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EXISTENCE PRINCIPLES FOR CARATHÉODORY DIFFERENTIAL EQUATIONS IN BANACH SPACES

MARLÈNE FRIGON[1] — JOHN W. LEE[2]

(Submitted by A. Granas)

Dedicated to the memory of Karol Borsuk

1. Introduction and Preliminaries

In [4], the topological transversality theorem of A. Granas [3] was used to establish existence principles for systems of differential equations in $\mathbb{R}^n$. The results developed in [4] extend rather easily to the case of Banach space-valued solutions. For initial results along these lines see [6]. The present paper extends the general existence results in the papers above in two directions. First, we do not ask that the right member of the differential equation be completely continuous. Instead, we introduce a new property, called K-Carathéodory, that is automatically satisfied when the right member is Carathéodory or continuous and the Banach space is finite dimensional and implies condition $(*)$ in [6] in the infinite dimensional case. With this new property, the basic existence principles have the same formulation in a Banach space as in $\mathbb{R}^n$. Second, we enlarge substantially the class of admissible boundary conditions considered in [4] and [6]. In effect, the results formulated below allow any linear boundary forms which together with the differential operator in question determine an invertible operator. In the papers above, the boundary conditions were required to

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