NSF-CBMS Regional Conference Series in Probability and Statistics Volume 3

### STOCHASTIC CURVE ESTIMATION

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#### Conference Board of the Mathematical Sciences

#### Regional Conference Series in Probability and Statistics

## Supported by the National Science Foundation

The production of the NSF-CBMS Regional Conference Series in Probability and Statistics is managed by the Institute of Mathematical Statistics: Paul Shaman, IMS Managing Editor; Jessica Utts, IMS Treasurer; and Jose L. Gonzalez, IMS Business Manager.

Library of Congress Catalog Card Number: 91-71236

International Standard Book Number 0-940600-22-6

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Printed in the United States of America

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### Preface

These notes are based on a regional set of lectures on curve estimation in the context of independent and dependent observations given at the University of California, Davis during June 1989. Much of these lectures is concerned with probability density or regression function estimation when observations are independent. The character of the asymptotic results (at least locally) is qualitatively the same if the observations are those of a dependent stationary sequence with short-range dependence. Research in the case of long-range dependence is recent and qualitatively of a different character. One should note that the spectral estimation (also discussed here) is of an older vintage and results there in part motivated later research on probability density estimates. Many of the results have the imprint both of stochastic methods as well as those of approximation theory.

The conference was organized by Professor George Roussas with the assistance of Professor Y. P. Mack. I should like to thank them both for their hospitality. The conference was sponsored by the Conference Board of the Mathematical Sciences and the University of California, Davis with the support of the National Science Foundation. I am indebted to Neola Crimmins who was so helpful in retyping these notes.

Murray Rosenblatt

La Jolla, April 1990