Perspectives in Mathematical Logic

Model-Theoretic Logics

Edited by J. Barwise S. Feferman

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Admissible Sets and Structures
An Approach to Definability Theory
by *J. Barwise*
1975. xiv, 394 pages.

Recursion-Theoretic Hierarchies
by *P.G. Hinman*
1978. xii, 480 pages.

Basic Set Theory
by *A. Levy*
1979, xiv, 391 pages.

Degrees of Unsolvability
Local and Global Theory
by *M. Lerman*
1983. xiii, 307 pages.

Constructibility
by *Keith J. Devlin*
1984. xi, 425 pages.

Model-Theoretic Logics
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Preface to the Series

Perspectives in Mathematical Logic

(Edited by the Ω-group for "Mathematische Logik" of the Heidelberger Akademie der Wissenschaften)

On Perspectives. *Mathematical logic arose from a concern with the nature and the limits of rational or mathematical thought, and from a desire to systematize the modes of its expression. The pioneering investigations were diverse and largely autonomous. As time passed, and more particularly in the last two decades, interconnections between different lines of research and links with other branches of mathematics proliferated. The subject is now both rich and varied. It is the aim of the series to provide, as it were, maps or guides to this complex terrain. We shall not aim at encyclopaedic coverage; nor do we wish to prescribe, like Euclid, a definitive version of the elements of the subject. We are not committed to any particular philosophical programme. Nevertheless we have tried by critical discussion to ensure that each book represents a coherent line of thought, and that, by developing certain* themes, *it will be of greater interest than a mere assemblage of results and techniques.*

The books in the series differ in level: some are introductory, some highly specialized. They also differ in scope: some offer a wide view of an area, others present a single line of thought. Each book is, at its own level, reasonably self-contained. Although no book depends on another as prerequisite, we have encouraged authors to fit their book in with other planned volumes, sometimes deliberately seeking coverage of the same material from different points of view. We have tried to attain a reasonable degree of uniformity of notation and arrangement. However, the books in the series are written by individual authors, not by the group. Plans for books are discussed and argued about at length. Later, encouragement is given and revisions suggested. But it is the authors who do the work; if, as we hope, the series proves of value, the credit will be theirs.

History of the Ω-Group. *During 1968 the idea of an integrated series of monographs on mathematical logic was first mooted. Various discussions led to a meeting at Oberwolfach in the spring of 1969. Here the founding members of the group (R. O. Gandy, A. Levy, G. H. Müller, G. E. Sacks, D. S. Scott) discussed the project in earnest and decided to go ahead with it. Professor F. K. Schmidt and Professor Hans Hermes gave us encouragement and support. Later Hans Hermes joined the group. To begin with all was fluid. How ambitious should we be? Should we write the books ourselves? How long would it take? Plans for authorless books were promoted, savaged and scrapped. Gradually there emerged a form and a method. At the end of an infinite discussion we found our name, and that of the series. We established our centre* *in Heidelberg. We agreed to meet twice a year together with authors, consultants and assistants, generally in Oberwolfach. We soon found the value of collaboration: on the one hand the permanence of the founding group gave coherence to the over-all plans; on the other hand the stimulus of new contributors kept the project alive and flexible. Above all, we found how intensive discussion could modify the authors' ideas and our own. Often the battle ended with a detailed plan for a better book which the author was keen to write and which would indeed contribute a perspective.*

Oberwolfach, September 1975

Acknowledgements. *In starting our enterprise we essentially were relying on the personal confidence and understanding of Professor Martin Barner of the Mathematisches Forschungsinstitut Oberwolfach, Dr. Klaus Peters of Springer-Verlag and Dipl.-Ing. Penschuck of the Stiftung Volkswagenwerk. Through the Stiftung Volkswagenwerk we received a generous grant (1970–1973) as an initial help which made our existence as a working group possible.*

Since 1974 the Heidelberger Akademie der Wissenschaften (Mathematisch-Naturwissenschaftliche Klasse) has incorporated our enterprise into its general scientific program. The initiative for this step was taken by the late Professor F. K. Schmidt and the former President of the Academy, Professor W. Doerr.

Through all the years, the Academy has supported our research project, especially our meetings and the continuous work on the Logic Bibliography, in an outstandingly generous way. We could always rely on their readiness to provide help wherever it was needed.

Assistance in many various respects was provided by Drs. U. Felgner and K. Gloede (till 1975) and Drs. D. Schmidt and H. Zeitler (till 1979). Last but not least, our indefatigable secretary Elfriede Ihrig was and is essential in running our enterprise.

We thank all those concerned.

Heidelberg, September 1982

R. O. Gandy *H. Hermes*
A. Levy *G. H. Müller*
G. E. Sacks *D. S. Scott*

Preface

The subject matter of this book constitutes a merging of several directions of work in general model theory over the last 25 years. Three main lines can be distinguished: first, that initiated by Andrzej Mostowski on *cardinality quantifiers* in the late 1950s; second, the work of Alfred Tarski, his colleagues and students on *infinitary languages* into the mid-1960s; and, finally, that stemming from the results of Per Lindström on *generalized quantifiers* and *abstract characterizations of first-order logic* in the late 1960s. The subject of *abstract model theory* blossomed from that as a unified and illuminating framework in which to organize, compare, and seek out the properties of the many stronger logics which had then come to be recognized.

Interest in abstract model theory and extended logics was intense in the early 1970s, particularly as a result of the work of Jon Barwise on *infinitary admissible languages*. Where the previous developments had largely connected up model theory with set theory, this added ideas from extended recursion theory in an essential way, e.g., to yield successful infinitary generalizations of the compactness theorem. It also turned out that proof theory—including such consequences as the interpolation and definability theorems—could be successfully generalized to these languages. Thus, one was here witness to an exciting confluence of all the main branches of mathematical logic. These successes led to promising research programs for further interactive development, but the hopes they raised, especially with respect to the treatment of uncountable languages, were not realized. A number of us (including Barwise and myself) who had been involved at that stage of the subject turned to other interests in the latter part of the 1970s and gave little attention to its ongoing progress. As it happens, all through that period (at least) the set-theoretic and model-theoretic aspects of the subject were continuing to develop at a rapid rate. Looking again at the field in 1980 we found a body of work that was quite staggering.

The re-examination of the area at that time had come in response to repeated urging by the editors of the Ω-group (particularly by Gert Müller) for Barwise and/or me to write a volume for the series on the general subject of model-theoretic logics. This might have been conceivable in the early 1970s, but owing to the intervening growth in the field, it was clearly beyond us eight years later. On the one hand, the field seemed to be in such a state of advanced and intense development that the idea of writing a relatively finished text did not seem appropriate—

even if undertaken by someone working directly in the subject. On the other hand, it became more and more apparent that unless some effort were made to provide an exposition of the field as currently understood, many potential researchers would simply be left behind.

An alternative idea then presented itself: namely, to knit together a number of individual contributions which would provide substantial coverage of the field and would constitute an introduction to the main ideas, examples, and results of the literature. Barwise and I made this proposal with some trepidation at the meeting of the Ω-group in Patras in August, 1980. We wanted it clearly understood that we were not suggesting writing a "handbook," each part of which would be reporting on a relatively finished or settled topic. Rather, we wanted to present a picture of a rapidly evolving subject, in which much that has been accomplished so far must be digested if one is to contribute to further progress. The aim of the project would be to give an entry into the field for anyone sufficiently equipped in general model theory and set theory, and thereby to bring them closer to the frontiers of research. This proposal, together with our preliminary table of contents and suggested list of individual authors of chapters, was agreed to enthusiastically by the editors of the Ω-group. As it happened, a number of those we had suggested as prospective authors were attending the Patras Logic Symposium. We quickly gathered from them enough expressions of willingness to participate so that the viability of the project could be assured.

Thanks to the support of the Ω-Group and the financial generosity of the Heidelberger Akademie der Wissenschaften, the organization of work on this book was able to proceed in a unique cooperative way. Almost all the authors and editors met together as a group on two occasions, first in Freiburg during the period 21–27 June 1981 and then at Stanford during the period 28 March–4 April 1982. At the first of these meetings, authors brought plans, outlines and, in many instances, first drafts of their chapters. These materials were explained, discussed, and circulated. In effect, this constituted a very high-level interchange on matters of substance, style, approach, and exposition. (It emerged that three additional chapters were needed to round out the coverage for which, fortunately, authors could be secured.) The participants found the Freiburg meeting extremely exciting and stimulating, and left with high confidence in the success of the project. Afterward, preparation continued at a quicker pace than originally expected. At the second meeting in Stanford, authors brought semi-final drafts of their chapters, and continued the process established in Freiburg. Then each chapter was circulated in the summer of 1982 to two other authors and/or editors for reading, detailed comments and suggestions. On the basis of these comments, chapters were brought to final form and submitted to the editors by early 1983. Soon after, a small working editorial group (meeting at Stanford) organized the manuscript in final form, touching up and smoothing out the chapters, preparing explanatory introductions on the various parts and completing the work on a unified bibliography. In doing so, no effort was made to impose uniformity of style or thought. The aim was to bring out the individual contributions in the best and most understandable and useful form possible.

The book is divided into six parts (A–F), each consisting of two to four chapters. Part A provides an introduction to the subject as a whole as well as to the basic theory and examples. In particular, Chapter I, by Barwise, presents a general discussion of the background and aims of our subject. Each part is preceded by a detailed introduction summarizing its contents. From that material the reader will learn which chapters can be read for general purposes and which for more special research interests, together with the background required in each case. It will be seen that many of the chapters can be read independently and that moving back and forth between them can be rewarding. Parts B–F of the book take up, in turn: finitary languages with additional quantifiers, infinitary languages, second-order logic, logics of topology and analysis, and advanced topics in abstract model theory.

An explanation is needed for the form of the bibliography for this book. As the chapters were being written, it soon became apparent that the individual bibliographies would be a valuable source of references and history taken in combination. Scott volunteered to oversee the collection of the materials to make a single, unified listing. What has been incorporated are all the contributions from the authors (with many additions and corrections which they sent in), an early bibliography started by Barwise, and many selections from the Omega Logic Bibliography at Heidelberg. At the start no one had any idea that the listing would have 1,261 items—or how vexing it would be to run down certain items. Fortunately, after coming to Carnegie-Mellon University in 1981, Scott was able to arrange that the bibliography be put on the computer, which was also used to make camera-ready copy; otherwise, without computer aids, a task of this size would have been virtually impossible. In the event, this project turned out to be more labor intensive than had been anticipated; it could not have been carried out without the full collaboration of Charles McCarty and John Horty, who spent many hours over many months checking sources, working in the library, typing into terminals, and proofreading many versions. The compilers were also aided by many other people at CMU: W. L. Scherlis and Roberto Minio gave us constant help with the TEX type-setting system, and Todd Knoblock, Lars W. Ericson, and John Aronis at various times served as programmers on the project; without their expertise and help on many small problems, nothing could have been done. Marko Petkovsek also very ably assisted with the final proofreading. The editors would thus like to take the opportunity to convey their warm thanks to all these people for their efforts, to the Computer-Science Department of Carnegie-Mellon University for the support of McCarty and Horty and the programmers and particularly for the generous use of their facilities, and to the authors, who helped assemble and check details of the bibliography. Alas, as it stands the bibliography is not complete historically, but, even so, the editors and compliers hope it will materially aid future students and researchers in learning about this work and continuing the investigations.

In addition to a unified bibliography, the idea of having a unified open problem section had also been given serious consideration. However, it was finally decided that such problems are best appreciated in the specific contexts in which they arise, and that no general rule need be followed as to their location in the individual chapters.

We also wish to express our thanks to the students Ian Mason and Sergio Fajardo, who read and made useful comments on various chapters; to Priscilla Feigen and Isolde Field, for their great assistance in many ways during and after the meetings at Stanford; likewise to Elfriede Ihrig for assistance at Freiburg and Heidelberg; to the secretarial staffs of the many institutions represented by the editors and authors of this book who helped in its preparation; to the University of Freiburg and Stanford University for providing us with facilities for our meetings; to the publisher, Springer-Verlag, and particularly the assistance of its editors; and finally (once more) to the Heidelberger Akademie der Wissenschaften, without whose support nothing like the present volume would have been possible.

Stanford, 1 *March* 1983 *Solomon Feferman*

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