Topological Methods in Nonlinear Analysis
Journal of the Juliusz Schauder Center
Volume 2, 1993, 55–74

A CONTINUATION APPROACH TO FOURTH ORDER SUPERLINEAR PERIODIC BOUNDARY VALUE PROBLEMS

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Dedicated to the memory of Juliusz Schauder

Consider the fourth order boundary value problem

$$(\mathrm{P}) \qquad \begin{cases} u'''' = g(u) + e(t), \\ u(\cdot) \quad T\text{ - periodic,} \end{cases}$$

where $g : \mathbb{R} \to \mathbb{R}$ is a continuous function satisfying

$$(\mathrm{i}) \qquad \lim_{|x| \to +\infty} g(x)\mathrm{sign}(x) = +\infty$$

and $e : \mathbb{R} \to \mathbb{R}$ is continuous and T-periodic (more general conditions for the function $e(t)$ will be considered in Section 5).

Various results for the solvability of problem (P) have been obtained in cases when asymptotically the ratio $g(x)/x$ does not interfere with the eigenvalues of the differential operator u'''' in the space of T-periodic functions. With this respect, we refer to the articles of Omari and Zanolin [10], De Coster, Fabry and Habets [5] and Gupta and Mawhin [7]. The possibility of a function g which grows faster than linear at $+\infty$ (or at $-\infty$) has been considered by Ward [11] and Afuwape, Mawhin and Zanolin [1]. In these latter papers, however, a rather strong restriction for the growth of g at $-\infty$ (respectively at $+\infty$) has to be assumed in order to obtain the a priori bounds for the solutions.

In this work we consider an example that, as far as we know, is new in the study of the periodic problem for higher order ordinary differential equations. Namely,