

SOLUTION SETS OF BOUNDARY VALUE PROBLEMS FOR NONCONVEX DIFFERENTIAL INCLUSIONS

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Dedicated to the memory of Karol Borsuk

1. Introduction and preliminaries

Topological properties of the solution set of Cauchy problems for differential inclusions have been investigated by several authors [16], [24], [14], [23], [10], [19], [3], [15]. Less attention has been, so far, devoted to analogous questions for boundary value problems.

In the present paper we consider boundary value problems of the type

$$(BV) \quad \begin{cases} x''(t) \in F(t, x(t), x'(t)), \\ x(0) = x(1) = 0, \end{cases}$$

where F is a multifunction from $I \times \mathbf{R}^q \times \mathbf{R}^q$, $I = [0, 1]$, to the non-empty compact subsets of \mathbf{R}^q . If F is Lipschitzean, we prove that the solution set S_F of (BV) is a retract of the Sobolev space $W^{2,1}(I, \mathbf{R}^q)$. In particular, S_F is contractible and hence arcwise connected. Whenever F is convex valued and Lipschitzean, S_F is a retract also of $C^1(I, \mathbf{R}^q)$. Finally, in the nonconvex case, under a continuity assumption on F , it is proved that S_F is non-empty.

To establish the retraction property of S_F , when F is Lipschitzean, we use some recent results due to Ricceri [21] and Bressan, Cellina and Fryszkowski [4],