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MULTIPLICITY OF POSITIVE SOLUTIONS FOR THE EQUATION $\Delta u + \lambda u + u^{2^*-1} = 0$ IN NONCONTRACTIBLE DOMAINS

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Dedicated to the memory of Juliusz Schauder

1. Introduction

Let Ω be a smooth bounded domain in $\mathbb{R}^n$ with $n \geq 3$, $2^* = 2n/(n-2)$ the critical exponent for the Sobolev embedding of $H_0^{1,2}(\Omega)$ in $L^p(\Omega)$, and λ a real parameter. In this paper we study the following problem:

$$P_\lambda(\Omega) \qquad \begin{cases} \Delta u + \lambda u + u^{2^*-1} = 0 & \text{in } \Omega, \\ u \in H_0^{1,2}(\Omega),\ u > 0, & \text{in } \Omega. \end{cases}$$

It is easy to verify (see [5]) that Problem $P_\lambda(\Omega)$ has no solution for $\lambda \geq \lambda_1$, where λ_1 is the first eigenvalue of $-\Delta$ in $H_0^{1,2}(\Omega)$.

If $\lambda \leq 0$, the well known Pokhozhaev identity (see [24], [5]) implies that there is no solution of $P_\lambda(\Omega)$ when Ω is starshaped.

In [5] Brézis and Nirenberg proved that, if $n \geq 4$, Problem $P_\lambda(\Omega)$ has a solution for every $\lambda \in]0, \lambda_1[$; the situation is more complex for $n = 3$ (see [5]) and a complete answer has been given only if Ω is a sphere: in this case $P_\lambda(\Omega)$ has a solution if and only if $\lambda \in]\lambda_1/4, \lambda_1[$. In [25] Rey proved that, for $\lambda > 0$ small enough, the number