# Niels Erik Nørlund in memoriam

by

THØGER BANG

The name of the chief editor N. E. Nørlund has been on the title page of Acta Mathematica through 55 years. The first volume where it was missing came 1982, exactly 100 years after the beginning of the journal. In this long period there had only been two chief editors, the founder Gösta Mittag-Leffler and then N. E. Nørlund. It ended with the death of Nørlund who reached the advanced age of almost 96 years.

Niels Erik Nørlund was born on 26 October 1885, the son of Alfred Nørlund, the licentiate of the pharmacy in Slagelse, a town in western Sealand. In this rater well-to-

do home he grew up with his younger sister Margrethe and younger brother Poul. They formed a trio that later attained a prominent position in the scientific and scholarly life of Denmark; Margrethe married the physicist Niels Bohr in 1912 and became a brilliant hostess for scientists from all over the world in Bohr's honorary residence at Carlsberg, while Poul, later director of the Nationalmuseum in Copenhagen, became well known for his excavations of Norse ruins in Greenland and of the Viking fortress at Trelleborg.

After attending the elementary school in his native town, Nørlund went to the old renowned public school Sorø Akademi in the neighbouring town of Sorø. The favourite subjects for his precocious mind were astronomy and mathematics. He has later in a letter to Mittag-Leffler told that he has probably been the youngest subscriber to Acta Mathematica, as he started as such at the age of twelve, and then read it simultaneously with Euler's *Institutiones Calculi Differentialis et Integralis,* given by his teacher in mathematics. When in 1903 he had finished school and left for Copenhagen University it was obvious that these subjects should be his field of study. However, he had difficulty in deciding which of them he preferred, and he therefore followed them both simultaneously.

At that time the two professors of mathematics at the university were Zeuthen and Julius Petersen. Zeuthen was a famous geometer, but at that time his interests had changed over to the history of mathematics in classical times and Nørlund felt that to be rather remote. Julius Petersen wrote on algebra and graph theory and what he taught in the theory of functions was felt to be old-fashioned by Nørlund. A third mathematician was the docent Niels Nielsen. From him Nørlund learned the theory of functions in the direction initiated by Weierstrass, but Nørlund would soon surpass him. Nørlund's preferred teacher was T. N. Thiele who held the chair of astronomy. As a result of an eye disease Thiele was unable to do any practical astronomical observation by this time, but he was a master in treating the numerical values derived from observations and his lectures on systematic and accidental errors were attended not only by students in astronomy, but also by many who wanted positions in insurance companies (at that time a chair in actuarial mathematics did not exist at Copenhagen).

Soon Nørlund took part in the work at the observatory and already in 1905 he could publish an astronomical discovery in *Astronomische Nachrichten.* As a result he obtained an appointment as assistant at the observatory and when in 1908 he wrote a prize paper answering questions about the observer errors occurring in astronomical observations, he was rewarded with the prize—a gold medal.

But already the year before he had received a gold medal for his entry for the mathematical prize. The topic was representations of functions in form of continued

fractions and in particular "reciprocal differences", a concept introduced by Thiele. The results were published in two notes in the French Comptes Rendus.

In the summer of 1910 he was awarded his master's degree in astronomy, and a few months later his doctor's degree in mathematics. The next day he could celebrate the successful defence of the thesis simultaneously with his 25th birthday, and he thus lived to see his 70 year doctor's jubilee. The thesis marked the beginning of his penetrating study of difference equations, to be described later.

The year 1910 was eventful for mathematics in Denmark. Early in the year Harald Bohr had defended his brilliant thesis; Nørlund later told how he remembered this occasion as a fanfare heralding a new era for the Danish study of mathematics. Now he gained his degree with similar honour (they had helped each other with the reading of proof-sheets and shortly after became related by the marriage of Nørlund's sister to Niels Bohr, the brother of Harald). But it was also a year with changes in the occupants of the chairs of mathematics at the university. The previous year Julius Petersen had retired and his position was filled by Niels Nielsen. Now Zeuthen came to his retiring age and as successor the university took the topologist Poul Heegaard; Harald Bohr got the docent position that Niels Nielsen had formerly held. In astronomy Thiele had retired 3 years earlier and Elis Strömgren had been appointed to the chair.

Hence it was difficult for Nørlund to look forward to a proper, suitable position, and for 2 years he stayed as assistant at the observatory. This he did not consider as a sinecure; he determined 7000 star positions and also published a catalogue of the proper motions of 140 stars calculated from old and new observations.

At the university in Lund in Sweden (not too far away from Copenhagen) there had until then been only one mathematician, Torsten Brodén, but in 1912 a new chair was established and Nørlund was appointed. In a letter from a later time Mittag-Leffler has told that he was not able to obtain this result without difficulties.

The same year Nørlund married Agnete Wæver. It was the beginning of a long and happy life together and after her death in 1959 he was lonely, the more so as their children, two daughters, had married and settled in Norway.

For the following 10 years Nørlund worked exclusively as a mathematician developing the many ideas about difference equations and interpolation that had started with his thesis. While in his work as astronomer he only needed to use real variables he now, just as in the thesis, treated the problems in a general frame with analytic functions in the complex plane; probably this was also more appealing to Mittag-Leffler who disdained applied mathematics.

Mittag-Leffler and Nørlund became close friends; often when Mittag-Leffler re-

turned to Stockholm from one of his many travels in Europe Nørlund received him in Trelleborg and followed in the train to Lund. In 1916 Nørlund became a member of the editorial board of Acta Mathematica. In France Nørlund was appreciated and when at this time the first volume of the collected works of Poincaré was published, it was with the comments of Nørlund; the subject-matter was mainly Fuchsian differential equations, in particular the brilliant treatise from the opening volume of Acta Mathematica in 1882.

Mittag-Leffler recognized the scientific and administrative capability of Nørlund and soon he got the idea that Nørlund could succeed him as director for the mathematical institute he planned to establish in Djursholm in the palatial villa with the enormous mathematical library; he often discussed his project with Nørlund.

At the same time in Copenhagen it became evident that the two chairs in mathematics at the university were insufficient to fulfil modern demands (the third position, the docentur of Harald Bohr, had been transferred to the Technical University). As also the plan of Mittag-Leffler was known, it was realized that if Nørlund was to be saved for Danish science it was necessary to act quickly; in 1919 a new chair in mathematics was established at the university and Nørlund declared himself willing to take it over. However, because of the difficult housing situation, it was not activated until September 1922.

Probably at this moment Nørlund and Mittag-Leffler considered it a temporary arrangement, but events to be mentioned below changed the situation and Nørlund's life should be filled by other assignments. He virtually gave up his mathematical research; what he wrote on mathematics in the mid-1920s is rather to be considered as a codification of his earlier work, in particular the impressive *Vorlesungen über Differenzenrechnung* from 1924 that gave a comprehensive exposition of the whole theory.

But he was faithful to Acta Mathematica. In 1925 Mittag-Leffler appointed him as the future chief editor, and the next year he was responsible for the 3 volumes dedicated to Mittag-Leffler on the occasion of his eightieth birthday; in 1927 Nørlund had the sad task to write the obituary for his old friend. Mittag-Leffler's dream of the mathematical institute in Djursholm could not, for financial reasons, be realized to its full extent at that time; Carleman was appointed as director, and Nørlund took him as coeditor for the Acta. As such he acted until he died in 1949. Since then Nørlund has been sole chief editor, but he did not take part in the editorial work after 1956.

In Copenhagen Nørlund, with his many international connections, was a highly active member of the Danish Academy and in 1928 he was elected President, the youngest president in the history of the Academy.

But what really changed his life was geodesy. In 1923 he was persuaded to take over the post as director of Gradmaalingen, i.e. the institution that performed the superior triangulation survey of Denmark. From now on Nørlund used most of his energy for the furtherance of geodesy. He organized guidance for students and after a few years the university had its first graduates with the official title of master of geodesy. He took up new topics, the first was seismology.

Next door to Gradmaalingen and in the same building existed the far larger topographical division of the General Staff that since 1842 had taken care of the ordnance survey of Denmark. The two divisions were closely related, as Gradmaalingen provided some fundamental data for use in the ordnance maps, and now Nørlund thought to have them united. With the great weight of his words it was possible to obtain a majority in the Danish Parliament, and in 1928 the Geodætisk Institut was founded as a civil institution under the Ministry of Defence, with Nørlund as director. Civil servants succeeded the "guides" of the General Staff and Nørlund became the leader of a large technical establishment. Here he could wholly use his capacity for organizing work, exacting a combination on a high level of mathematics, physics, astronomy and mechanics, but it also gave him a large burden of labour.

He carried out a new survey of the Faroe Islands and in Greenland he used aerial photogrammetric survey as a substantial aid to terrestrial measurement, so troublesome in mountain regions; this initiative was to be a valuable contribution to the advantageous issue for the Danes of the lawsuit at the World Court in The Hague in 1933, after the Norwegians had proclaimed occupation of part of Eastern Greenland. An interesting experiment was the hydrostatic levelling across the Great Belt, where by means of a 20 kilometre pipe filled with water and laid across the bottom of the Belt it became possible to compare the indications of height in Funen and Sealand with an accuracy of less than a millimetre; as a by-product one could observe the tidal movement of the firm (but weakly elastic) crust of the Earth. During the German occupation the institute had to reduce its activity, but instead Nørlund published a series of stately atlases containing a detailed history of the mapping of Denmark, the Faroe Islands and Iceland.

Nørlund managed the Geodætisk Institut in such a way that it fulfilled the demands from society, and the demands were increasing. However, it must be emphasized that at the same time Nørlund created an institution that in its spirit and its work is a scientific research institute. Even if it does not belong to the university there is a close collaboration in the education of scientists, and as masters of geodesy usually obtain employment in the institute it means continued research activity for most of them.

Nørlund was more an administrator than a practical geodesist, and what he published in the many years until his retirement in 1955 is mainly general reports on the activity of the institute. In between he lectured on mathematics, mainy on analytic functions—what he called higher analysis. His rather formal lectures were followed by only a few students, who nevertheless acquired an advanced supplement to their general education from the other and more attractive professors of mathematics, Harald Bohr and Hjelmslev and later Jessen.

As mentioned, Nørlunds mathematical work is a penetrating study of linear difference equations in the complex domain. It started with his thesis and after the many years in Geodætisk Institut he took parts of the topic up again in his retirement.

In the thesis Nørlund used the new methods from the theory of analytic functions developed in the preceding years, and most of what at that time had been written on difference equations and interpolation was felt obsolete by him as he thought that it did not reach the core of the problems. In his review (1914) of the textbook of Wallenberg & Guldberg on difference equations he expressed it in this way:

"The authors do not possess any general method for solving the equation

$$F(x+1)-F(x)=f(x),$$

where $f(x)$ is a given function, and therefore the book must be characterized as a very audacious experiment. Perhaps many will think that it saw the light too early. It was probably not the intention of the authors to write a work of lasting influence, but rather a work which by its imperfection could encourage further investigations. And, if it is understood in this way, then I for my part will give it a welcome. The book treats a theory that is on the point of being born."

The work of Nørlund from the following 10 years was devoted to the creation of this theory.

Nørlund emphasized that although a difference equation in its direct nature seems more elementary than a differential equation, it is really much more difficult owing to the fact that the solution is not uniquely determined; in the equation above the solution $F(x)$ is only determined apart from a periodic function with period 1, and the real problem is to choose what should be called the best solution or "main solution" and to give a characterization of this main solution. Besides the more practical applications he was also fascinated by the fact that using difference equations it is possible to create analytic functions which can not be obtained by more standard operations, including differential equations. An example is the gamma function which by Hölder's theorem does not satisfy any algebraic differential equation.

A simple difference equation like that above has a formal solution

$$F(x) = \int_a^\infty f(z)\,dz - \sum_{s=0}^{\infty} f(x+s).$$

Usually both terms are divergent, but it is often possible to obtain a solution by multiplying $f(z)$ with a proper summability factor, i.e. replace $f(z)$ by $f(z)\cdot e^{-\lambda\cdot\mu(z)}$, where $\mu(z)$ tends to infinity, and then let $\lambda$ tend to 0. If $f(x)$ is analytic, this can give a main solution, characterized by a simple behaviour at infinity and Nørlund designates it as "sum" of $f(x)$ or

$$F(x) = \sum_a^x f(z)\,\Delta z\cdot$$

For more complicated difference equations like

$$\sum_{j=0}^{k} F(x+j)\cdot p_j(x) = f(x),$$

where the $p_j(x)$ are analytic functions, $x=\sigma+it$, it is possible in similar ways, and also otherwise, to find a fundamental system of solutions (like a fundamental system of solutions to a linear differential equation). Important in this connection are factorial series

$$\Omega(x) = \sum_{s=0}^{\infty} \frac{a_s\cdot s!}{x(x+1)\dots(x+s)}.$$

Nørlund lays stress on their importance for difference equations, it is like the importance of power series for differential equations. They can be used for representation of the solutions and they are allowed to occur in the coefficients $p_j(x)$.

A factorial series is convergent in a right halfplane $\sigma>m$, and here $\Omega(x)$ is bounded, but $m$ is not immediately determined by the properties of $\Omega(x)$. Nørlund shows how it is possible to transform the series in such a way that the terms get the form

$$\frac{b_s\cdot s!}{x(x+c)\dots(x+sc)},$$

if then $c$ tends to infinity convergence is obtained in the largest halfplane where $\Omega(x)$ is bounded.

He determines fundamental systems of solutions in right halfplanes and left halfplanes and finds how they interfere, and uses them to determine singularities and regularity domains for $F(x)$ in the complex plane. He also constructs asymptotic series

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for the solutions. All this considered, it is not amazing that the articles of Nørlund are stamped by an impressive abundance of formulas.

Important for the simple difference equations with only two terms are the Bernoulli polynomials, and now Nørlund introduces Bernoulli polynomials (and analogous Euler polynomials) of higher order for difference equations with many terms and he constructs tables for their coefficients.

His name is often met with in the connection "Nørlund summation". That is summability defined by a sequence of positive numbers $p_0, p_1, p_2, \ldots$ where

$$\lim_{n\to\infty} \frac{p_n}{p_0+p_1+\ldots+p_n} = 0,$$

a (divergent) sequence $s_0, s_1, s_2, \ldots$ can then be ascribed the generalized limit

$$\lim_{n\to\infty} \frac{p_0 s_n+p_1 s_{n-1}+\ldots+p_n s_0}{p_0+p_1+\ldots+p_n}$$

if this limit exists. The method comprises the better of the usually used summability methods, e.g. Cesaro summability but not Hölder summability. In a short paper Nørlund provided the important result, that if the method works for two different sequences $p_0, p_1, p_2, \ldots$ and $q_0, q_1, q_2, \ldots$, then the limits must be the same.

Nobody who met Nørlund could doubt that he was a distinguished personality, a conclusion in keeping also with his tall aristocratic stature. He was a man of few words, not immediately obliging to strangers, and even his closer collaborators often felt his taciturnity embarrassing. This coolness could be felt as an aloofness; maybe it was due to some sort of shyness, but on the other hand his words thus gained more importance and one could feel how he exerted himself to find the right solution to problems. Students or employees who came closer to him could rejoice in his warmhearted interest.

Nørlund appreciated tasteful surroundings and had a beautiful home enriched by many books in fine bindings. Some years before his death he disposed of his impressive mathematical library and now it is the principal part of the mathematical library in the new University of Odense.

His very active life ended on 4 July 1981.

## Bibliography

Détermination de l'orbite de $\xi$ ursae majoris = 1523. *Astr. Nachr.*, 170 (1905), 117–132.
Sur les différences réciproques. *C. R. Acad. Sci. Paris*, 147 (1908), 521–524.

Sur la convergence des fractions continues. *C. R. Acad. Sci. Paris,* 147 (1908), 585–587.
Note om opstigende Kædebrøk. *Nyt Tidsskr. Mat. B,* pp. 25–29, (1909).
Untersuchung der persönlichen Gleichung und der Helligkeitsgleichung bei Beobachtungen mit dem Repsoldschen selbstregistrierenden Mikrometer. *Astr. Nachr.,* 183 (1909), 17–44.
Sur les équations aux différences finies. *C. R. Acad. Sci. Paris,* 149 (1909), 841–843.
Bestimmung der Rektaszensionen von 184 Sternen mit dem Passageinstrument der Kopenhagener Universitäts-Sternwarte. *Astr. Nachr.,* 183 (1909), 49–54.
Fractions continues et différences réciproques. *Acta Math.,* 34 (1910), 1–108.
Sur les fractions continues d'interpolation. *Overs. K. danske Vidensk. Selsk. forh.,* pp. 57–68, (1910).
*Bidrag til de lineære differentialligningers Theori.* Thesis. (71 pages.) København (1910).
Halleys Komet. *Fys. Tidsskr.,* 9 (1910), 13–22.
*Astrofysik, Grundrids ved folkelig Universitetsundervisning.* 178 (1910), København.
T. N. Thiele (Obituary). *Fys. Tidsskr.,* 9 (1910), 1–7.
Über lineare Differenzengleichungen. *K. danske Vidensk. Selsk. Skr.,* 6 (1911), 307–326.
Beobachtungen am Meridiankreis der Kopenhagener Universitäts-Sternwarte. *Astr. Nachr.,* 189 (1911), 17–26.
Beobachtungen am Meridiankreis der Kopenhagener Universitäts-Sternwarte. *Astr. Nachr.,* 189 (1911), 277–288.
Untersuchungen über die Eigenbewegungen für 140 Sterne des IV Secchischens Typens. *K. danske Vidensk. Selsk. Skr.,* 6 (1912), 329–371.
Sur les équations aux différences linéaires à coefficients constants. *Nyt Tidsskr. Mat. B,* pp. 53–65, (1912).
Sur les équations linéaires aux différences finies. *C. R. Acad. Sci. Paris,* 155 (1912), 1485–1487.
Sur les équations linéaires aux différences finies. *C. R. Acad. Sci. Paris,* 156 (1913), 51–54.
Sur le problème de Riemann dans la théorie des équations aux différences finies. *C. R. Acad. Sci. Paris,* 156 (1913), 200–203.
Sur l'intégration des équations linéaires aux différences finies par des séries de facultés. *Rend. Circ. Mat. Palermo,* 35 (1913), 177–216.
Sur une classe d'integrales définies. *J. Math. Pures Appl.,* 9 (1913), 77–88.
Sur une classe de fonctions hypergéométriques. *Overs. K. danske Vidensk. Selsk. Forh.,* pp. 135–153, (1913).
Om Fakultetrækker og deres Anvendelser. *Den tredie skandinaviske Matematikerkongres i Kristiania 1913,* pp. 77–82.
Sur les séries de facultés. *Acta Math.,* 37 (1914), 327–387.
Wallenberg & Guldberg, Theorie der linearen Differenzengleichungen (review). *Bull. Sci. Math.,* 38 (1914), 23–24. (Also in *Nyt Tidsskr. Mat. B,* pp. 84–87, (1914).)
Sur l'existence de solutions d'une équation linéaire aux différences finies. *Ann. Sci. Ecole Norm. Sup.,* 31 (1914), 205–221.
Sur les séries de facultés. *C. R. Acad. Sci. Paris,* 158 (1914), 1252–1253.
Sur les séries de facultés et les méthodes de sommation de Cesaro et de M. E. Borel. *C. R. Acad. Sci. Paris,* 158 (1914), 1325–1327.
Sur les équations linéaires aux différences finies à coefficients rationels. *Acta Math,* 40 (1915), 191–249.
Notes. *Œuvres de Henri Poincaré,* vol. 2, pp. 619–629. Paris, Gauthier-Villars, 1916.
P. J. Myrberg, Zur Theorie der Konvergenz der Poincaréschen Reihen (review). *Bull. Sci. Math,* 42 (1918), 9–11.
Tycho Brahe, Opera omnia I–III (review). *Bull. Sci. Math.,* 42 (1918), 201–209.
S. Stadler, Sur les systèmes d'équations aux différences finies linéaires et homogènes (review). *Bull. Sci. Math.,* 42 (1918), 223–224.

Sur le calcul aux différences finies. *Acta Univ. Lund.* (2), 14:15 (1918).
H. Bohr, Mollerup & Andersen, Nyere Undersøgelser over Integralregning (review). *Nyt Tidsskr. Mat. A*, pp. 64–68, (1918).
Der Mond. In *Vorträge für Kriegsgefangene herausgegeben von Dänischen Roten Kreuz, ser. A*, 10 (1918), 1–14.
De Bernoulli'ske Polynomier. *Mat. Tidsskr. B*, pp. 33–41, (1919).
De Euler'ske Polynomier. *Mat. Tidsskr. B*, pp. 49–55, (1919).
Sur les polynomes d'Euler. *C. R. Acad. Sci. Paris*, 169 (1919), 166–168.
Sur les polynomes d'Euler. *C. R. Acad. Sci. Paris*, 169 (1919), 221–223.
Sur les équations aux différences finies. *C. R. Acad. Sci. Paris*, 169 (1919), 372–375.
Sur la solution principale d'une certaine équation aux différences finies. *C. R. Acad. Sci. Paris*, 169 (1919), 462–465.
Sur les polynomes de Bernoulli. *C. R. Acad. Sci. Paris*, 169 (1919), 521–524.
Sur une extension des polynomes de Bernoulli. *C. R. Acad. Sci. Paris*, 169 (1919), 608–610.
Sur le calcul aux différences finies. *C. R. Acad. Sci. Paris*, 169 (1919), 770–773.
Sur le calcul aux différences finies. *C. R. Acad. Sci. Paris*, 169 (1919), 894–896.
Sur une application des fonctions permutables. *Acta Univ. Lund.* (2), 16:8 (1919).
Mémoire sur les polynomes de Bernoulli. *Acta Math.*, 43 (1920), 121–196.
Sur la convergence de certaines séries. *C. R. Acad. Sci. Paris*, 170 (1920), 506–509.
Sur un théorème de Cauchy. *C. R. Acad. Sci. Paris*, 170 (1920), 715–718.
Sur l'état actuel de la théorie des équations aux différences finies. *Bull. Sci. Math.*, 4 (1920), 174–192; 200–220.
Sur les équations aux différences finies. In *Comptes rendus du Congrès international des mathématiciens, Strasbourg*, pp. 98–119, (1920).
Sur les équations aux différences finies. *Enseign. Math.*, 21 (1920), 202–203.
Sur la formule d'interpolation de Stirling. *C. R. Acad. Sci. Paris*, 174 (1922), 919–921.
Sur la formule d'interpolation de Newton. *C. R. Acad. Sci. Paris*, 174 (1922), 1108–1110.
Sur les formules d'interpolation de Stirling et de Newton. *Ann. Sci. École Norm., Sup.*, 39 (1922), 343–403.
Nogle Bemærkninger angaaende Interpolation med ækvidistante Argumenter. *K. danske Vidensk. Selsk. mat. fys. Medd.*, 4 (1922), 1–34.
Mémoire sur le calcul aux différences finies. *Acta Math.*, 44 (1922), 71–212.
Sur les formules d'interpolation de Stirling et de Newton. *Ann. Sci. Ecole Norm. Sup.*, 40 (1923), 35–54.
Neuere Untersuchungen über Differenzengleichungen. *Enzykl. math. Wiss.*, C7 (1923), 675–721.
Summen af en given Funktion. *Mat. Tidsskr. B*, pp. 65–68, (1923).
Sur certaines équations aux différences finies. *Trans. Amer. Math. Soc.*, 25 (1923), 13–98.
On certain difference equations. *Bull. Amer. Math. Soc.*, 29 (1923), 212.
Remarques diverses sur le calcul aux différences finies. *J. Math. Pures Appl.*, 2 (1923), 193–214.
Extrait d'un mémoire inédit d'Henri Poincaré sur les fonctions fuchsiennes. *Acta Math.*, 39 (1923), 58–93.
*Videnskabelige causerier*. (125 pages.) København, 1923.
*Bland siffror och tal* [Swedish translation of the preceding]. (113 pages.) Stockholm, 1923.
Sunto dei suoi lavori sul calcolo delle differenze finite. *Boll. Un. Mat. Ital.*, 2 (1923), 182–186.
Sunto dei suoi lavori sul calcolo delle differenze finite. *Boll. Un. Mat. Ital.*, 3 (1924), 28–31.
*Vorlesungen über Differenzenrechnung*. (551 pages.) Berlin, Springer-Verlag, 1924.
Sur l'interpolation. *Bull. Soc. Math. France*, 52 (1924), 114–132.
Stirlings Interpolationsrække. *K. danske Vidensk. Selsk. Skr.*, 7:2 (1924), 212–280.
Indledningstale ved Aabningsmødet. In *Beretning om VI$^{e}$ skandinaviske Matematikerkongres i København 1925*, pp. 15–25.

Om Interpolationsrækker. *Beretning om VI^e skandinaviske Matematikerkongres i København 1925*, p. 251.

Stirlings Interpolationsrække. *Mat. Tidsskr B*, pp. 20–24, (1925).

*Leçons sur les séries d'interpolation.* (236 pages.) Paris, Gauthier-Villars, 1926.

J. L. W. V. Jensen (Obituary), *Overs. K. danske Vidensk. Selsk. Forh.*, pp. 43–51, (1926).

J. L. W. V. Jensen (Obituary). *Mat. Tidsskr. B*, pp. 1–7, (1926).

Sur la 'somme' d'une fonction. *Méml. Sci. math.*, 24 (1927), 1–54.

G. Mittag-Leffler (Obituary). *Acta Math.*, 50 (1927), I–XXIII.

J. L. E. Dreyer (Obituary). *Overs. K. danske Vidensk. Selsk. Forh.*, pp. 51–59, 1927.

*Leçons sur les équations linéaires aux différences finies.* (152 pages.) Paris, Gauthier-Villars, 1929.

Rapport sur les travaux géodésiques exécutés de 1924 à 1930. In *Meddr Danm. geod. Inst., Copenhague 1930.*

Rapport sur les travaux géodésiques exécutés de 1930 à 1933. In *Meddr Danm. geod. Inst., Copenhague 1933.*

Observations de l'intensité de la pesanteur avec le nouveau modèle de pendule Holweck–Lejay. *Mémoires de l'Inst. Géod. de Danemark*, 1 (1934), 1–163.

Untersuchungen über die Genauigkeit relativer Schweresmessungen mit dem Holweck–Lejay Pendel. In *Verhandl. Tagung Baltischen geod. Kommission.* Helsinki, 1935.

Über die Wahl von Sternen bei Zeit- und Längenbestimmungen. In *Verhandl. Tagung Baltischen geod. Kommission.* Helsinki, 1935.

Some methods and procedure developed during recent expeditionary surveys in Greenland, *Geogrl J.*, 86 (1935), 317–329.

Kongen og den videnskabelige Forskning. *Christian den Tiende Konge af Danmark og Island, Aarene 1912–37.* København, pp. 196–213, (1937).

Om Længdebestemmelser. *Mat. Tidsskr. B*, pp. 72–74, (1937).

Presidential Address. In *Third general Assembly of International Council of Scientific Unions, Cambridge*, 1937.

The Beacon lamp of the Danish Geodetic Institute. *Meddr Danm. geod. Inst.*, 8 (1937), 1–12.

Rapport général du Danemark. In *5ième congrès int. de photogrammétrie à Rome.* Copenhague, 1938.

J. P. Koch (Obituary). *Geogr. Tidsskr.*, 42 (1939), 3–6.

*Vermessungsarbeiten in Grönland, Island und Dänemark.* Darmstadt, 1939.

Kortlægningen af Grønland og Island. *Geogr. Tidsskr.*, 42 (1939), 21–30.

The survey work of the Danish Geodetic Institute in Greenland and Iceland. *Polar Rec.*, 17 (1939), 38–45.

Rapport sur les travaux géodésiques exécutes de 1933 à 1939. In *Meddr Danm. geod. Inst., Copenhague 1939.*

Ausgleichung nach der Methode der kleinsten Quadrate bei gruppenweiser Anordnung der Beobachtungen. *Acta Math.*, 72 (1940), 283–353.

*Johannes Mejers Kort over det danske Rige I–III.* (294 plates.) København, Munksgaard, 1942.

*Danmarks Kortlægning I.* (77 pages, 105 plates.) København, Munksgaard, 1942.

*Bestemmelse af Vægtene paa de Ubekendte ved Elementudjœvning.* København, Munksgaard, 1943.

De gamle danske Længdeenheder. *Mémoires de l'Inst. Géod. de Danemark*, 3 (1944), 1–80.

*Islands Kortlægning.* (110 pages, 170 plates.) København, Munksgaard, 1944.

*Færøernes Kortlægning.* (74 pages, 90 plates.) København, Munksgaard, 1944.

Hydrostatisk Nivellement over Store Bælt. *Mémoires de l'Inst. Géod. de Danemark*, 6 (1945), 1–122.

Hydrostatisk Nivellement over Øresund. *Mémoires de l'Inst. Géod. de Danemark,* 7 (1946), 1–84.
Hypergeometriske Funktioner. *Mat. Tidsskr. B,* pp. 18–21, (1950).
Harald Bohr (Obituary). *Overs K. danske Vidensk. Selsk. Forh.,* pp. 61–67, (1951).
Séries hypergéométriques. *K. fysiogr. Sällsk. Lund Förh.,* 21: 15 (1952).
F. Lösch & Schoblik, die Fakultät (Gammafunktion) und verwandte Funktionen (review). *Mat. Tidsskr. B,* p. 105, (1952).
Opmaalingen af Danmarks Veje. *Meddr Danm. geod. Inst.,* 26 (1953), 1–14 (also in *Geografisk Tidsskr.,* 52 (1953), 232–241.
Théorie des fonctions—sur les fonctions hypergéométriques. *C. R. Acad. Sci. Paris,* 237 (1953), 1371–1373; 1466–1468.
Report on the geodetic works executed in the period 1939–1953. In *Meddr Danm. Geod. Inst., Copenhagen 1954.*
Über hypergeometrische Funktionen. *Arch. Math.,* 5 (1954), 258–266.
Hypergeometric functions. *Acta Math.,* 94 (1955), 289–349.
Tanker om tre Videnskaber. *Nord. Mat. Tidsskr.,* pp. 1–19, (1956).
Sur les fonctions hypergéométriques d'ordre supérieur. *Mat. fys. Skr. K. danske Vidensk. Selsk.,* 1: 2 (1956), 1–47.
Sur les valeurs asymptotiques des nombres et des polynomes de Bernoulli. *C. R. Acad. Sci. Paris,* 251 (1960), 2269–2271.
Sur la convergence de certaines séries de facultés. *Atti Accad. Naz. Lincei,* 28 (1960), 532–538.
Sur les valeurs asymptotiques des nombres et des polynomes de Bernoulli. *Rend. Circ. Mat. Palermo,* 10 (1961), 1–18.
The logarithmic solutions of the hypergeometric equation. *Mat. fys. Skr. K. danske Vidensk. Selsk.,* 5 (1963), 1–58.