen broer mellem funksjonsteori og tallteori. *Norsk Mat. Tidsskr.* 19. årg. (1937), 89–100. Fig.

Åpningstale ved Matematikerkongressen 1936. *Comptes rendus du congrès internationale des mathématiciens Oslo* 1936, 40–42 (1937).

Remark on a paper "On a trigonometric series", by T. W. Chaundy. *J. Lond. Math. Soc.* 13 (1938), 195.

Sur une généralisation de la constante d'Euler. (*Grave, D. A.*) *Sbornik posvjaščennyj pamjati akademika* . . . Moskva (1940), 316–319.

Sur une recherche qualitative et quantitative d'un système d'équations différentielles jouant un role important dans la physique cosmique. (*Grave, D. A.*) *Sbornik posvjaščennyj pamjati akademika* . . . Moskva (1940), 310–315.

Sur un problème curieux de la théorie des nombres concernant les fonctions elliptiques. *Arch. Math. Naturvid.* 47 (1943), 83–86.

The Polar Aurora. Oxford, Clarendon Press, 1955.

(A complete list of Carl Störmers works before 1944 has been given by Knut Thalberg: *Fredrik Carl Mülertz Störmer*, Oslo, 1944.)