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ON THE STRUCTURE OF NILPOTENT GROUPS OF A CERTAIN TYPE

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Dedicated to the memory of Karol Borsuk

1. Introduction

Let $\mathcal{N}$ be the class of nilpotent groups. Then, given any group N in $\mathcal{N}$ and any family of primes P we may construct the P-localization N_p of N (see [4]). Thus N_p is *P-local*, meaning that it admits unique q^{th} roots for q outside P, and there is a homomorphism $e : N \to N_p$ which is universal for homomorphisms from N into P-local nilpotent groups.

Now let $N \in \mathcal{N}$ be finitely generated (fg). Mislin [5] defined the *genus* of N to be the set $\mathcal{G}(N)$ of isomorphism classes of fg nilpotent groups M such that $M_p \cong N_p$, for all primes p. He showed that the genus is, in general, non-trivial, but gave no means of calculating it in this generality. He also demonstrated its relevance for the discussion of genus in the collection of homotopy types of nilpotent polyhedra of finite type (see [4]).

Let $\mathcal{N}_0$ be the class of finitely generated, but not finite, nilpotent groups with finite commutator subgroup $[N, N]$. Then for any N in $\mathcal{N}_0$ the genus $CalG(N)$ (see [5, 3]) has the structure of a finite abelian group. This *genus-group* was calculated in [1] in the case that N belongs to a certain subclass $\mathcal{N}_1$ of $\mathcal{N}_)$.