

## REMARKS ON MULTIPLE SOLUTIONS FOR ASYMPTOTICALLY LINEAR ELLIPTIC BOUNDARY VALUE PROBLEMS<sup>1</sup>

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*Dedicated to Jean Leray*

### 1. Introduction

It is known that the critical groups are useful in distinguishing critical points (cf. [5]). We shall present here a few examples from semilinear elliptic boundary value problems showing how they work in the study of multiple solutions. Let us consider the problem

$$(1.1) \quad \begin{cases} -\Delta u = g(x, u), \\ u|_{\partial\Omega} = 0, \end{cases}$$

where  $\Omega$  is a smooth bounded domain in  $\mathbb{R}^n$ . Let  $\lambda_j$  be the  $j$ -th eigenvalue of  $-\Delta$  with zero Dirichlet boundary data. We assume:

$$(g_1) \quad g \in C^1(\overline{\Omega} \times \mathbb{R}^1, \mathbb{R}^1), \quad g(x, 0) = 0,$$

$$(g_2) \quad g'(x, 0) < \lambda_1 \quad \forall x \in \overline{\Omega}$$

$$(g_3) \quad \lim_{|t| \rightarrow \infty} g(x, t)/t \triangleq g_\infty > \lambda_2.$$

$g_\infty$  satisfies one of the following three conditions:

$$(i) \quad g_\infty \notin \sigma(-\Delta), \text{ the spectrum of } -\Delta.$$

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