POSITIVE SOLUTIONS OF NONLINEAR ELLIPTIC PROBLEMS APPROXIMATING DEGENERATE EQUATIONS

Monica Musso — Donato Passaseo

Dedicated to Louis Nirenberg on the occasion of his 70th birthday

1. Introduction

In recent years there has been an increasing interest in positive solutions of some nonlinear elliptic problems, where some concentration phenomena enable one to relate the number of positive solutions to the geometrical properties of the domain.

Phenomena of this type occur, for example, in some nonlinear problems involving critical or supercritical Sobolev exponents like the following:

$$(1.1) \qquad \begin{cases} \Delta u + u^{p-1} = 0 & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where Ω is a smooth domain in $\mathbb{R}^N$, $N \geq 3$, and $p \geq 2N/(N-2)$ (the critical Sobolev exponent for the embedding $H_0^{1,2}(\Omega) \hookrightarrow L^p(\Omega)$).

Many papers have been devoted to such problems (see [2], [5], [6], [10]–[12], [14], [15], [18]–[23], [25], [27], [28], and the references therein).

Here the lack of compactness, due to the presence of the critical exponent, is just associated with concentration phenomena and, when it is possible to overcome the difficulties due to the lack of compactness, one can often obtain multiplicity results for positive solutions.