

MULTIPLICITY OF NODAL SOLUTIONS FOR ELLIPTIC EQUATIONS WITH SUPERCRITICAL EXPONENT IN CONTRACTIBLE DOMAINS

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Introduction

This paper is concerned with existence and multiplicity of solutions for the problem

$$P(\Omega, p) \quad \begin{cases} \Delta u + |u|^{p-2}u = 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \\ u^+ \not\equiv 0, \quad u^- \not\equiv 0 & \text{in } \Omega, \end{cases}$$

where Ω is a bounded domain in $\mathbb{R}^n$, with $n \geq 3$ and $p > 2^* = 2n/(n-2)$ (2^* is the critical exponent for the Sobolev embedding $H_0^{1,2}(\Omega) \hookrightarrow L^p(\Omega)$). Following the notation introduced in [3], we shall refer to the solutions of $P(\Omega, p)$ as *nodal solutions*.

It is well known that the main difficulty in problems of this type is related to the lack of compactness due to the presence of the exponent $p > 2n/(n-2)$. Therefore the classical topological methods of calculus of variations cannot be applied in a straightforward way. When $2 < p < 2n/(n-2)$, the existence of positive and nodal solutions for the problem

$$(*) \quad \begin{cases} \Delta u + |u|^{p-2}u = 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \\ u \not\equiv 0 & \text{in } \Omega, \end{cases}$$

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